Young Research Leaders Group Workshop

Transport and transfer of angular momentum: magnons, chiral phonons and beyond

Ingelheim, 9-11 June 2026

Extrinsic thermal Hall transport in ordered and disordered magnets

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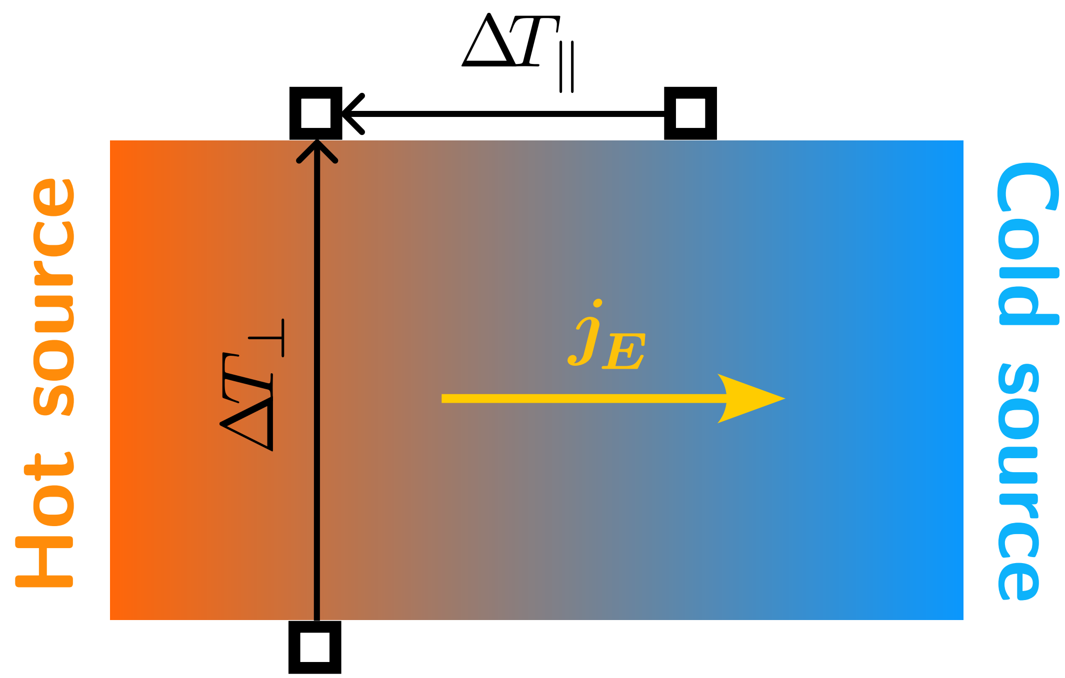
Thermal Hall conductivity reminders



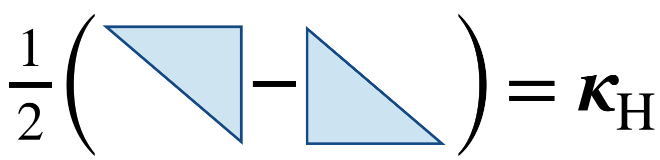
$$\boldsymbol{j}_E = -\boldsymbol{\kappa}\boldsymbol{\nabla}T$$

$$\boldsymbol{\kappa} = \boldsymbol{\kappa}(T, \boldsymbol{B}, \cdots)$$

$$\boldsymbol{\kappa} = \begin{bmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{yx} & \kappa_{yy} & \kappa_{yz} \\ \kappa_{zx} & \kappa_{zy} & \kappa_{zz} \end{bmatrix}$$

Longitudinal = κ_{L}

Hall conductivity:



$\boldsymbol{\kappa}_{\mathrm{H}}$ is time reversal (TR)-odd $\Rightarrow$ **chiral** energy carriers

Experimental results in insulating magnets

Pyrochlore FM — $Lu_2V_2O_7$, $Ho_2V_2O_7$, $In_2Mn_2O_7$	*(2010, 2012)*	magnons?
Honeycomb FM — VI_3	*(2021)*	
Kagome ferro/paramagnet — Cu(1,3-bcd)	*(2015)*	
Cubic AF — Cu_3TeO_6	*(2022)*	phonons?
Pyrochlore AF/paramagnet — $Tb_2Ti_2O_7$, diluted Tb → Y	*(2015,2019)*	
Kagome AF/paramagnet — volborthite, (Ca/Cd)-kapellasite	*(2016, 2018, 2020)*	???
Honeycomb AF/paramagnet — α-$RuCl_3$	*(2018 +)*	phonons/magnons /majoranas?

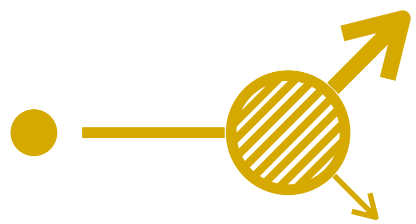
Energy carriers: certainly **neutral** — bosons, fermions? — **what mechanism?**

Mechanisms for anomalous Hall effects

Intrinsic —

$$\begin{cases} \dot{p} = \mathsf{q}(-\partial_X \phi + \dot{X} \times B) \\ \dot{X} \;=\; \partial_p \mathcal{E} - \dot{p} \times \Omega \end{cases}$$

Ω = Berry curvature



Collisions — especially skew-scattering

Wish list for neutral (non-conserved) bosons: want to describe both simultaneously

→ Structure of a kinetic equation:

intrinsic $\quad \mathcal{D}_t F_n(p, X) \quad = \quad \mathcal{I}_n(p, X)\left[F_{n'}(p', X')\right] \quad$ collisions

« $\mathcal{D}_t = \partial_t + \dot{X}\,\partial_X + \dot{p}\,\partial_p$

« $\mathcal{I}_n(p) = \sum_{n',p'} \mathcal{W}_{np\to n'p'}\,(F_{n'p'} - F_{np}) \times 2\pi\,\delta(\mathcal{E}_{np} - \mathcal{E}_{n'p'})$ »

Intrinsic AHE: what is different for neutral bosons?

Problem n°1: whence $-\dot{p}\times\boldsymbol{\Omega}$ for neutral particles?

Semiclassical (wavepacket) equations of motion:

$$\begin{cases} \dot{p} = \sout{q(-\partial_X \phi + \dot{X}\times B)} \; -\partial_X \mathcal{E} \quad ? \\ \dot{X} = \partial_p \mathcal{E} - \dot{p}\times\Omega \end{cases}$$

anomalous velocity

Problem n°2: what is the energy current?

For $U(1)$ charge-carrying conserved particles:

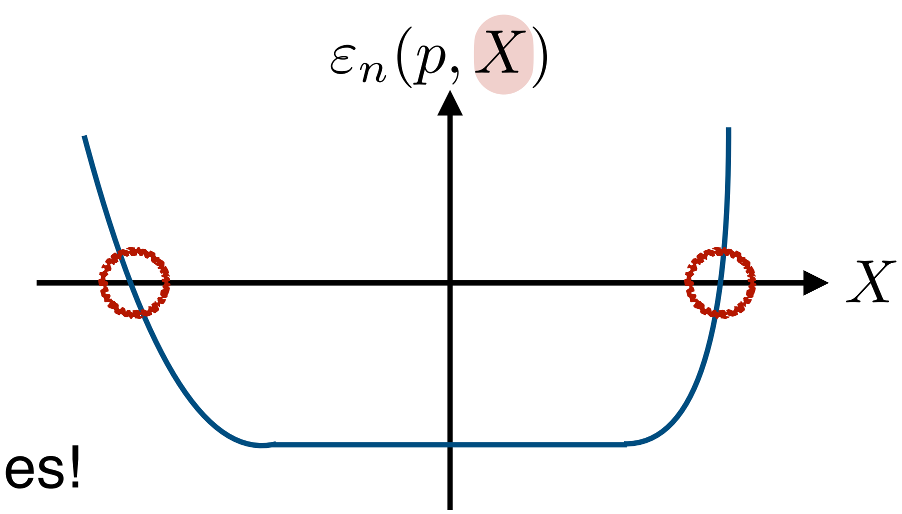
$$\sout{\boldsymbol{j}_{\mathrm{c}}(X) = \sum_n \int_{\boldsymbol{p}} (-e)\,\dot{X}_n F_n(X,p)}$$

From the vector potential: $\sout{\boldsymbol{j}_{\mathrm{c}} = -\frac{\delta \mathcal{S}}{\delta \boldsymbol{A}}}$ (Ward identity) ?

