ALTERING QUANTUM HALL FERROMAGNETISM WITH CAVITY FIELDS

Luis Brey

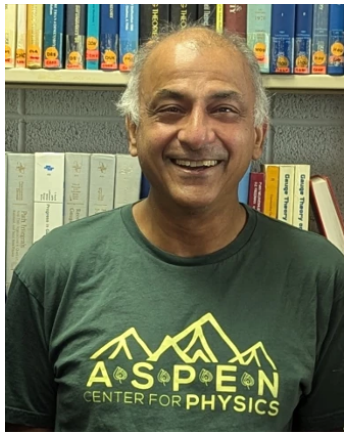
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arxiv.org/2609.14838



Ceren B.Dag



Ganpathy Murthy



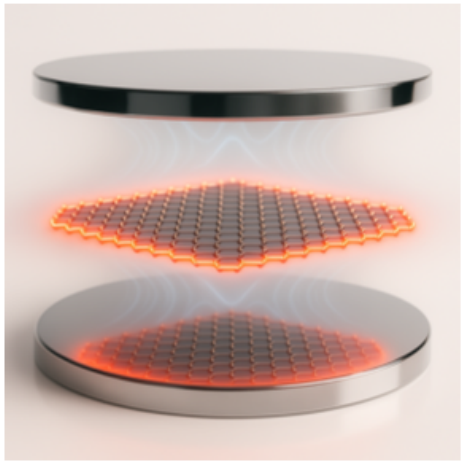
H.A.Fertig

L.B. thanks Ministerio de Ciencia de Innovación de España
Severo Ochoa Centres of Excellence program.

OPTICAL CAVITIES.

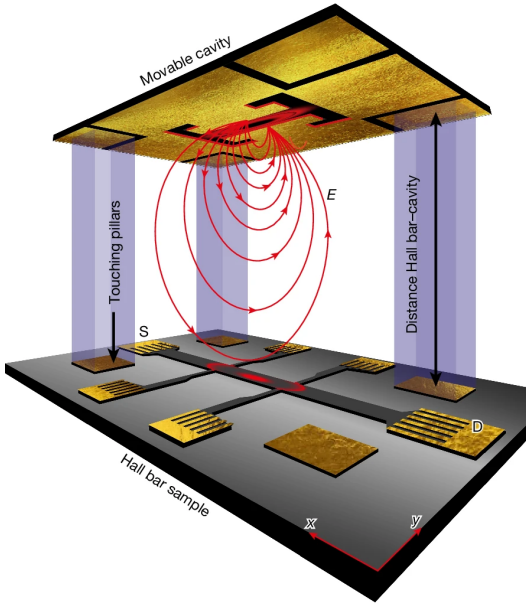
Optical cavities are basically a setup to confine light in some region of space. This quantizes the EM modes that can exist in the cavity. They allow a number of tuning knobs, such as the mode frequency, the profile of the E and B fields of the mode etc.

In optical cavities it is no necessary to send light, the confinement induces zero point electromagnetic fluctuations.



Artist's impression by V. Karle

Parallel mirrors



Split-ring resonator

J.Enkner et al. Nature '25

MOTIVATION.

Optical cavities could be a new way to control electronic materials.

Even in the absence of external light the vacuum electromagnetic field inside the optical cavity can influence the properties of a material.

Coupling could destroy electron collective states or modify phase boundary between different many-body states.

Quantum Hall Systems in an Optical Cavity

QUANTUM OPTICS , *Science* **375**, 1030–1034 (2022)

Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect

Felice Appugliese[1]*, Josefine Enkner[1], Gian Lorenzo Paravicini-Bagliani[1]†, Mattias Beck[1], Christian Reichl[2], Werner Wegscheider[2], Giacomo Scalari[1], Cristiano Ciuti[3], Jérôme Faist[1]*

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Tunable vacuum-field control of fractional and integer quantum Hall phases

https://doi.org/10.1038/s41586-025-08894-3

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Josefine Enkner[1,2]✉, Lorenzo Graziotto[1,2], Dalin Boriçi[3], Felice Appugliese[1,2], Christian Reichl[2,4], Giacomo Scalari[1,2], Nicolas Regnault[5,6,7], Werner Wegscheider[2,4], Cristiano Ciuti[3] & Jérôme Faist[1,2]✉

- Reduction of the spin splitting gap at odd integer filling factor.
- Gaps at even integer filling factor are practically unaffected.
- Enhancement of fractional quantum Hall gaps.

Cavity vacuum fields can strongly influence many-body electronic states.

