

Magnetic toroidics and unconventional responses

From toroidal metals to *S*-type altermagnets and *p*-wave magnets

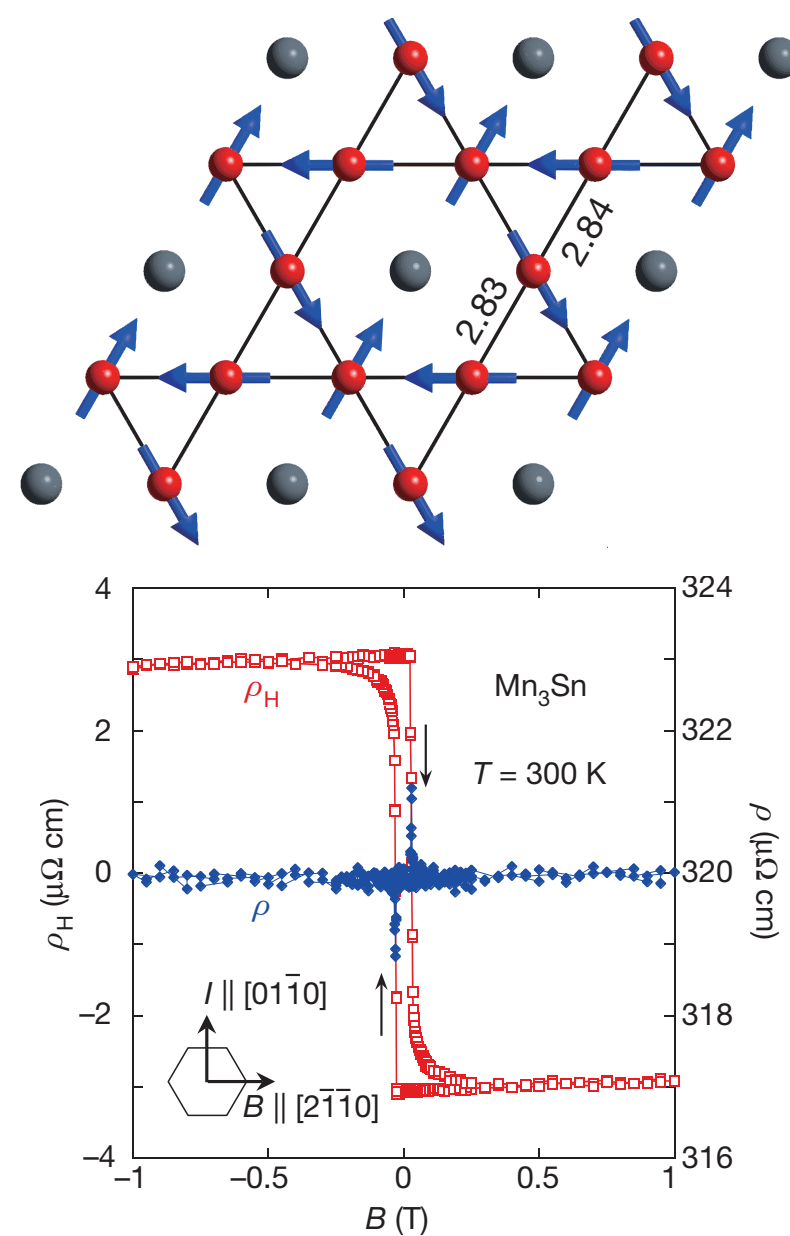
Yukitoshi Motome



Why magnetic toroidics?

- Strong Hall or ME responses can arise even when the net magnetization M is vanishingly small.