Problem n°3: how to enforce boundaries?

Landauer's picture: edge modes with velocity

$$\dot{X}\big|_{\mathrm{R/L}} = \partial_X \varepsilon_n(p,X)\big|_{\mathrm{R/L}} \times \boldsymbol{\Omega}$$



However: potential fixed to zero for non-conserved particles!

Inhomogeneous setup

Ingredients:

- Any set of real bosonic fields $\Phi_a(\boldsymbol{r})$
- Hamiltonian: any hermitian quadratic form $\mathrm{H} = \frac{1}{2}\int_{\boldsymbol{r},\boldsymbol{r}'} \Phi_a(\boldsymbol{r})\, \mathsf{H}_{ab}(\boldsymbol{r},\boldsymbol{r}')\, \Phi_b(\boldsymbol{r}')$

Inhomogeneous theory

Free fields: means $i\hbar\,\partial_t \Phi_a(\boldsymbol{r}) = \int_{\boldsymbol{r}'} \mathsf{K}_{ab}(\boldsymbol{r},\boldsymbol{r}')\, \Phi_b(\boldsymbol{r}')$

dynamical matrix (obtained directly from H)

Examples:

- Phonons: $\Phi(\boldsymbol{r}) = (u_x, u_y, u_z, \pi_x, \pi_y, \pi_z)$ — *u* displacement, *π* momentum

- Magnons in an AF: $\Phi(\boldsymbol{r}) = (n_x, n_y, m_x, m_y)$ — *n* staggered, *m* net magnetization fluctuations
  (order along z)

Goal: obtaining the kinetic equation and intrinsic thermal Hall conductivity of the Φ bosons.

Inhomogeneous kinetic theory for neutral bosons

We want the time evolution of the density matrix $\mathsf{F}_{ab}(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{2}\langle\{\Phi_a(\boldsymbol{r}),\Phi_b(\boldsymbol{r}')\}\rangle$

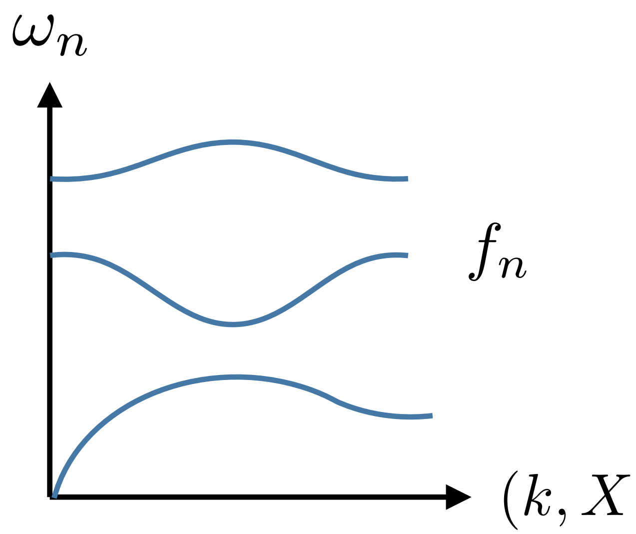
$$i\hbar\partial_t \mathsf{F}(\boldsymbol{r},\boldsymbol{r}') = \int_{\boldsymbol{r}''}\left[\mathsf{K}(\boldsymbol{r},\boldsymbol{r}'')\,\mathsf{F}(\boldsymbol{r}'',\boldsymbol{r}') - \mathsf{F}(\boldsymbol{r},\boldsymbol{r}'')\,\mathsf{K}^\dagger(\boldsymbol{r}'',\boldsymbol{r}')\right]$$

THE END?

Formal wish list

→ A proper kinetic theory (cf Boltzmann's equation) should be made local in phase space *(k,X)* coordinates

→ F, K are matrices (size 2*N*, *N* = number of modes). Intuition: free particles ⇒ should be able to *diagonalize*



Technicalities

Inhomogeneous theory ⇒ cannot diagonalize "as usual" — need $\star = e^{\frac{i}{2}(\overleftarrow{\partial}_X \overrightarrow{\partial}_k - \overleftarrow{\partial}_k \overrightarrow{\partial}_X)\hbar}$

Inhomogeneous kinetic theory for neutral bosons

Result:

$$i\hbar\,\partial_t \underline{\mathsf{F}}_{\rm d} = \left[\underline{\mathsf{K}}_{\rm d} \star \underline{\mathsf{F}}_{\rm d} - \underline{\mathsf{F}}_{\rm d} \star \underline{\mathsf{K}}_{\rm d}\right] \qquad @(k, X)$$

$$\star = e^{\frac{i}{2}(\overleftarrow{\partial}_X \overrightarrow{\partial}_k - \overleftarrow{\partial}_k \overrightarrow{\partial}_X)\hbar}$$

Ω = Berry curvature

Explicitly to leading order:

anomalous velocity

$$(\partial_t + \dot{\mathsf{X}}_\mu \partial_{X_\mu} + \dot{\mathsf{p}}_\mu \partial_{p_\mu})\underline{\mathsf{F}}_{\rm d}(X, p, t) = 0$$

$$\begin{cases} \dot{\mathsf{X}} = \partial_{\boldsymbol{p}} \underline{\mathsf{K}}_{\rm d} - \dot{\mathsf{p}} \times \Omega + \dots \\ \dot{\mathsf{p}} = -\partial_{\boldsymbol{X}} \underline{\mathsf{K}}_{\rm d} + \dots \end{cases}$$

from the **inhomogeneous** theory

What is new there?

✓ Derived (not postulated) the semiclassical equations of motion.
Identified the proper eigenbasis, phase-space covariant coordinates, etc.

⟶ Will be useful later!

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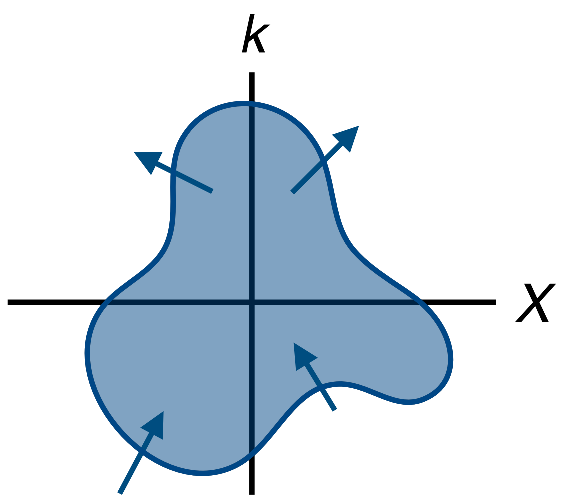
Phase-space density of energy current



Energy density: $\mathrm{H} = \int_{k,X} \frac{1}{2}\,\mathrm{Tr}[\mathsf{H} \star \mathsf{F}] \;=: \int_{k,X} \mathcal{H}(k,X)$

Continuity equation *in phase space*: $\partial_t \mathcal{H} + \partial_\alpha \mathcal{J}_\alpha = 0 \qquad \alpha = X_\mu, p_\mu$