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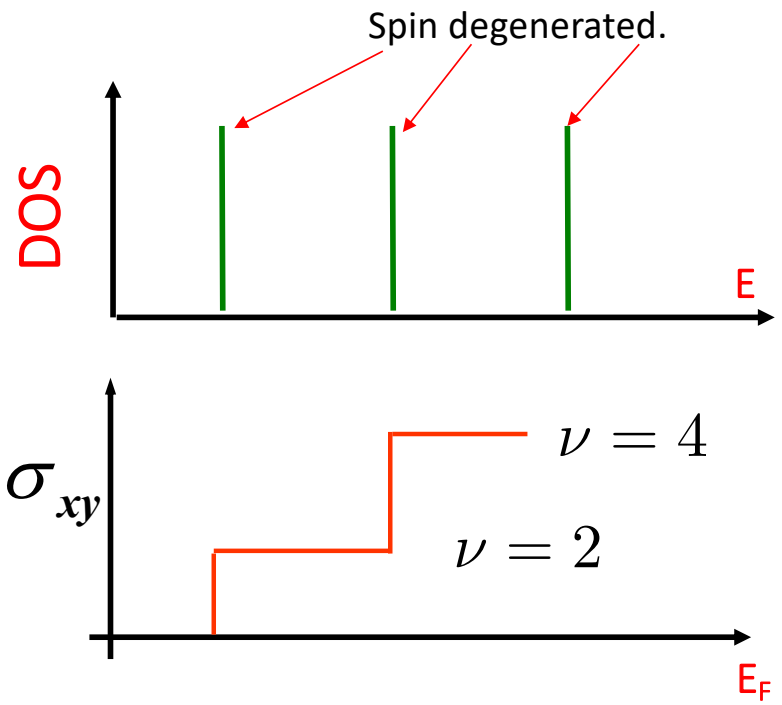
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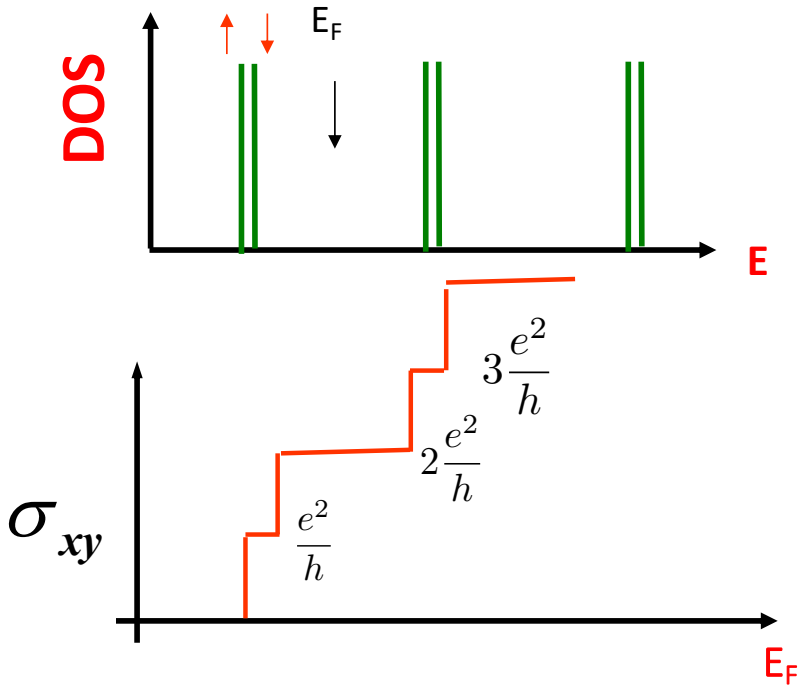
QUANTUM HALL FERROMAGNETISM.

2DEG + Magnetic field $E = (n + \frac{1}{2})\hbar\omega_c$

$$\sigma_{xy} = \nu \frac{e^2}{h}$$



e-e spontaneously breaks SU(2) spin symmetry, spin polarized an open an exchange gap.
Odd integer quantum Hall effect.
QUANTUM HALL FERROMAGNET, GS at $\nu = 1$



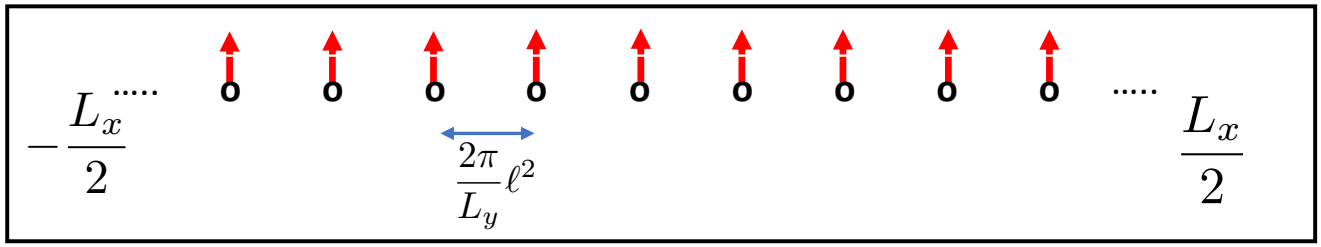
$\mathbf{A} = (0, -Bx, 0)$

QUANTUM HALL FERROMAGNET. ν=1

k_y is a good quantum number, In the Lowest Landau Level $\phi_{k_y}(\mathbf{r}) \propto e^{ik_y y} e^{-\frac{(x/\ell - k_y \ell)^2}{2}}$

$$X = k_y \ell^2 = n \frac{2\pi}{L_y} \ell^2$$

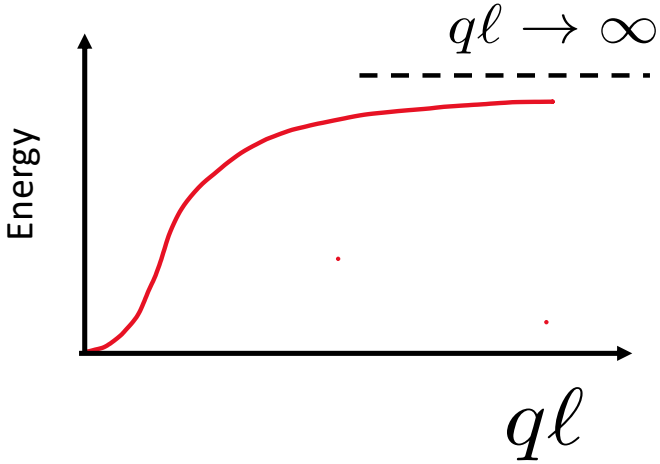
$$\ell = \sqrt{\hbar / eB}$$



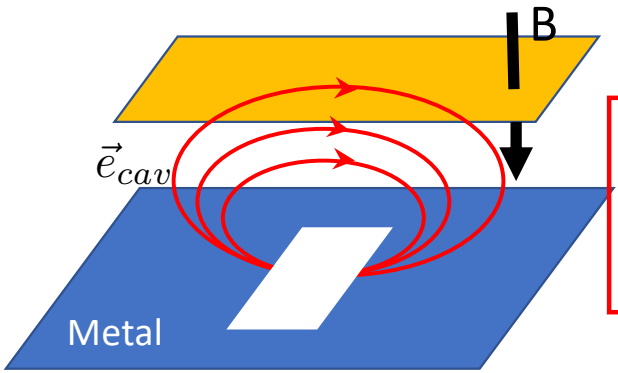
$$N_\phi = \frac{L_x L_y}{2\pi \ell^2}$$

At $\nu = 1$, all states occupied and spins aligened along a direction.

Interaction polarizes the electron spins.
Also, it introduces a local spin stiffness, leading to low-lying spin wave excitations and skyrmions.



MODEL HAMILTONIAN.



$$H = \frac{1}{2m_e}\sum_{i=1}^{N_e}\left(\vec{p}_i - e\vec{A}(\vec{r}_i) - e\vec{A}_c(\vec{r}_i)\right)^2 + \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) + H^{ee}{}_{int}$$

$$\vec{A} = -Bx\hat{e}_y; \qquad \vec{A}_c = A_0 u(x)\hat{e}_x(a + a^\dagger)$$

Here $u(x)$ is the field profile, which we assume varies only in the x-direction in the plane of the sample, and A_0 is a normalization constant which includes the effective cavity volume.