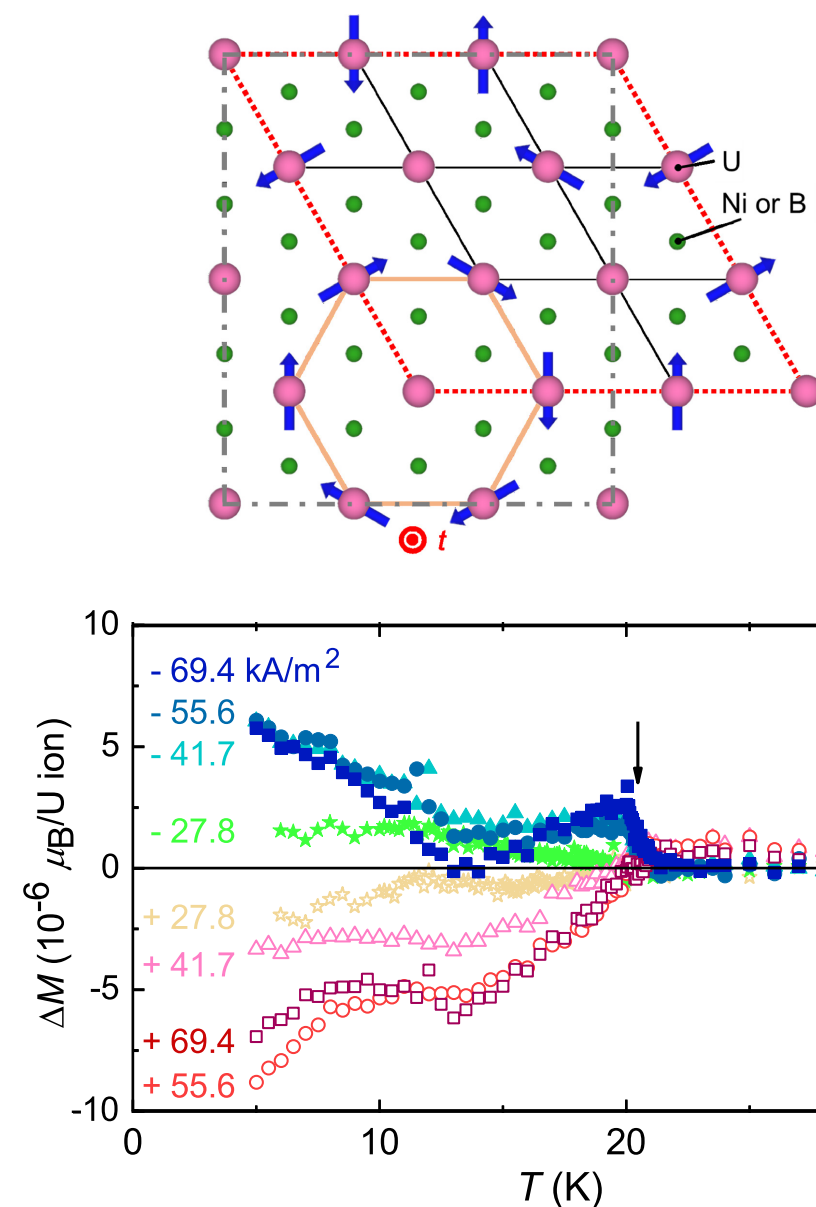
Mn_3Sn



tiny net M , large AHE signals

S. Nakatsuji *et al.*, Nature **527**, 212 (2015)

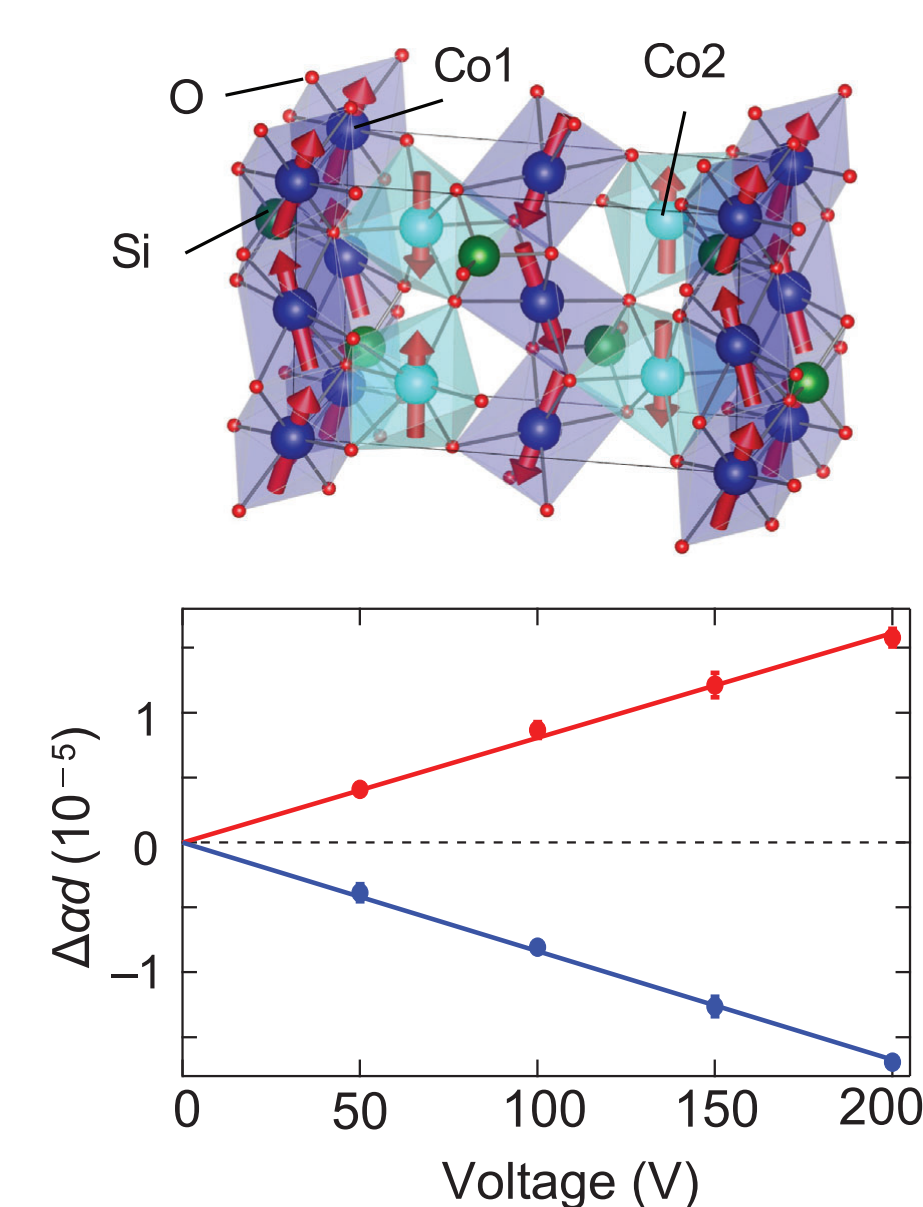
UNi_4B



current-induced magnetization

H. Saito *et al.*, J. Phys. Soc. Jpn. **87**, 033702 (2018)

Co_2SiO_4



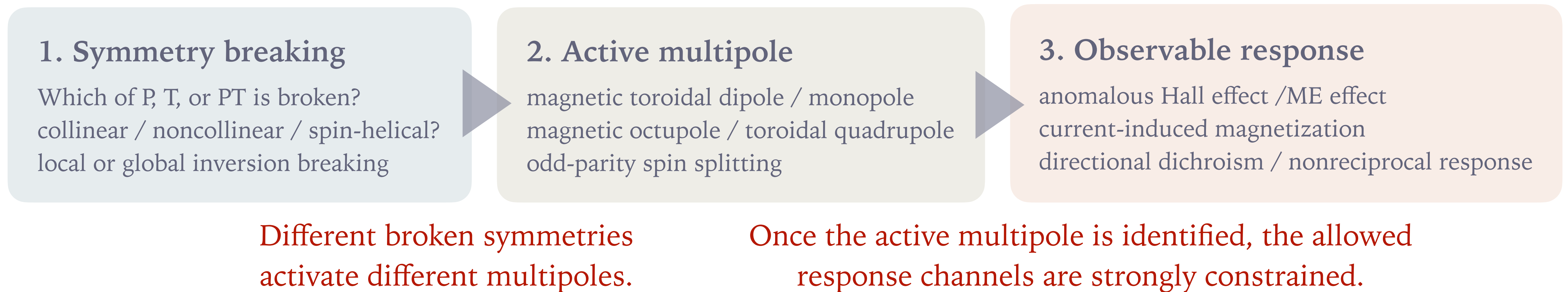
electric-field-induced NDD

T. Hayashida *et al.*, Adv. Mater. **37**, 2414876 (2025)

Need a language beyond ferromagnetism → toroidal and related hidden multipoles

Why magnetic toroidics?

© In compensated magnets, unconventional response follows multipole symmetry—not net M .

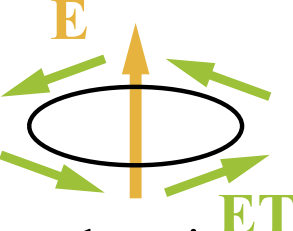