Can put the phase space current into a diagonal, gauge-invariant form:

Result:

$$\mathcal{J}_{\boldsymbol{X}} = \frac{1}{2}\mathrm{Tr}\left[\underline{\mathsf{F}}_d \left(\partial_{\boldsymbol{p}} + \hbar\Omega \times \partial_{\boldsymbol{p}}\right) \underline{\mathsf{K}}_d\right] + \ldots \qquad @(p, X)$$

In fact: $\mathcal{J}_{X_\mu} = \mathcal{J}^{(1)}_{X_\mu} + \mathcal{J}^{(2)}_{X_\mu}$ transport + magnetization

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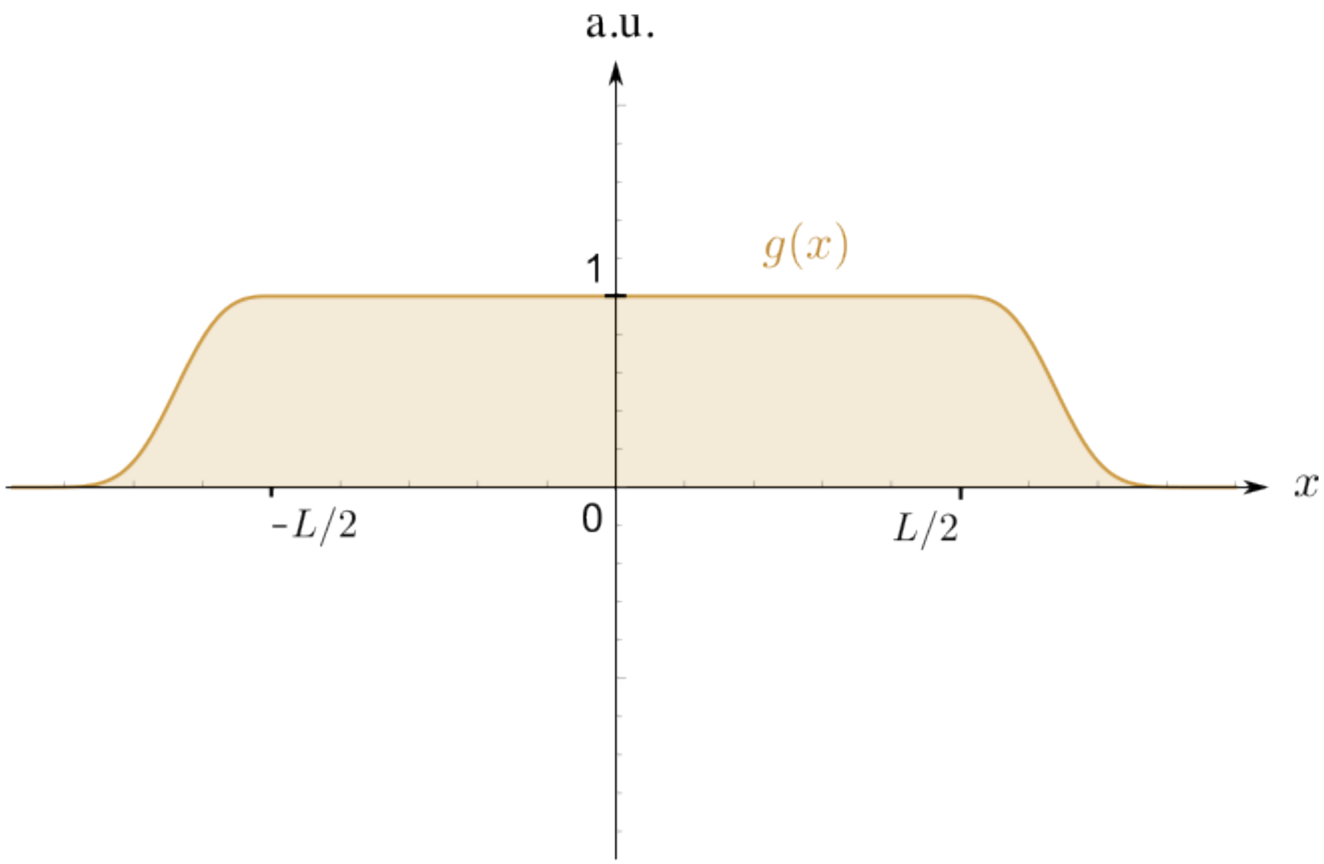
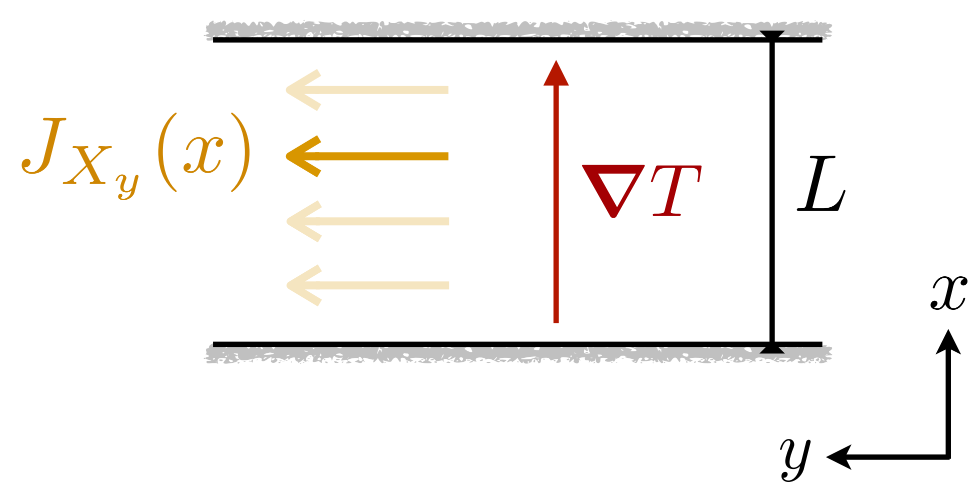
Local real-space energy current

Local current: $$J_{X_\mu} = \int_{\boldsymbol{k}} \left(\mathcal{J}^{(1)}_{X_\mu} + \mathcal{J}^{(2)}_{X_\mu} \right)$$