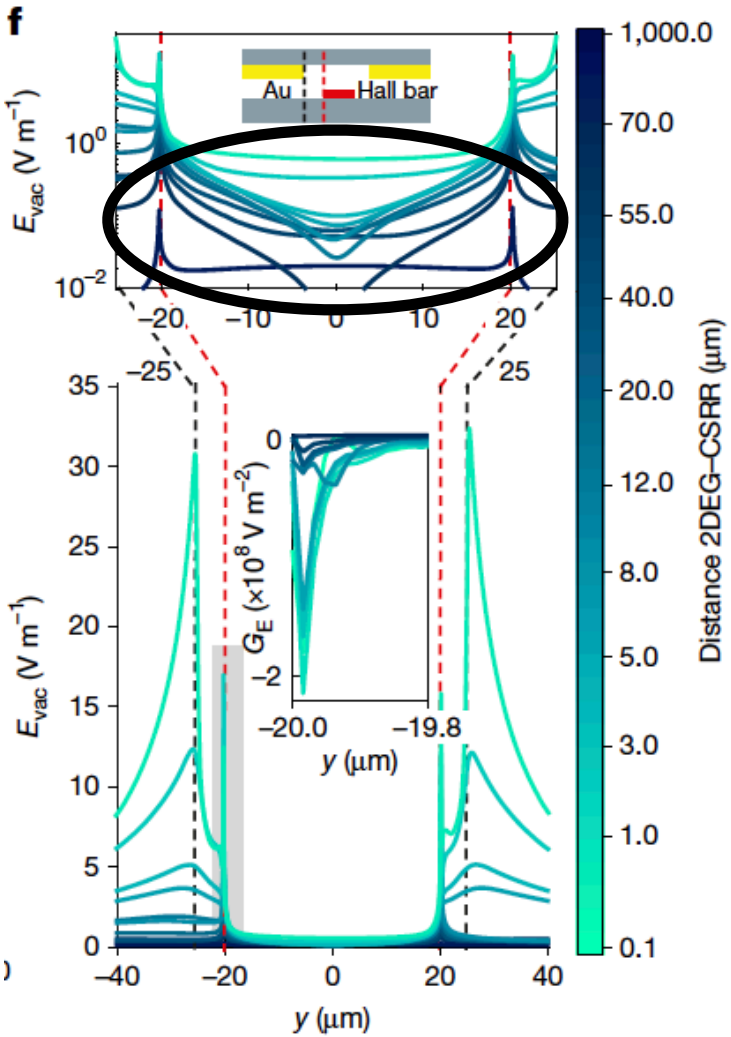
We consider the electrons interact with only a single mode of the cavity a.

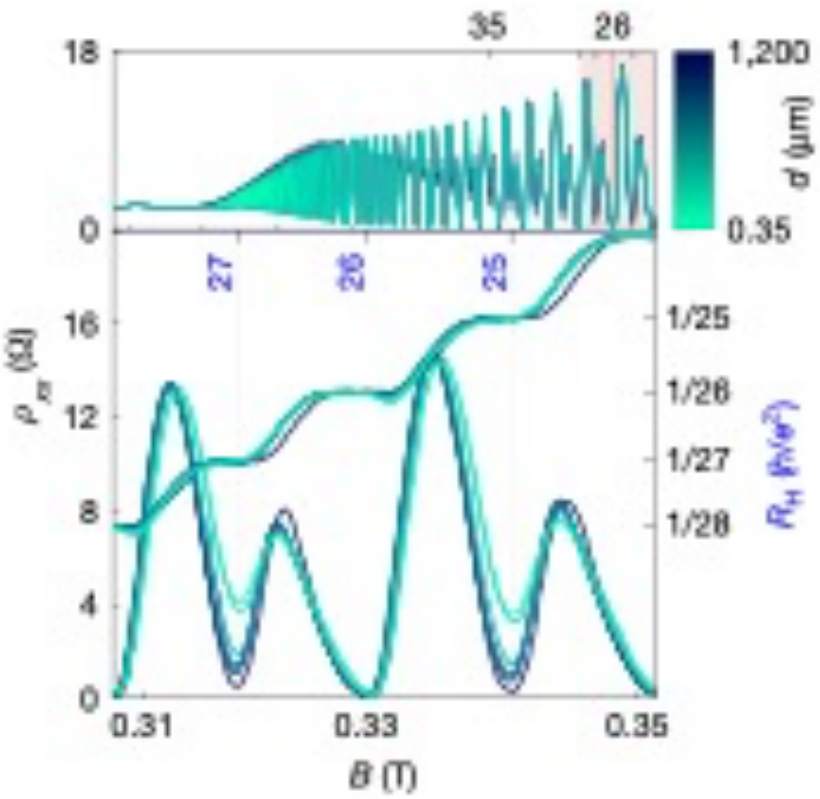
It is known that if the field profile is a constant, only the center-of-mass mode of the electrons (Kohn mode) talks to the cavity. We will assume that the field is varying linearly with x.

2DEG at $\nu = 1$ and second Landau level very high in energy $\rightarrow$ we make LLLA.

F. Appugliese, …, J. Faist, *Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect,* Science 375, 1030 (2022)

Enkner, J., Graziotto, L., Boriçi, D. *et al. Tunable vacuum-field control of fractional and integer quantum Hall phases.* Nature 641, 884–889 (2025).





DIPOLE GAUGE AND LLLA (1).