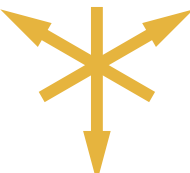
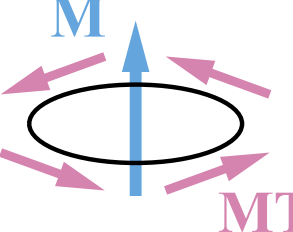
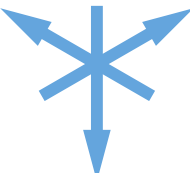
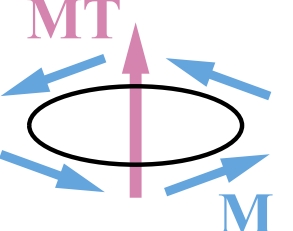
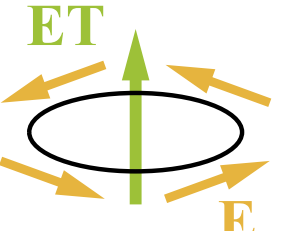


- Mn_3Sn : coplanar AFM $\rightarrow$ hidden octupole $\rightarrow$ large AHE with tiny magnetization
- UNi_4B : vortex-like order $\rightarrow$ magnetic toroidal dipole $\rightarrow$ current-induced magnetization
- Co_2SiO_4 : collinear AFM $\rightarrow$ magnetic toroidal monopole $\rightarrow$ E -induced toroidal response and NDD

Understanding active multipoles provides a key framework for elucidating unconventional responses.