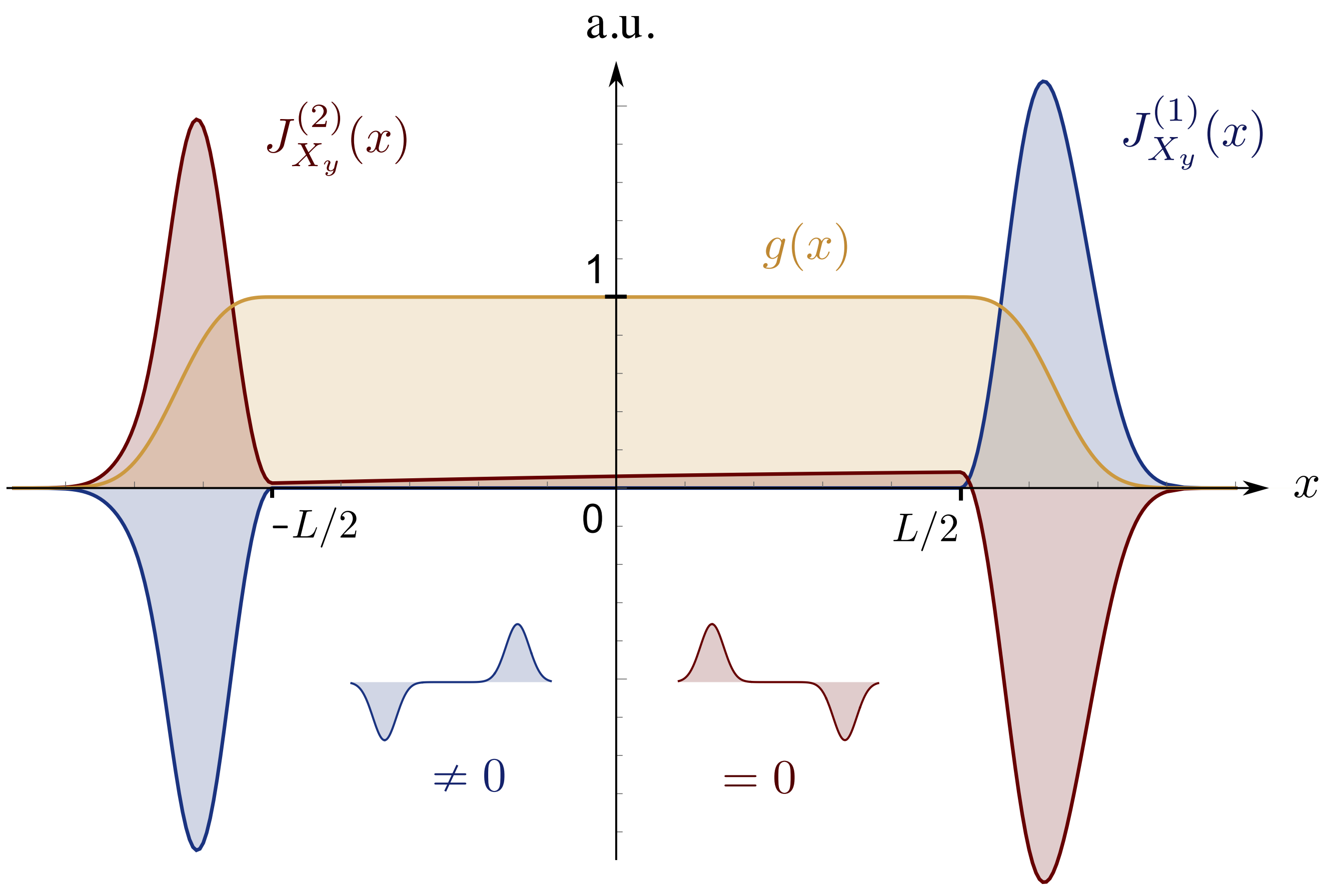


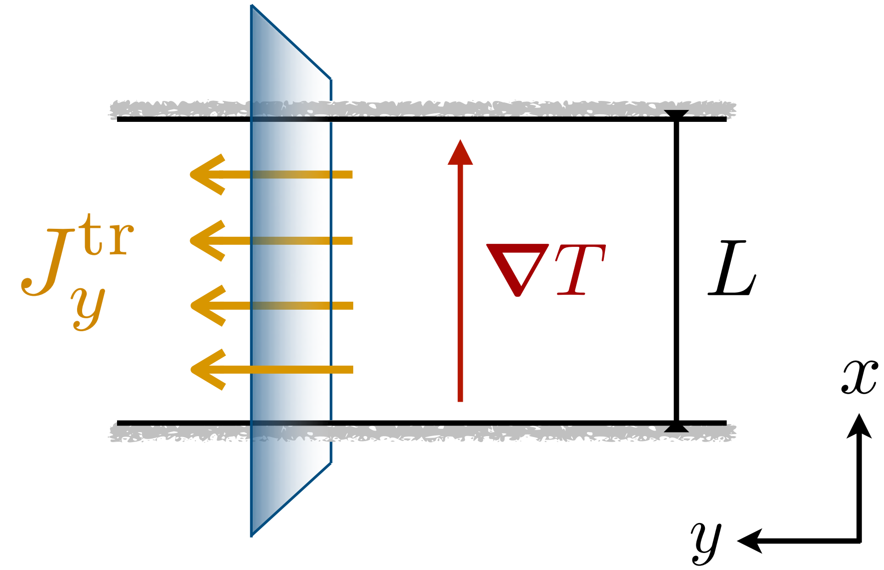
For boundaries: use an **inhomogeneous** theory

$$\mathrm{H}(X, k) = g(X)\, \mathrm{H}^{\mathrm{h}}(k)$$

Local real-space energy current

Local current: $J_{X_\mu} = \int_{\boldsymbol{k}} \left(\mathcal{J}^{(1)}_{X_\mu} + \mathcal{J}^{(2)}_{X_\mu} \right)$





For boundaries: use an **inhomogeneous** theory

$$\mathrm{H}(X, k) = g(X)\,\mathrm{H}^{\mathrm{h}}(k)$$

the local energy current is a physical quantity

Key properties:

- $\mathcal{J}^{(2)}_{X_\mu}$ is a magnetization current
- J_{X_μ} satisfies *locally* thermodynamics' third law

Thermal Hall conductivity

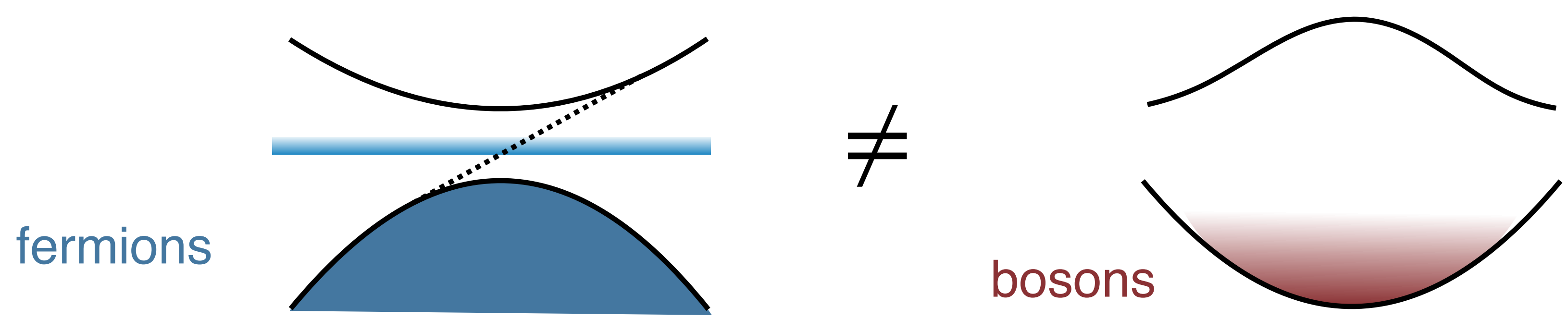
Just a byproduct

Thermal Hall conductivity: $\kappa_{xy}^{\mathrm{tr}} = \frac{1}{\partial_x T} \lim_{L\to\infty} \frac{1}{L} \int \mathrm{d}x \, J_{X_y}(x)$

Analytical result: $$\kappa_{xy}^{\mathrm{tr}} = -\frac{1}{T} \sum_n \int_{\boldsymbol{p}} \Omega^{(n)}_{p_x p_y} \int_{\mathcal{E}_n(\boldsymbol{p})}^{\infty} \mathrm{d}\varepsilon \, \varepsilon^2 \partial_\varepsilon n_{\mathrm{B}}(\varepsilon, T)$$ intrinsic

QUESTION:

Role of disorder?



- Thermal activation: for bosons, bulk states *always* contribute
- Boson-boson interactions are usually weak: more prominent role of disorder?