For $\nu = 1$, we restrict the Hilbert space to the LLL because $\omega_c = \frac{eB}{m_e} \gg \omega_0, E_{ee}$

We want the limit $\omega_c \longrightarrow \infty$ mantainig $\ell = \sqrt{\hbar/eB}$ constant. The way to do it is to take $m_e \to 0$.

$$H = \frac{1}{2m_e}\sum_{i=1}^{N_e}\left(\vec{p}_i - e\vec{A}(\vec{r}_i) - e\vec{A}_c(\vec{r}_i)\right)^2 + \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) + H^{ee}{}_{int}$$

The problem is that the coupling of the electrons to the cavity also comes with a $\frac{1}{m_e}$, and the limit $m_e \to 0$ induces a divergence coupling. We therefore make a unitary transformation to dipole gauge

DIPOLE GAUGE AND LLLA (2).

$$U = e^{i\frac{e}{\hbar}A_0(a+a^+)\sum_i \varphi(x_i)}; \quad \frac{d\varphi(x)}{dx} = u(x) = 2\varphi^{(2)}x\,; \varphi(x) = \varphi^{(2)}x^2$$

This gets rid of the cavity coupling in the kinetic term.

$$UHU^\dagger = H_d = \frac{1}{2m_e}\sum_{i=1}^{N_e}\left(\vec{p}_i - e\vec{A}_i\right)^2 + H_{int} + \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) -$$

$$iA_0 e\omega_0\left(a - a^\dagger\right)\sum_{i=1}^{N_e}\varphi(x_i) + \frac{{A_0}^2e^2}{\hbar}\omega_0\left(\sum_{i\neq j}^{N_e}\varphi(x_i)\,\varphi(x_j)\right)^2 + \frac{{A_0}^2e^2}{\hbar}\omega_0\sum_i^{N_e}\varphi(x_i)^2$$

'Dipolar' Term — Many-Body Term — One-Body Term

Now we can go ahead and project to the LLL by setting $m_e = 0$.

DIPOLE GAUGE HAMILTONIAN IN THE LLL

$$\overline{H}_d = \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) + ig(a - a^\dagger)\sum\left(\frac{X^2}{\ell^2} + 1\right)c_{sX}^\dagger c_{sX} + 2\frac{g^2}{\omega_0}\sum\left(\frac{X^2}{\ell^2} + \frac{1}{2}\right)c_{sX}^\dagger c_{sX}$$

'Dipolar' Term

One-Body Term

$$+\frac{g^2}{\omega_0}\sum\left(\frac{X_1^2}{\ell^2} + 1\right)\left(\frac{X_2^2}{\ell^2} + 1\right)c_{s_1X_1}^\dagger\, c_{s_2X_2}^\dagger c_{s_2X_2} c_{s_1X_1} + H_{int}$$

Many-Body Term

$$g = \frac{A_0 e}{\hbar}\varphi^{(2)}\omega_0$$

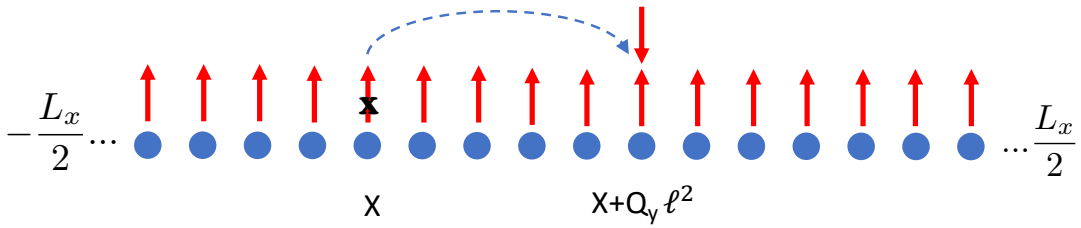
- $c_{S,X}$ destroys an electron with spin s, and momentum k_y. =X/ℓ^2.
- The 'One-Body term' represents an effective one-body potential induced by the cavity, which "wants" the electrons to be at the node of E_x and is the source of ground state phase transitions.

SKETCH OF THE CALCULATIONS

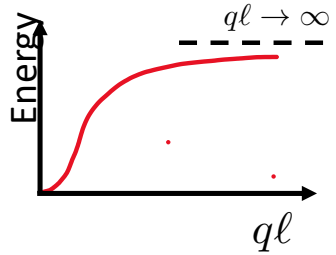
- We assume that the ground state for weak-coupling is the QHFM and find the spin-wave excitation spectrum. There is an instability as the coupling to the cavity grows.
- We carry out a DMRG calculation to find the ground state. The weak-coupling state is indeed the QHFM, while the strong-coupling state is a wedding-cake structure with double occupation in the center. The cavity is not entangled with the electrons.
- Inspired by the DMRG we carry out a HF assuming a “wedding cake” structure for the ground state and find the critical coupling to the nonuniform phase.

SPIN WAVE OF A QHF COUPLED TO A CAVITY.

- Electron ground state is FM at g=0, $|\Omega>=\Pi c^{+}_{\uparrow,X}|vac.>$
- When g=0, SW are formed by linear combinations of $\sum_X \beta_X |X,Q_y>$ $\boxed{|X,Q_y>=c^{+}_{\downarrow,X+Q_y\ell^2}c_{\uparrow,X}|\Omega>}$