Magnetic toroidal multipoles as a symmetry language

Electronic order can be organized by four multipole families distinguished by P and T.

type	symbol	$\mathcal{P}$	$\mathcal{T}$	monopole	dipole	quadrupole	octupole
Electric (E)	$\mathbb{Q}_{l,m}$	$(-1)^l$ polar	+	 electric charge	 electric polarization	 elasticity	
Magnetic (M)	$\mathbb{M}_{l,m}$	$(-1)^{l+1}$ axial	-	 magnetic charge	 magnetization		
Magnetic toroidal (MT)	$\mathbb{T}_{l,m}$	$(-1)^l$ polar	-	 magnetic polar charge	 electric current		
Electric toroidal (ET)	$\mathbb{G}_{l,m}$	$(-1)^{l+1}$ axial	+	 chiral charge (chirality)	 electric axiality (rotational distortion)		

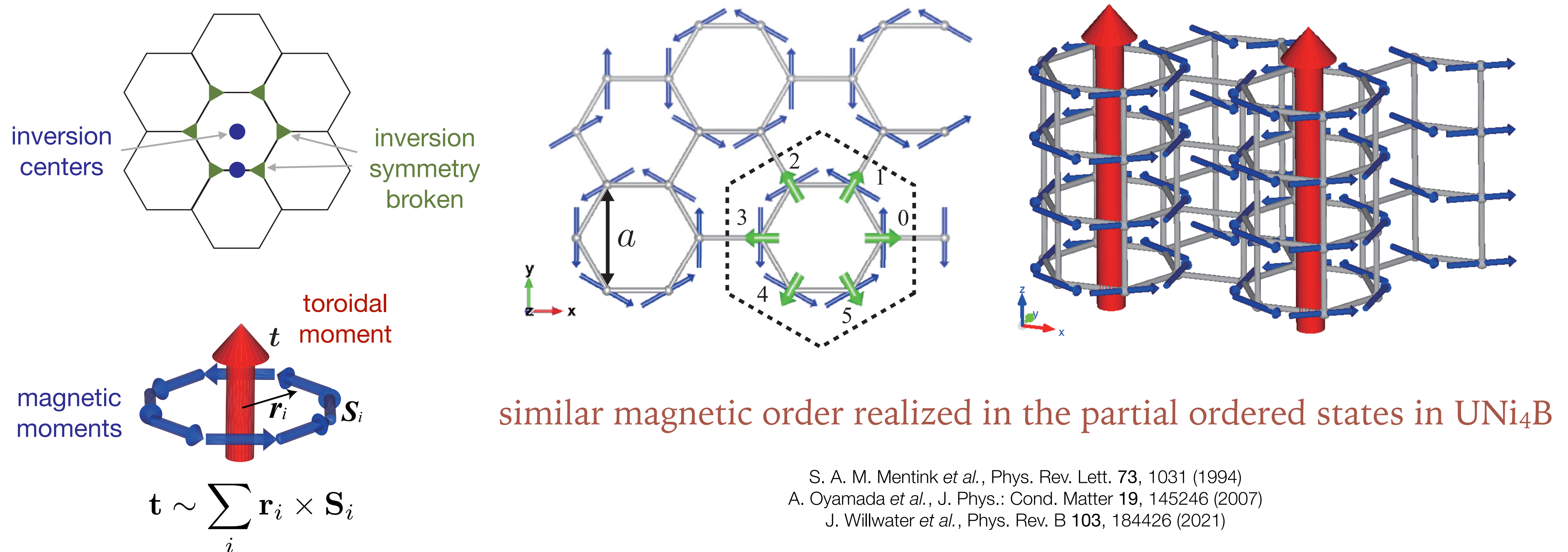
broken symmetry		typical response
T only		anomalous Hall
P only		Rashba / Dresselhaus
P+T		ME / nonreciprocal

Magnetic toroidal multipoles
are especially useful
when both P and T are broken
in compensated magnets.