Extrinsic effects: adding disorder

(customary)

$\neq$ **electrons !**

Now: $\mathsf{H}_{ab}(\boldsymbol{r}, \boldsymbol{r}') \rightarrow \mathsf{H}_{ab}(\boldsymbol{r}, \boldsymbol{r}') + \hat{V}_{ab}(\boldsymbol{r}, \boldsymbol{r}')$

Disorder ~~potential~~ operator:

- *any* (hermitian) matrix structure
- it is a *random variable*

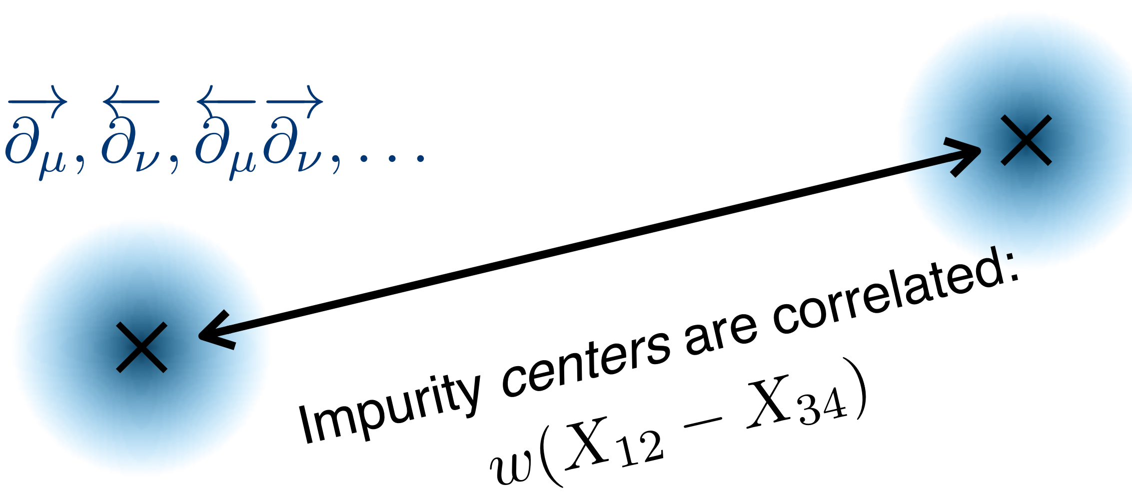
Physical picture for disorder correlations:

$$\hat{W}_{cd} \sim 1, \overrightarrow{\partial_\mu}, \overleftarrow{\partial_\nu}, \overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}, \ldots$$

$$\hat{W}_{ab} \sim 1, \overrightarrow{\partial_\mu}, \overleftarrow{\partial_\nu}, \overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}, \ldots$$

$$X_{34} = \frac{x_3 + x_4}{2}$$

$$X_{12} = \frac{x_1 + x_2}{2}$$



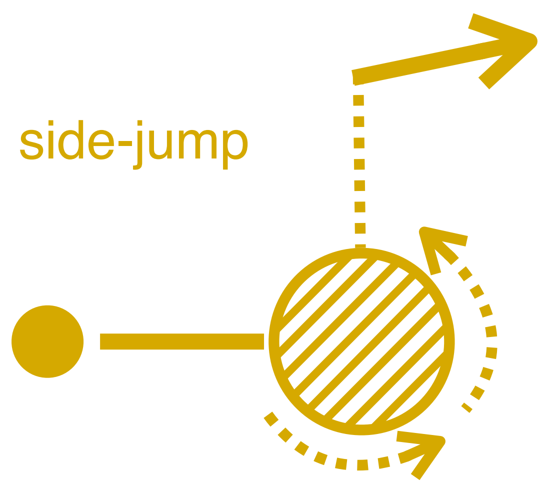
Intuition: local modulation of elasticity, magnetic exchange, etc — bond disorder

$$\langle \hat{V}_{ab}(x_1, x_2)\hat{V}_{cd}(x_3, x_4)\rangle = \hat{W}_{ab}(x_1 - x_2)\hat{W}_{cd}(x_3 - x_4)\; w\left(\frac{x_1+x_2}{2} - \frac{x_3+x_4}{2}\right)$$

The semiclassical position shift

Upshot: $$(\partial_t + \dot{\mathsf{X}}_\mu \partial_{X_\mu} + \dot{\mathsf{p}}_\mu \partial_{p_\mu}) \underline{\mathsf{F}}_{\mathrm{d}}(X, p, t) = \mathcal{I}_{\mathrm{coll}}\left[\underline{\mathsf{F}}_{\mathrm{d}}\right]$$

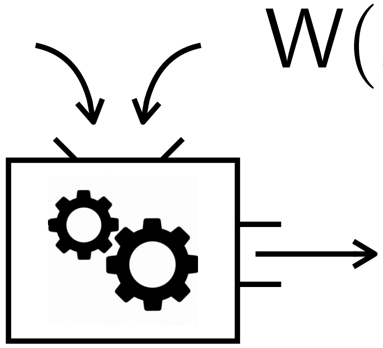
$$\mathcal{I}_n(p, X) = \sum_{n', p'} \mathcal{W}_{np \to n'p'} \left(F_{n'p'}(X + \Delta) - F_{np}(X)\right) \times 2\pi\, \delta\left(\mathcal{E}_{n'p'}(X + \Delta) - \mathcal{E}_{np}(X)\right)$$



Position shift: $$\Delta(p, p') = (p' - p) \times \Omega^W$$

$\Omega^W_{p_\mu p_\nu}$ is real, gauge-invariant, depends on $\hat{W}$,
is (generally) not $\Omega_{p_\mu p_\nu}$

A_{p_μ} $\hat{\mathsf{W}}(\boldsymbol{k})$



$\Omega_{p_\mu p_\nu}$, $\Omega^W_{p_\mu p_\nu}$

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Extrinsic contribution to the thermal Hall conductivity

New term in the current, non-equilibrium distribution, etc…

Result: disorder-induced thermal Hall conductivity:

$$\kappa_{xy}^{\mathrm{dis}} = -\frac{1}{T}\sum_n \int_{\boldsymbol{p}} \mathcal{E}_n(\boldsymbol{p})^2\, \partial_\varepsilon n_{\mathrm{B}}(\mathcal{E}_n(\boldsymbol{p}), T)\; \Omega^{W\,(n)}_{p_y p_x}(\boldsymbol{p})\, p_x\, \partial_{p_x} \mathcal{E}_n(\boldsymbol{p})$$

Recall the intrinsic thermal Hall conductivity:

$$\kappa_{xy}^{\mathrm{tr}} = -\frac{1}{T}\sum_n \int_{\boldsymbol{p}} \Omega^{(n)}_{p_x p_y} \int_{\mathcal{E}_n(\boldsymbol{p})}^{\infty} \mathrm{d}\varepsilon\, \varepsilon^2 \partial_\varepsilon n_{\mathrm{B}}\left(\varepsilon, T\right)$$

The extrinsic contribution depends on the nature of disorder (as a local hermitian operator).

Any reason to believe it should be neglected?

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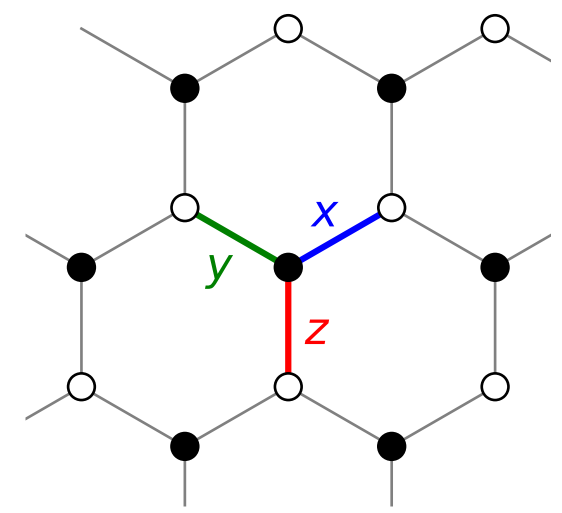
Concrete example

Honeycomb $KJ\Gamma\Gamma'$ spin model in a field:

$$H_{\rm spin} = \sum_{\gamma\in\{x,y,z\}} \sum_{\langle \mathsf{i},\mathsf{j}\rangle\in\gamma} \mathbf{S}_{\mathsf{i}}^\top H_\gamma \mathbf{S}_{\mathsf{j}} - \sum_{\mathsf{i}} \boldsymbol{h}\cdot\mathbf{S}_{\mathsf{i}},$$