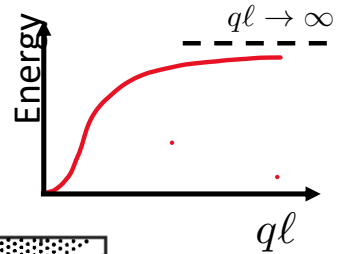
$|Q_y|\ell^2 < L_x$

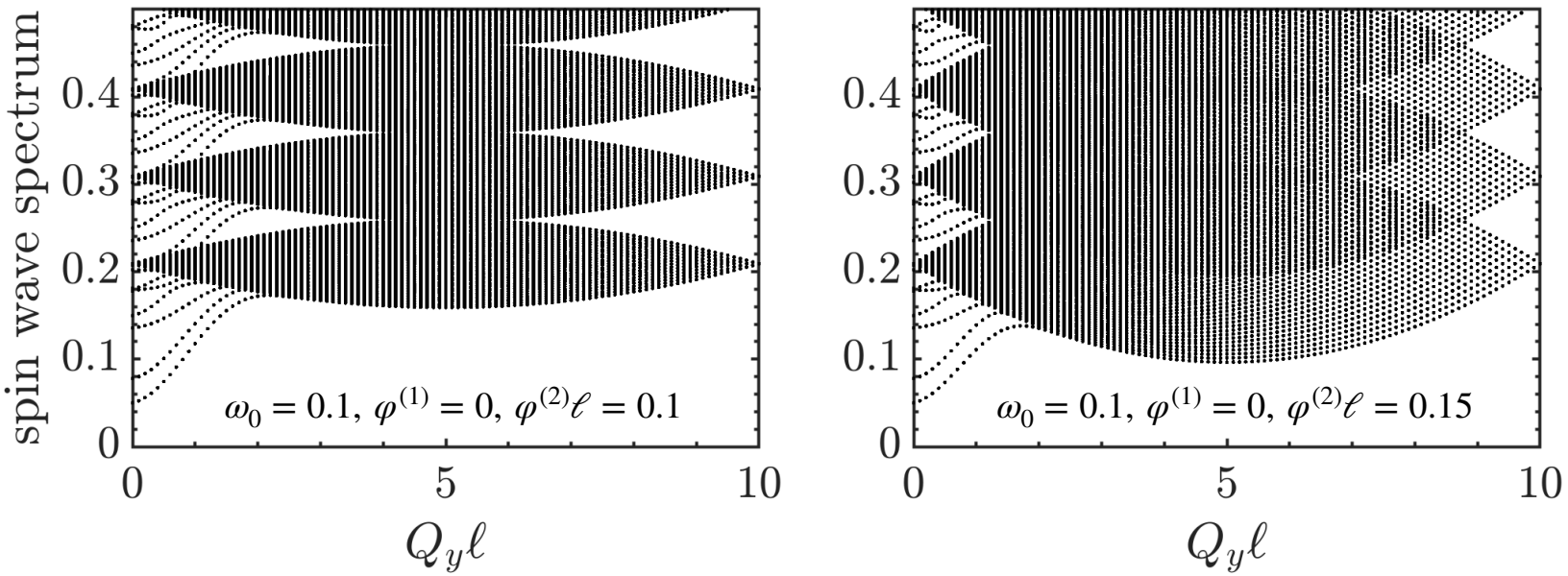


- Assuming the electronic part is a QHF, look for SW excitations as function of g and look for instabilities.
- The excitations of the system are the lineal combinations of a tensorial product $|X,Q_y>\otimes|p>$

$L_x = 10\ell,\ L_y = 20\pi\ell,\ N = 101,$ Up to 20 photons $\quad |Q_y|\ell^2 < L_x$

PBC in *y*-direction.



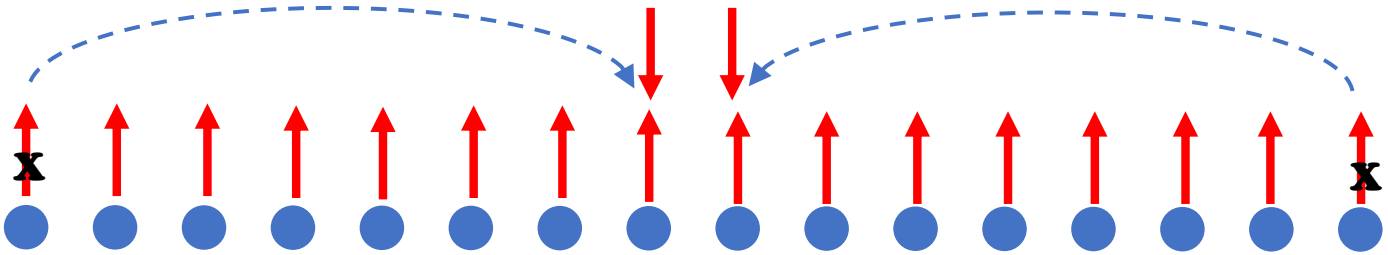


$$\overline{H}_d = \hbar\omega_0\left(a^\dagger a + \frac{1}{2}\right) + ig\left(a - a^\dagger\right)\sum\left(\frac{X^2}{\ell^2} + 1\right)c_{sX}^\dagger c_{sX} + 2\frac{g^2}{\omega_0}\sum\left(\frac{X^2}{\ell^2} + \frac{1}{2}\right)c_{sX}^\dagger c_{sX}$$

$$+\frac{g^2}{\omega_0}\sum\left(\frac{X_1^2}{\ell^2} + 1\right)\left(\frac{X_2^2}{\ell^2} + 1\right)c_{s_1X_1}^\dagger\, c_{s_2X_2}^\dagger c_{s_2X_2} c_{s_1X_1} + H_{int}$$