N. Spaldin, M. Fiebig, and M. Mostovoy, J. Phys.: Condens. Matter **20**, 434203 (2008)
X. Xu, F.-T. Huang, and S.-W. Cheong, J. Phys.: Condens. Matter **36**, 203002 (2024)

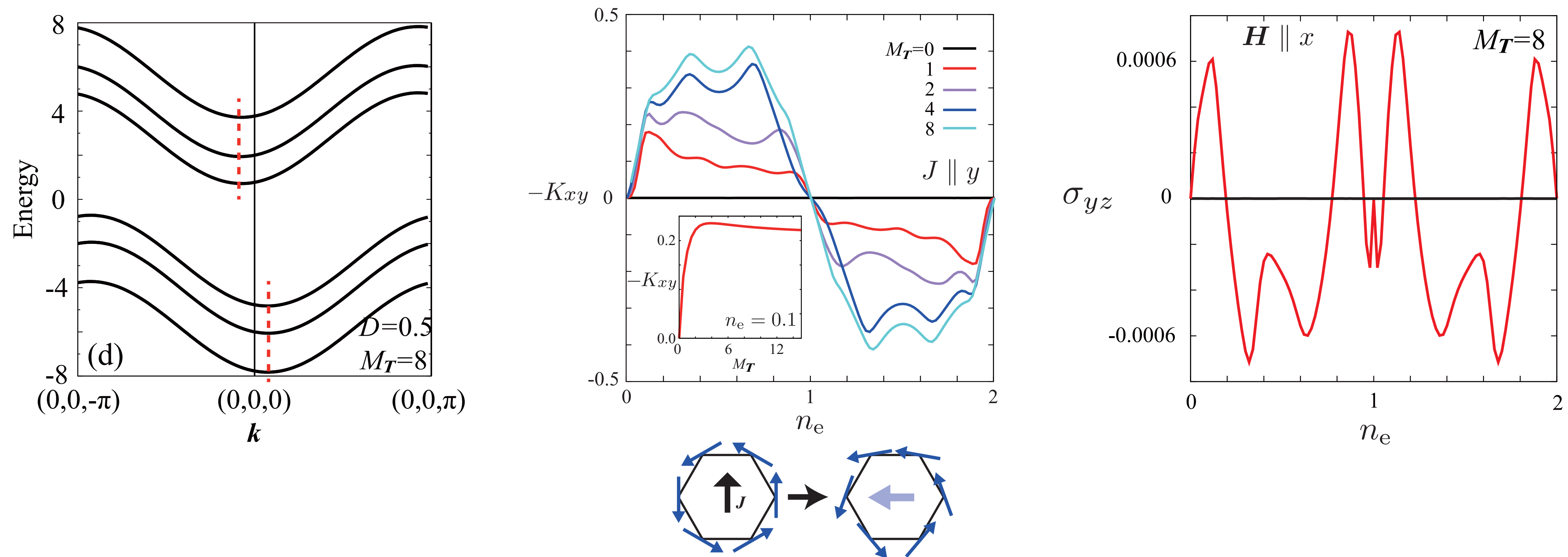
Canonical example: toroidal dipole in a metal

◎ vortex-like magnetic texture on local P broken sites → magnetic toroidal dipole order



Canonical example: toroidal dipole in a metal

- toroidal dipole order $\rightarrow$ band bottom shift, current-induced magnetization (Edelstein effect), anisotropic magneto-transport, characteristic ME responses, nonlinear Hall response, ...



Plan of this talk

- Magnetic toroidal multipoles provide an effective handle for controlling unconventional responses in compensated magnets. Rather than being merely a characteristic magnetic texture, it functions as a generator of responses.

○ Two interesting extensions of this concept:



magnetic toroidal monopole and E -induced responses in antiferromagnetic olivines

T breaking collinear AFM + local P breaking $\rightarrow$ magnetic toroidal monopole
electric-field-induced magnetic toroidal dipole, nonlinear ME responses



commensurate spin-spiral metal and unconventional AHE

p -wave type odd-parity spin splitting + SOC (P breaking) + magnetic field (T breaking)
 $\rightarrow$ gapped spin nodal lines $\rightarrow$ spiral-induced AHE

magnetic toroidal monopole and E -induced responses

Y. Kato, T. Hayashida, K. Matsumoto, T. Kimura, and Y. Motome, Phys. Rev. B **113**, 094411 (2026)

What is magnetic toroidal monopole?