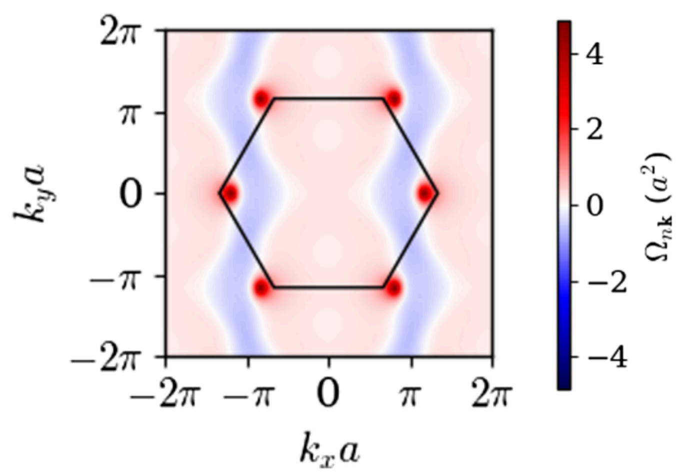


$\mu,\nu \in \{x,y,z\}$

$$(H_\gamma)_{\mu\nu} = J\,\delta_{\mu\nu} + K\,\delta_{\mu\gamma}\delta_{\nu\gamma} + \Gamma\,|\epsilon_{\mu\nu\gamma}| + \Gamma'\,(\delta_{\mu\gamma}+\delta_{\nu\gamma})(1-\delta_{\mu\gamma}\delta_{\nu\gamma})$$

SOC + explicit TR breaking

Spin wave expansion ⇒ two free magnon bands with Berry curvature



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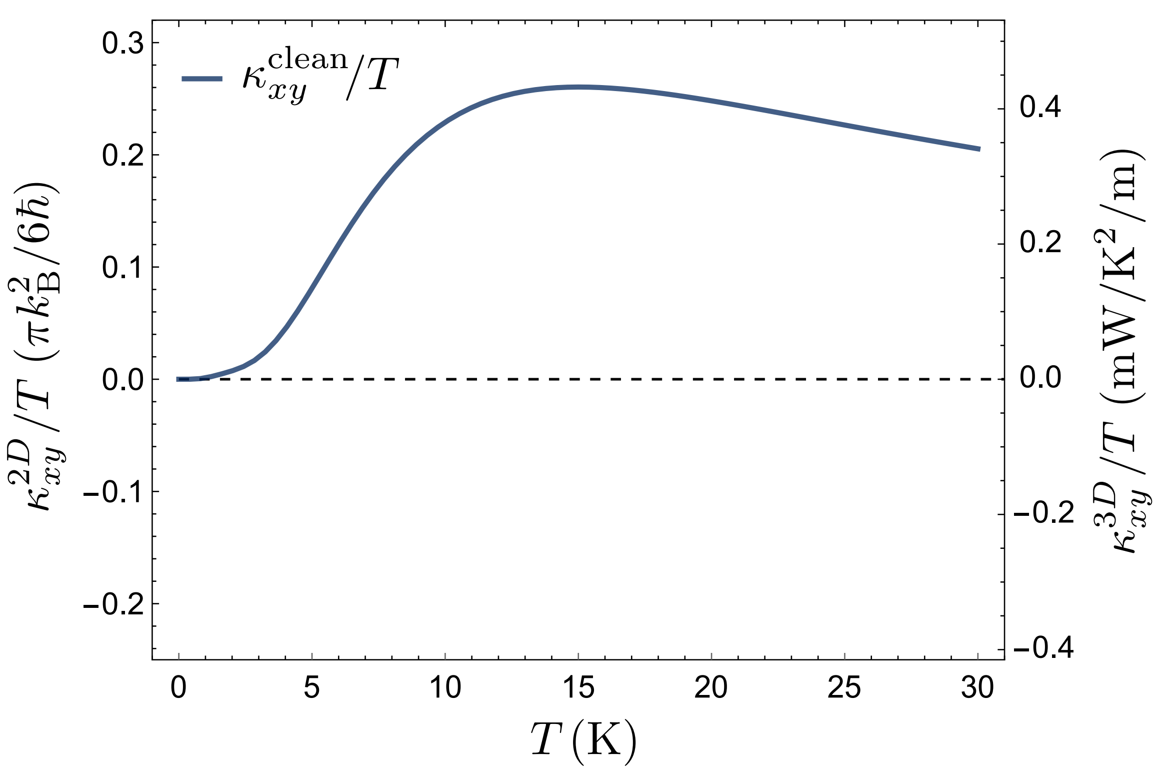
Intrinsic



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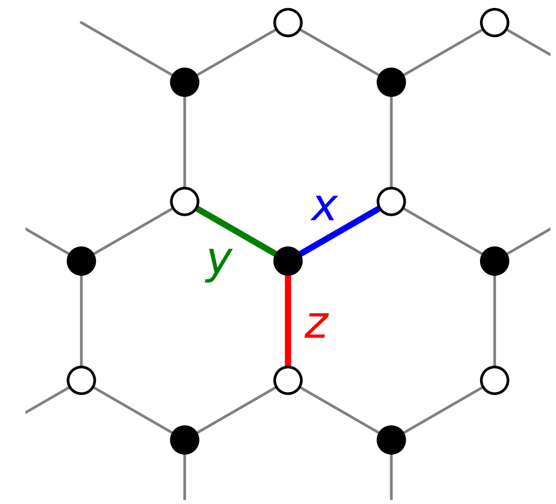
$\mu,\nu \in \{x,y,z\}$

$$(H_\gamma)_{\mu\nu} = J\,\delta_{\mu\nu} + K\,\delta_{\mu\gamma}\delta_{\nu\gamma} + \Gamma\,|\epsilon_{\mu\nu\gamma}| + \Gamma'\,(\delta_{\mu\gamma}+\delta_{\nu\gamma})(1-\delta_{\mu\gamma}\delta_{\nu\gamma})$$

K, J, Γ, Γ' $\quad$ g

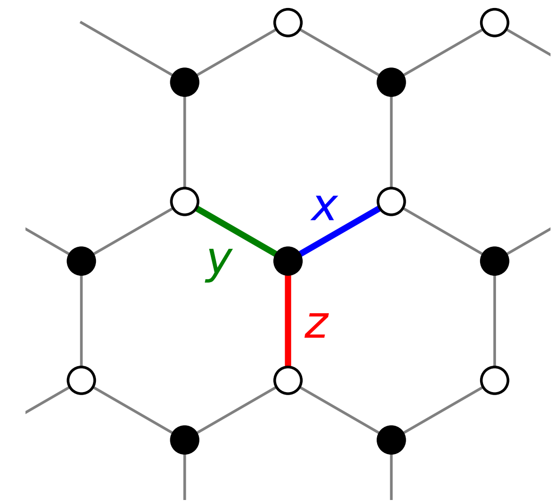


Introducing disorder: local (random) modulation of parameters K, J, Γ, Γ', g

$\Rightarrow$ This determines the disorder matrix $\hat{W}$ whence $\Omega^{W}_{p_\mu p_\nu}$

Concrete example

Honeycomb $KJ\Gamma\Gamma'$ spin model in a field:



$$H_{\text{spin}} = \sum_{\gamma\in\{x,y,z\}} \sum_{\langle \mathsf{i},\mathsf{j}\rangle\in\gamma} \mathbf{S}_{\mathsf{i}}^{\top} H_\gamma \, \mathbf{S}_{\mathsf{j}} - \sum_{\mathsf{i}} \boldsymbol{h}\cdot\mathbf{S}_{\mathsf{i}},$$