A QUANTUM PHASE TRANSITION

due to the competition between light-matter and e-e interactions

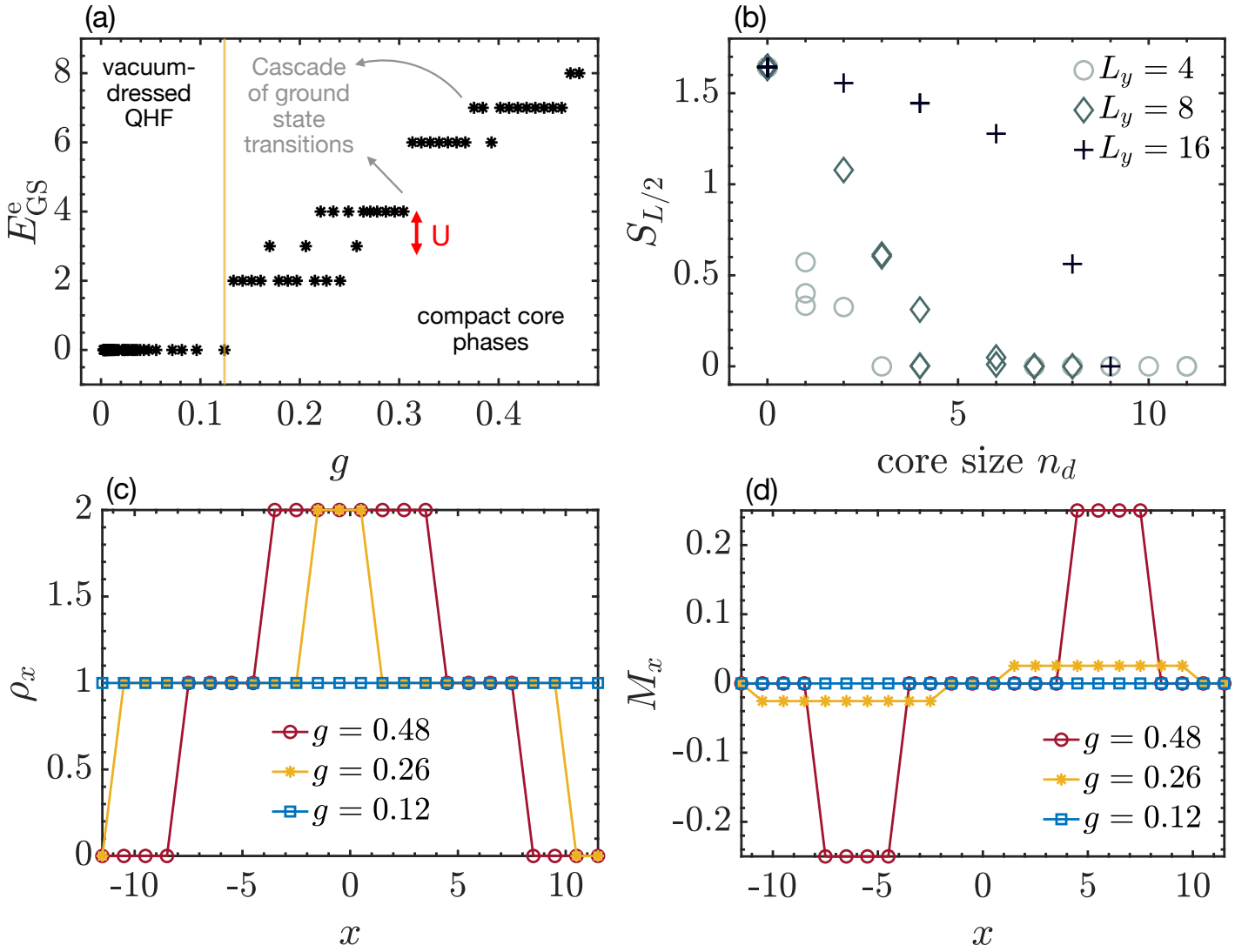


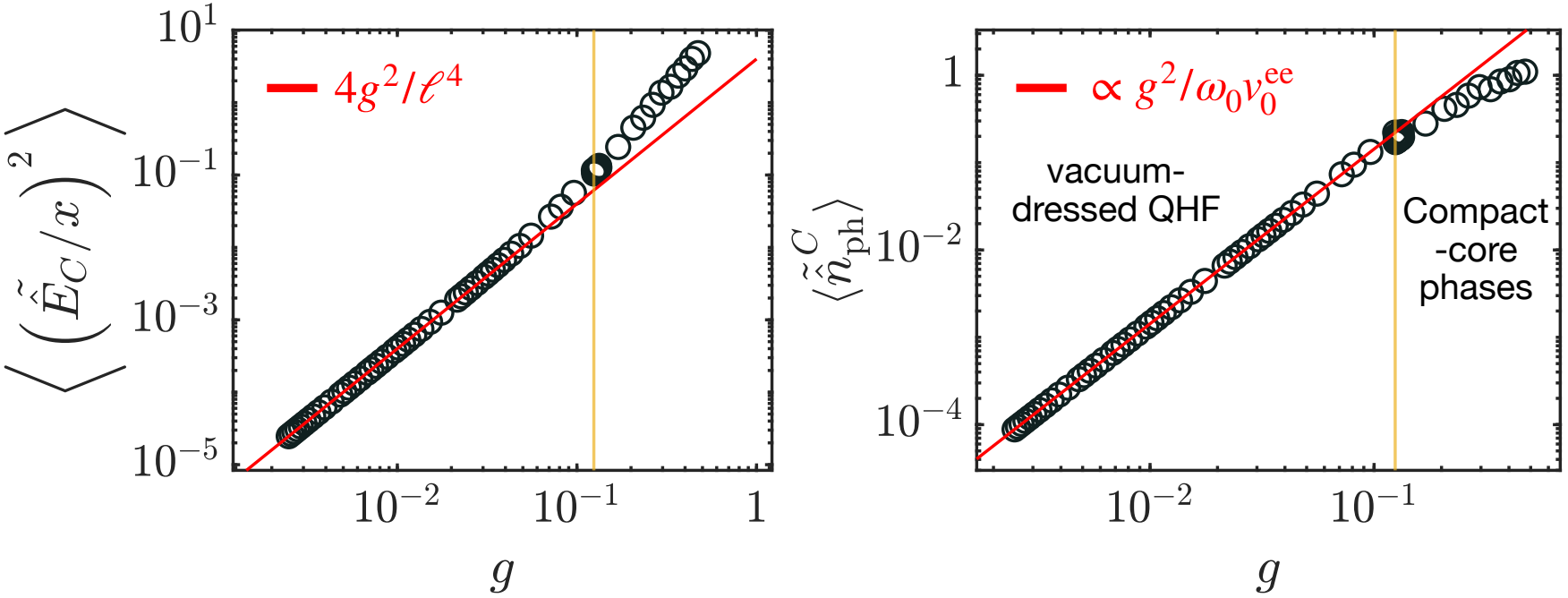
The instability occurs at $|Q_y \ell^2| \sim L_x/2$, this suggests that that the new ground state will involve electrons being displaced from the system edge to the system center, creating a non-magnetic core state.