● “magnetic polar charge”: P-even T-odd scalar

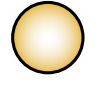
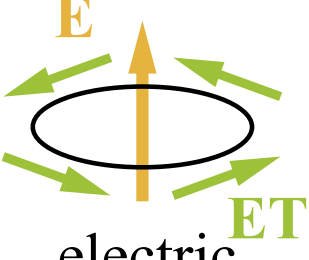
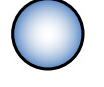
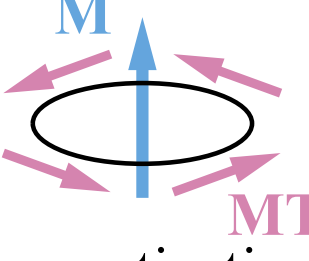
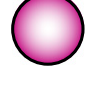
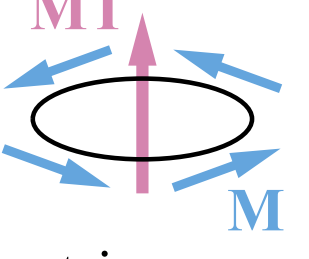
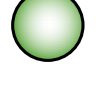
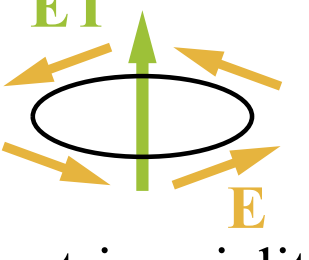
cf. electric monopole (electric charge): P-even T-even
 magnetic monopole (magnetic charge): P-odd T-odd
 electric toroidal monopole (chirality): P-odd T-even

● It can couple to both an electric field $\mathbf{E}$ (P-odd and T-even) and a magnetic toroidal moment $\mathbf{t}$ (P-odd and T-odd).