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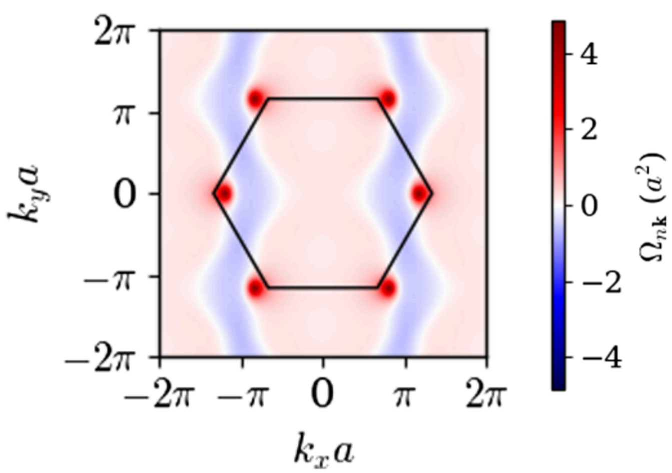
K, J, Γ, Γ' g

Spin wave expansion ⇒ two free magnon bands with Berry curvature



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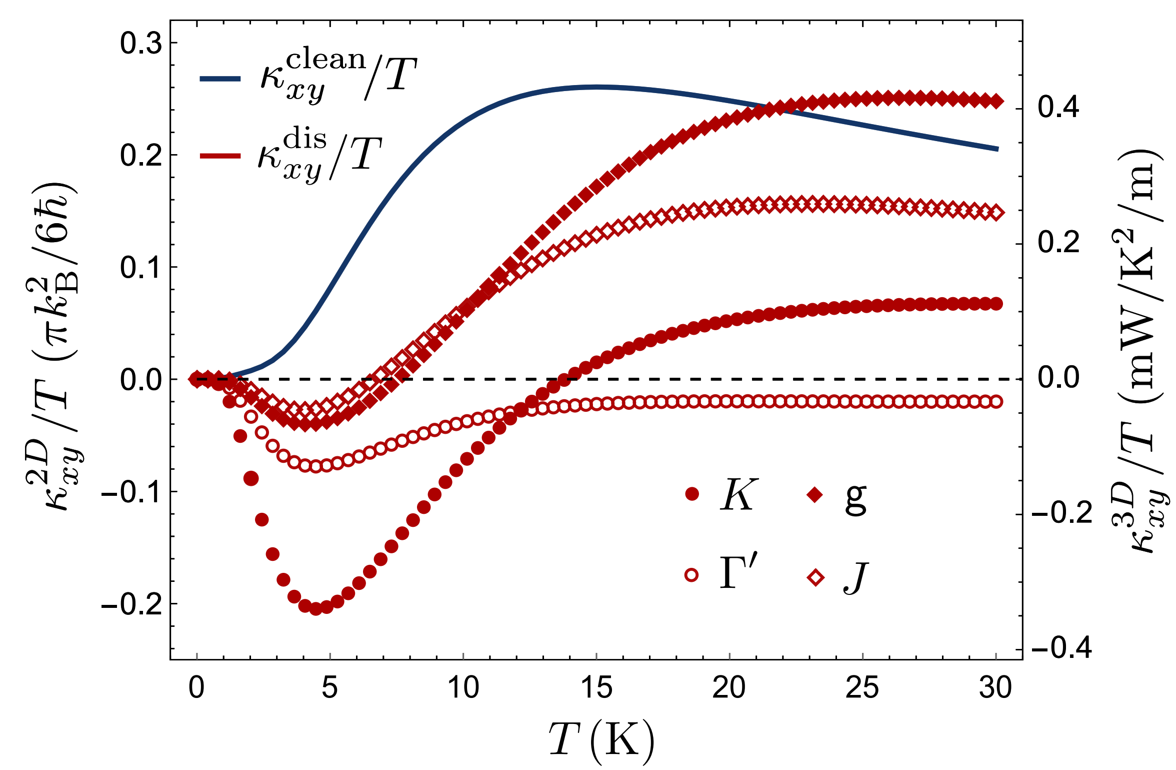
Intrinsic
+
Extrinsic



Concrete example

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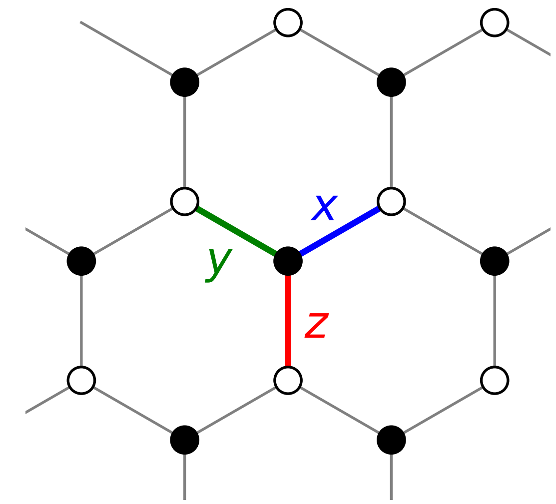

$$H_{\text{spin}} = \sum_{\gamma\in\{x,y,z\}} \sum_{\langle \mathrm{i,j}\rangle\in\gamma} \mathbf{S}_\mathrm{i}^\top H_\gamma \mathbf{S}_\mathrm{j} - \sum_\mathrm{i} \boldsymbol{h}\cdot\mathbf{S}_\mathrm{i},$$

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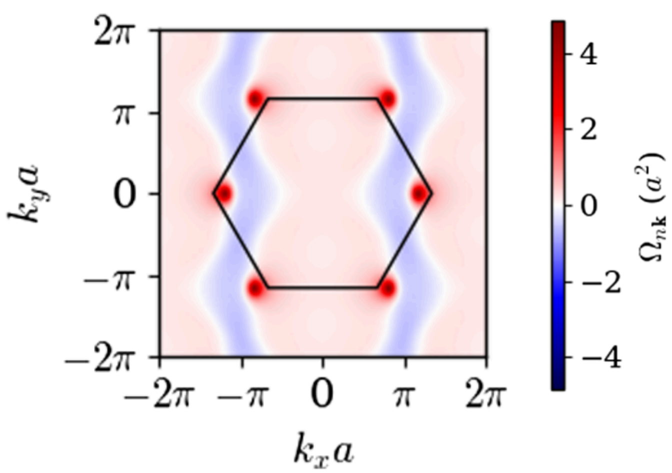
K, J, Γ, Γ' g