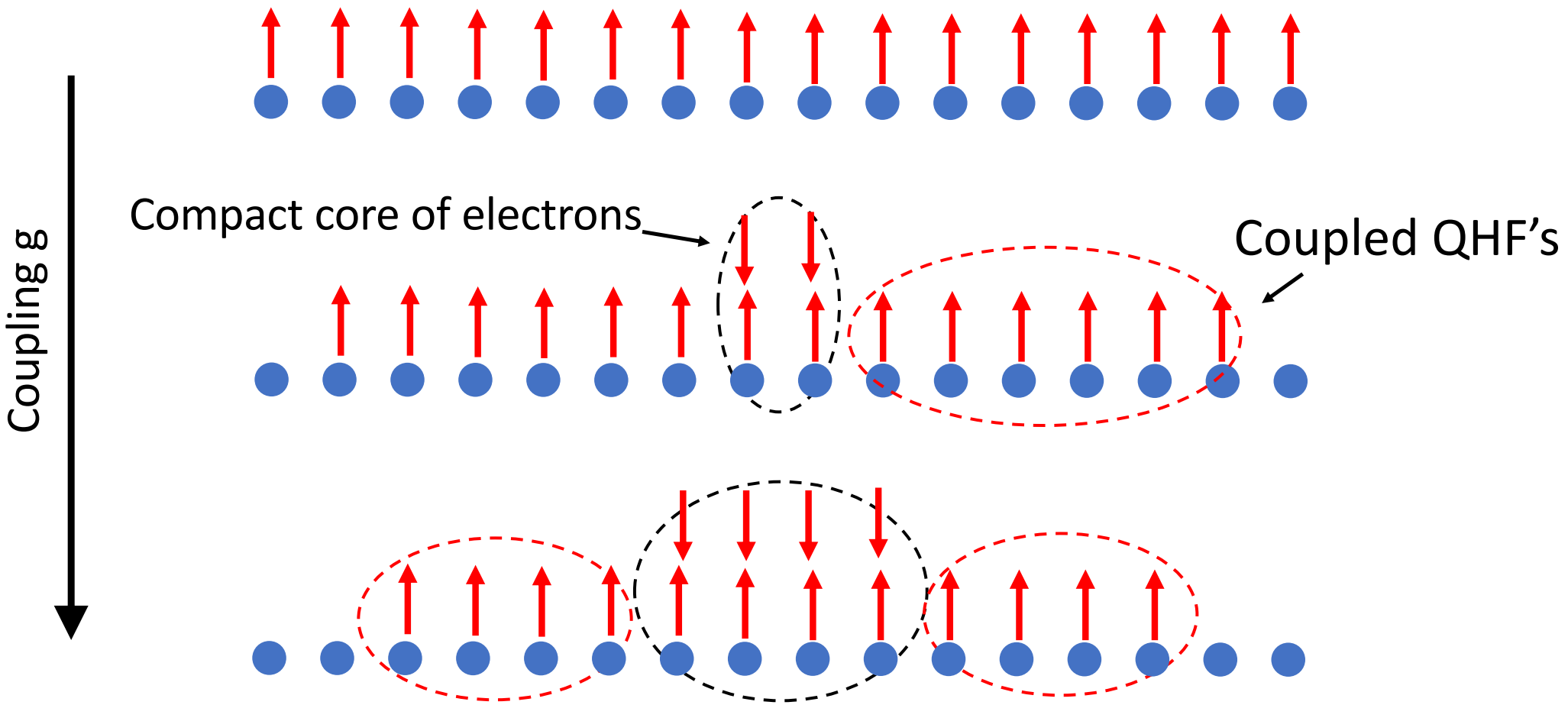
HOW IS THE NEW GROUND STATE?

DENSITY MATRIX RENORMALIZATION GROUP.

N=24, S_z=0, $V = v_0 \ell^2 \delta(\vec{r})$




$4g^2/\ell^4$
$\left\langle \left(\tilde{\hat{E}}_C/x\right)^2 \right\rangle$
10^1
10^{-1}
10^{-3}
10^{-5}
10^{-2}
10^{-1}
1
g
$\propto g^2/\omega_0 v_0^{\mathrm{ee}}$
$\langle \tilde{\hat{n}}_{\mathrm{ph}}^C \rangle$
1
10^{-2}
10^{-4}
vacuum-
dressed QHF
Compact
-core
phases
10^{-2}
10^{-1}
g


Coupling g
Compact core of electrons
Coupled QHF's

DENSITY MATRIX RENORMALIZATION GROUP. Results.

- The many-body GS is always a tensor product $|\psi_e> \otimes |\psi_p>$ (Dipole Gauge)

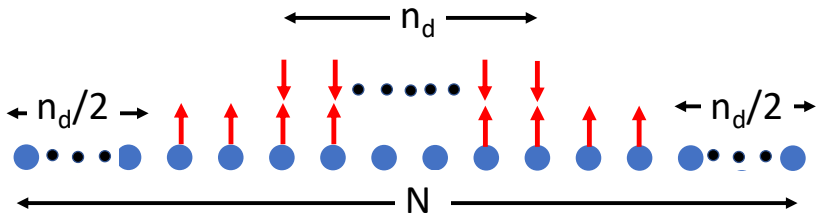
  No entanglement between the 2DEG and the cavity. S=Tr$[\rho_e \log \rho_e] = 0$.
- However the coupling modifies both the properties of the 2DEG and the cavity.
- The QHF is broken into a central compact core of double occupied states, edge states forming QHF flanks and empty states.
- In the dipole gauge the cavity ground state is a coherent state $|\Psi p> \sim a|0>$
- The expectation value of the number of photons in the system increases with the coupling. Change slope at the transition.