➡ Free energy includes a term $\sim T_0 \mathbf{E} \cdot \mathbf{t}$

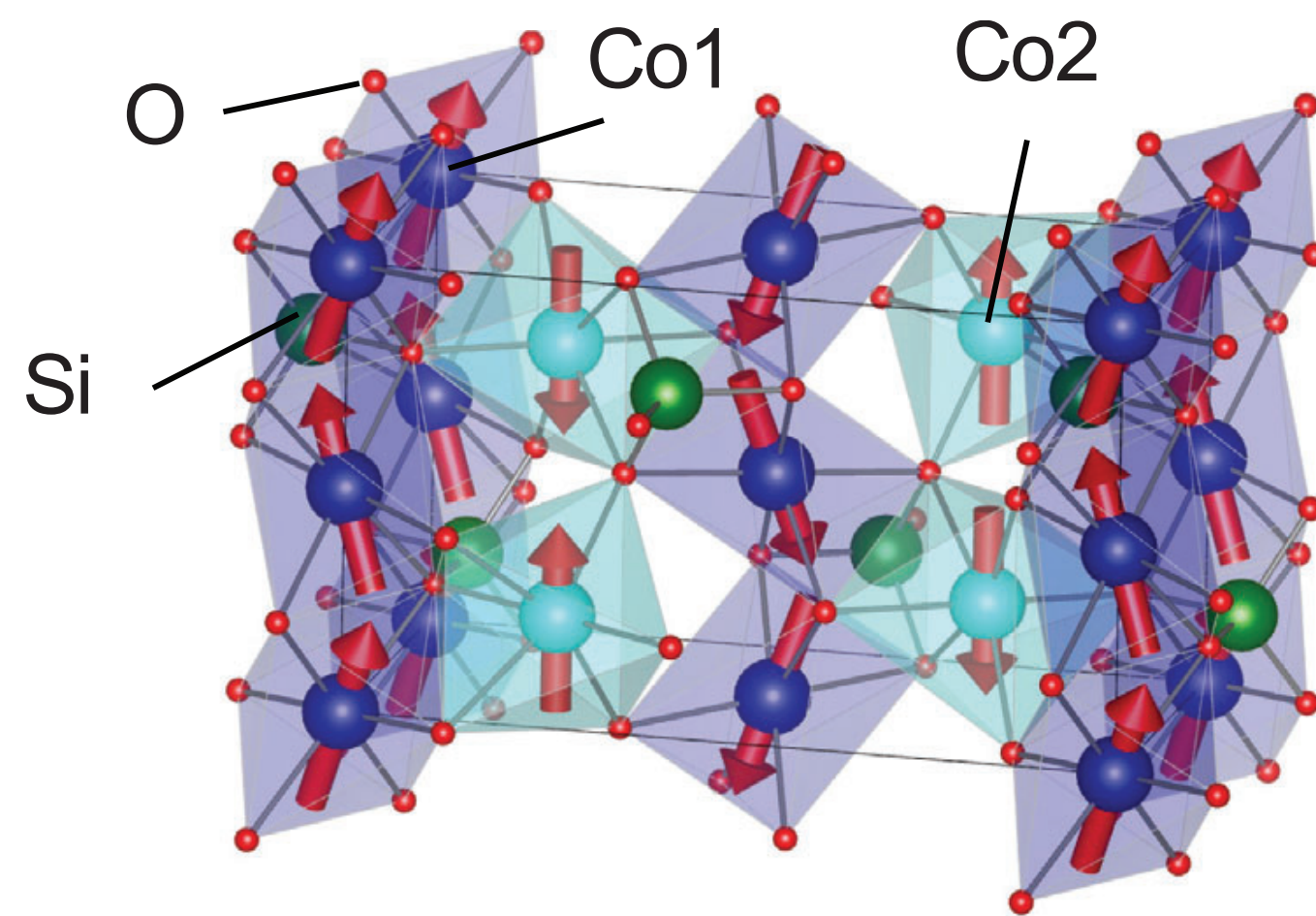
T_0 : magnetic toroidal monopole

In the presence of T_0 , $\mathbf{E}$ can induce $\mathbf{t}$.
 (electro-toroidic effect)

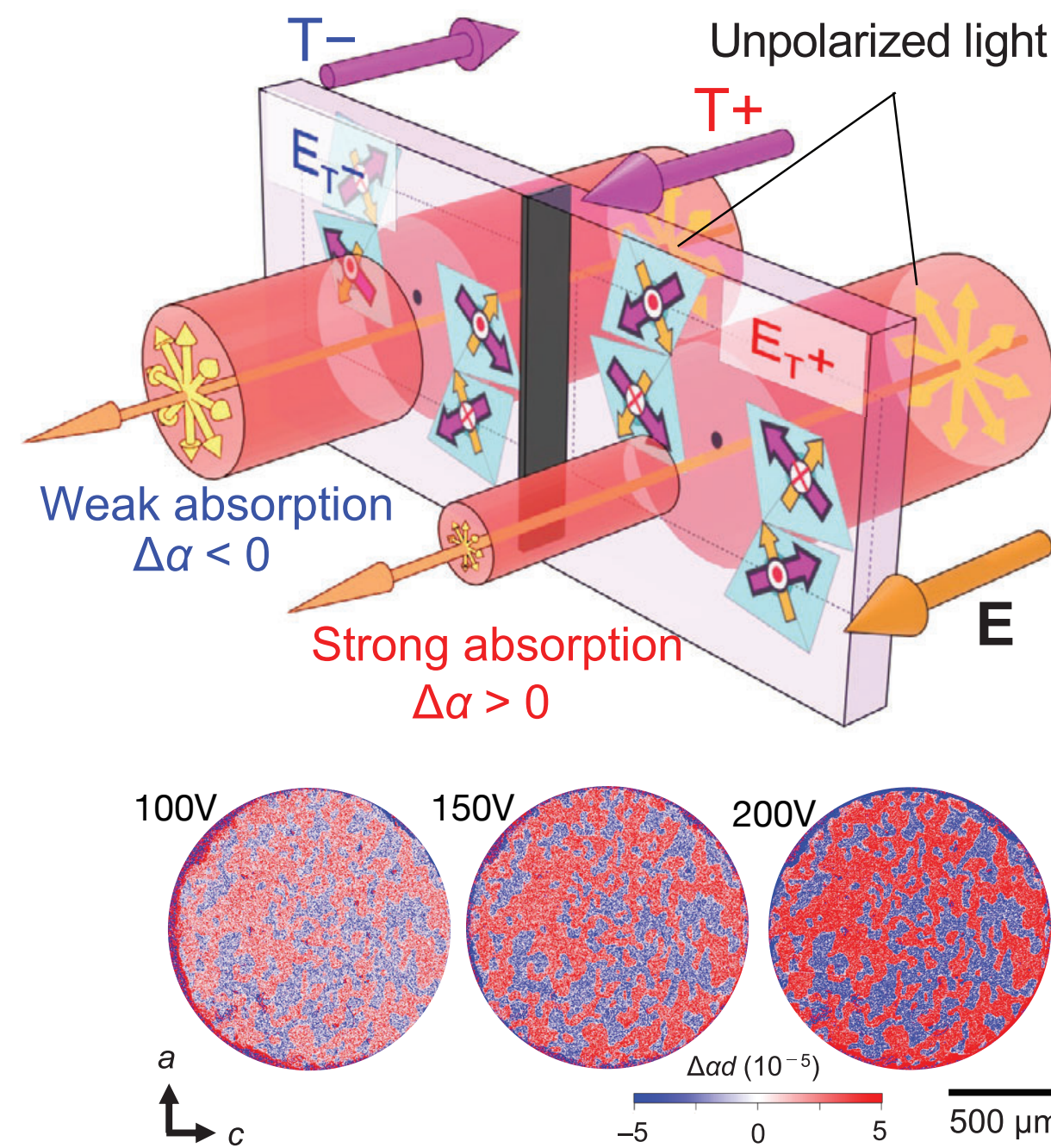
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Electro-toroidic effect in Co_2SiO_4