Spin wave expansion ⇒ two free magnon bands with Berry curvature



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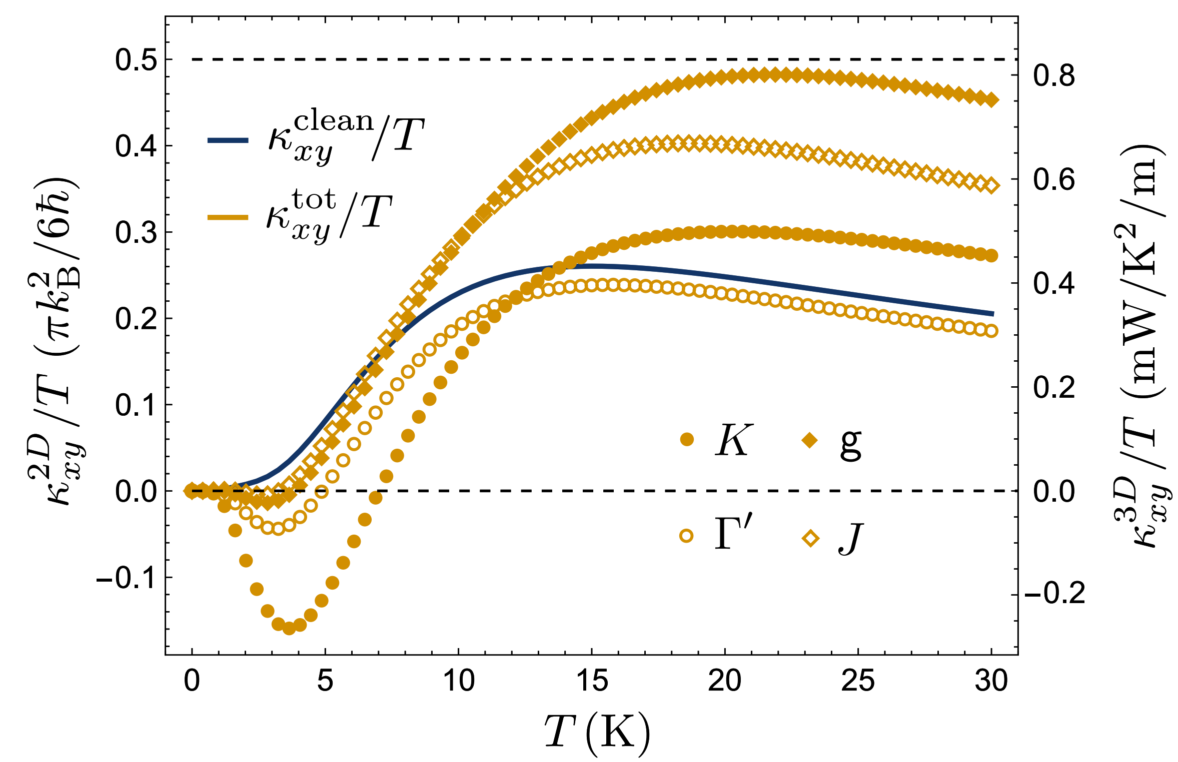
Strong effect of disorder!



Range of applicability

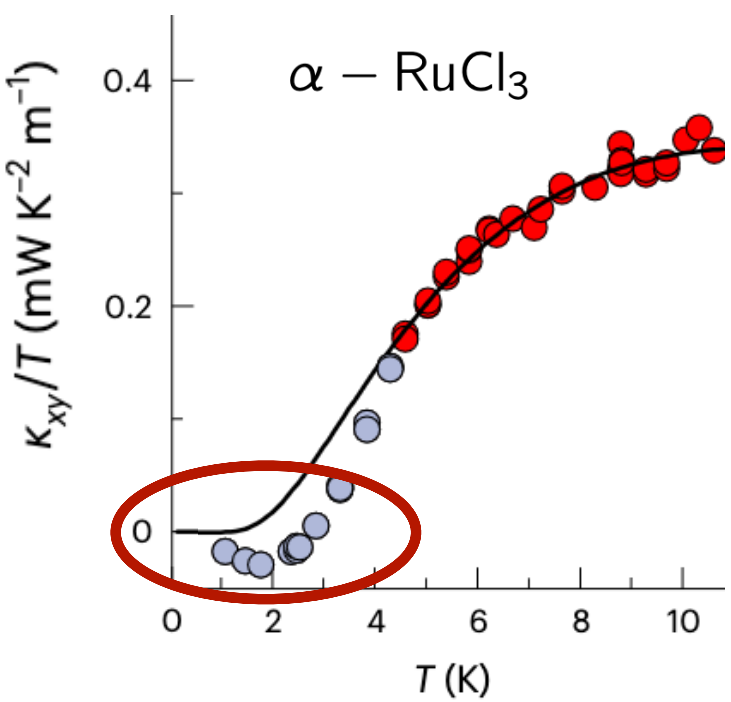
Question: role of the disorder strength?

$$\kappa_{\mathrm{H}}^{\mathrm{dis}} \approx \frac{1}{3}\, c_{\mathrm{V}}\, v^{\mathrm{sj}}\, \ell_{\mathrm{net}} \propto \ell_{\mathrm{net}}/\ell_{\mathrm{dis}}$$

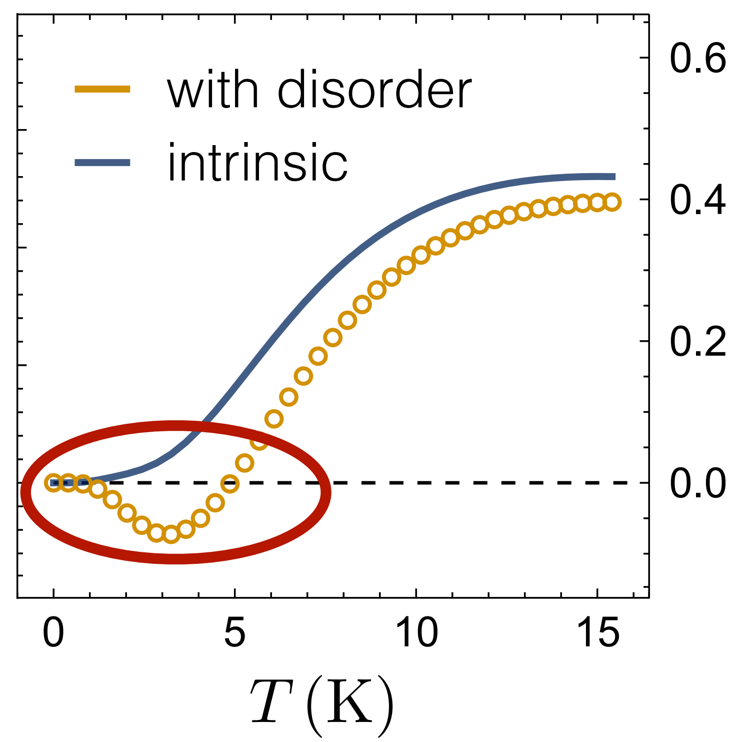
$\mathcal{I}_{\mathrm{coll}}$

- disorder dominated ($\ell_{\mathrm{net}} \approx \ell_{\mathrm{dis}}$) → side-jump $\sim$ intrinsic
- interaction dominated ($\ell_{\mathrm{net}} \ll \ell_{\mathrm{dis}}$) → skew-scattering / intrinsic / ?

One optimistic example:



Czajka et al. (2022)



L.M., J.Knolle (2026)

To be continued…

Conclusion

$$\kappa_{xy}^{\text{dis}} = -\frac{1}{T}\sum_n \int_{\boldsymbol{p}} \mathcal{E}_n(\boldsymbol{p})^2\, \partial_\varepsilon n_{\text{B}}(\mathcal{E}_n(\boldsymbol{p}), T)\; \Omega^{W\,(n)}_{p_y p_x}(\boldsymbol{p})\, p_x\, \partial_{p_x} \mathcal{E}_n(\boldsymbol{p})$$

Possible picture for thermal Hall effects:

κ_{H} disorder dominated	intrinsic κ_{H}	κ_{H} skew-scattering dominated	→ κ_{L}
PRX 16 (1), 011048 (2026)	PRB 109, 235137 (2024)	PRX 12, 041031 (2022) PRB 106, 245139 (2022)	

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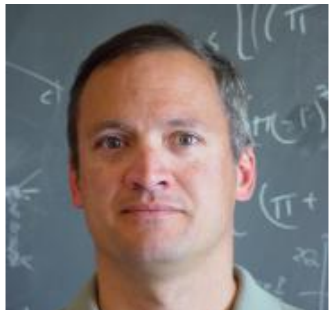

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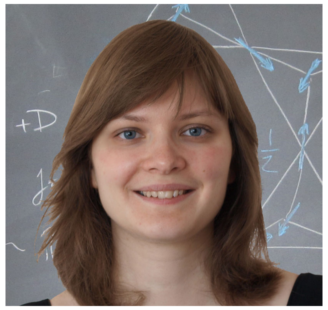

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