MEAN FIELD CALCULATIONS

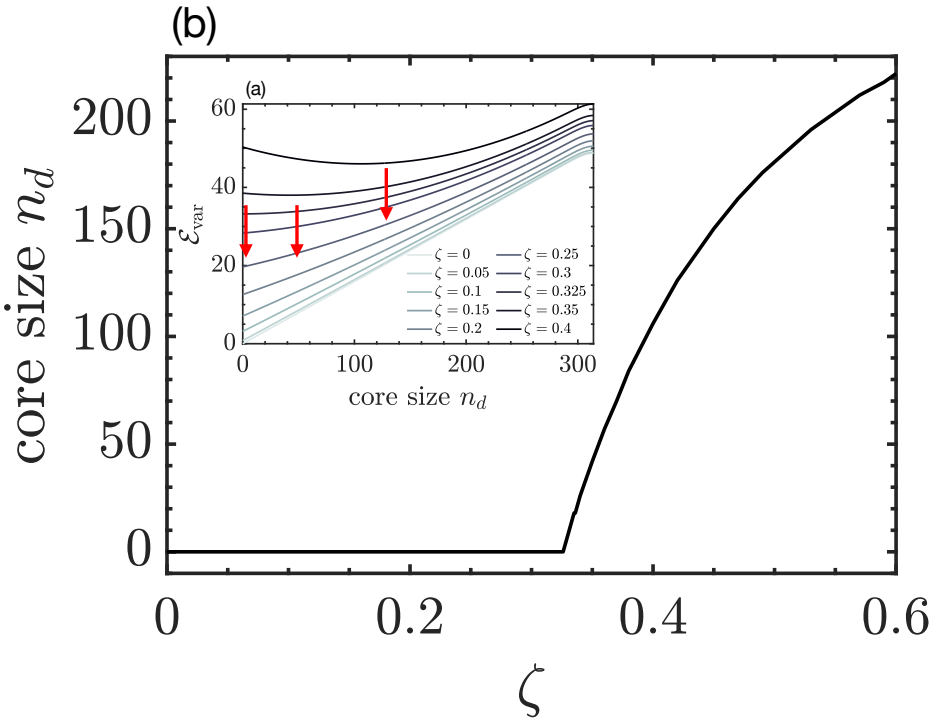
$L_x=L_y=20\pi\ell, N = 629$

$$< c^{+}_{X,s} c_{X,s} >= 0, 1$$

$$|\Psi_p >= e^{ip(a+a^{+})}|0_p >$$

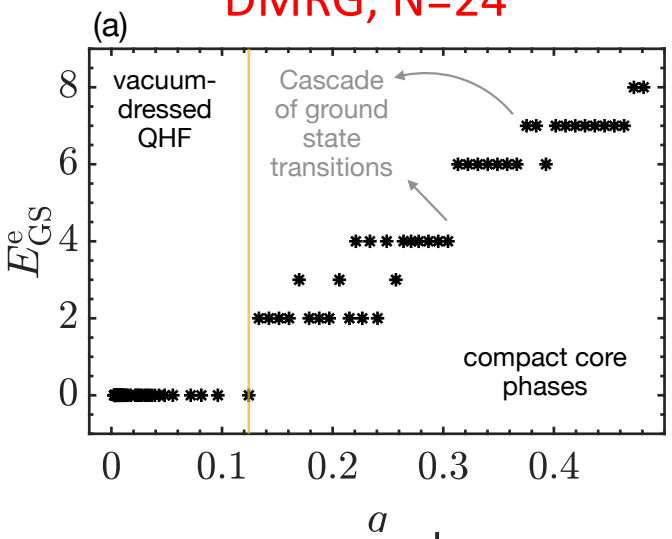


Continuous transition, order parameter n_d

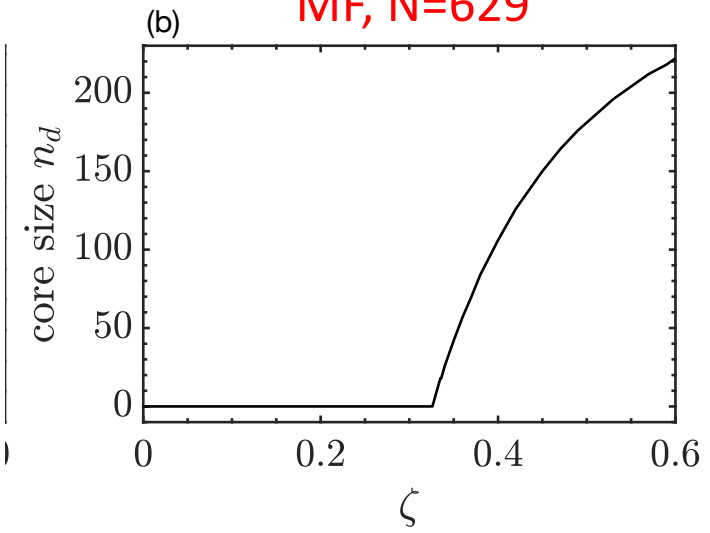


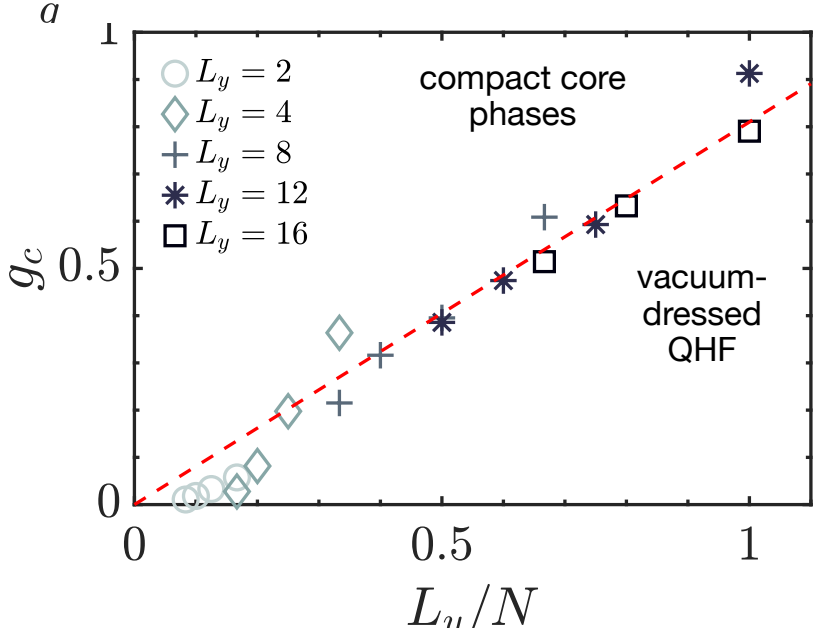
Critical coupling value coincides with spin-wave calculation

DMRG, N=24



MF, N=629

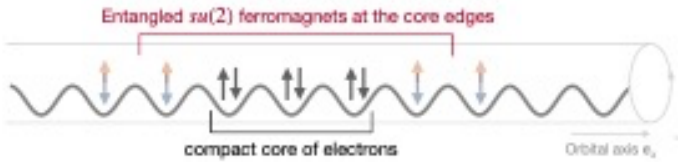




$$g_c = \sqrt{\frac{v_0^{ee}\omega_0}{\pi}}\frac{\ell}{L_x}$$

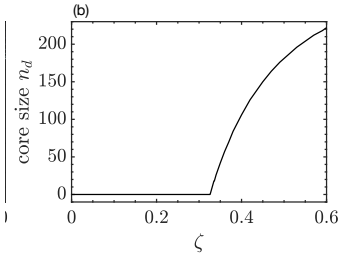
SUMMARY

- Optical cavity with non-uniform electric field coupled to a QHF.
- Instability in the SDW indicates uniform QHF reorganizes.
- DMRG calculations non entanglement electrons-photons (Dipole Gauge).
- DMRG results indicate charge transfer to region where the magnitude electric field is low.
- QHF-Non Magnetic Core-QHF



- Hartree-Fock calculations describes a second order phase transition and properties of the new state.

$$g_c = \sqrt{\frac{v_0^{ee}\omega_0}{\pi}}\frac{\ell}{L_x}$$



- The photon state is also modified at the electronic transiti