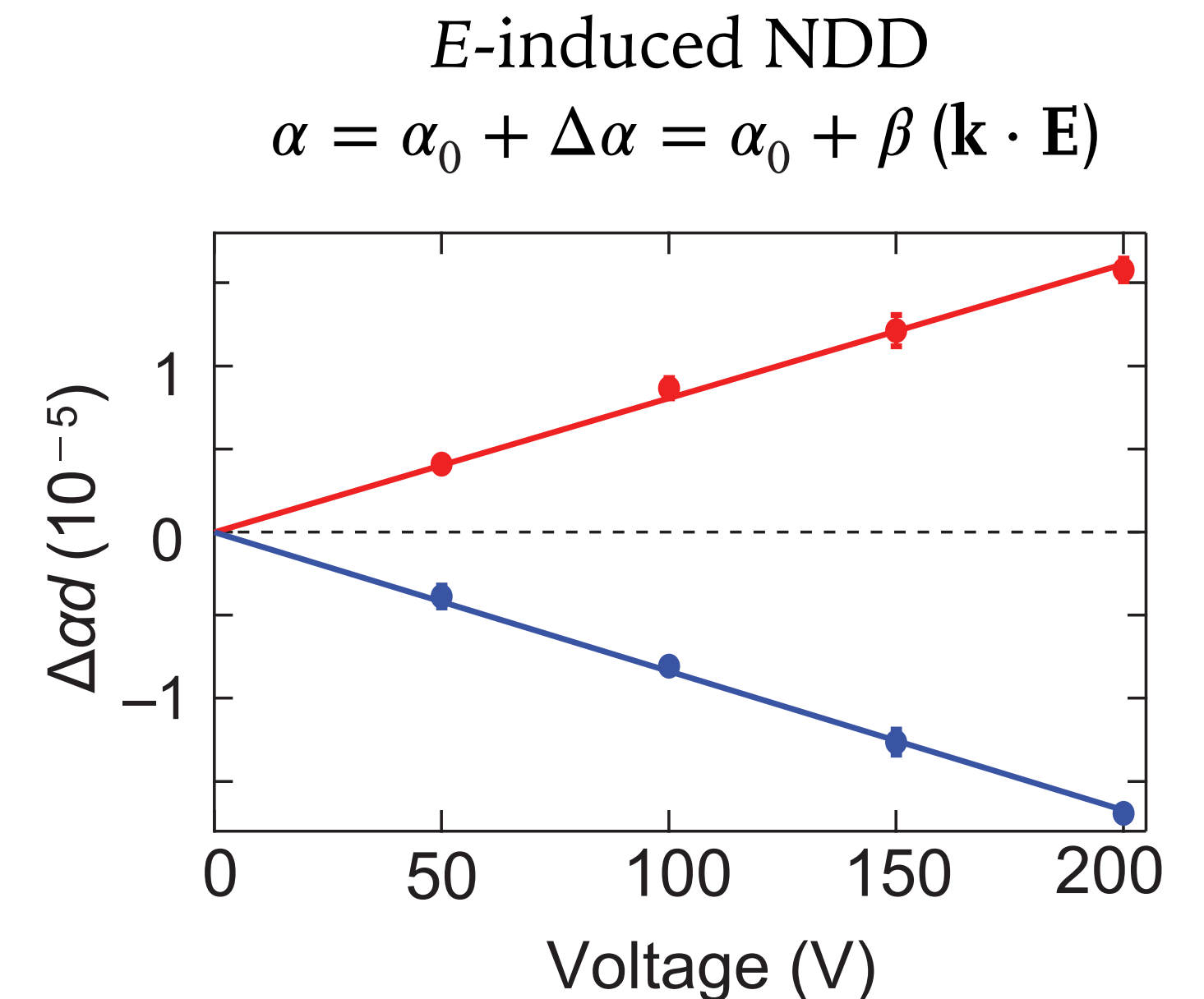
- AFM olivine Co_2SiO_4 exhibits nonreciprocal directional dichroism (NDD) in an applied $\mathbf{E}$.



T-broken antiferromagnet
(magnetic point group mmm ,
which allows a finite T_0)



T. Hayashida *et al.*, Adv. Mater. 37, 2414876 (2025)



suggesting an $\mathbf{E}$ -induced magnetic toroidal dipole from a magnetic toroidal monopole
— *microscopic origin?*

Model

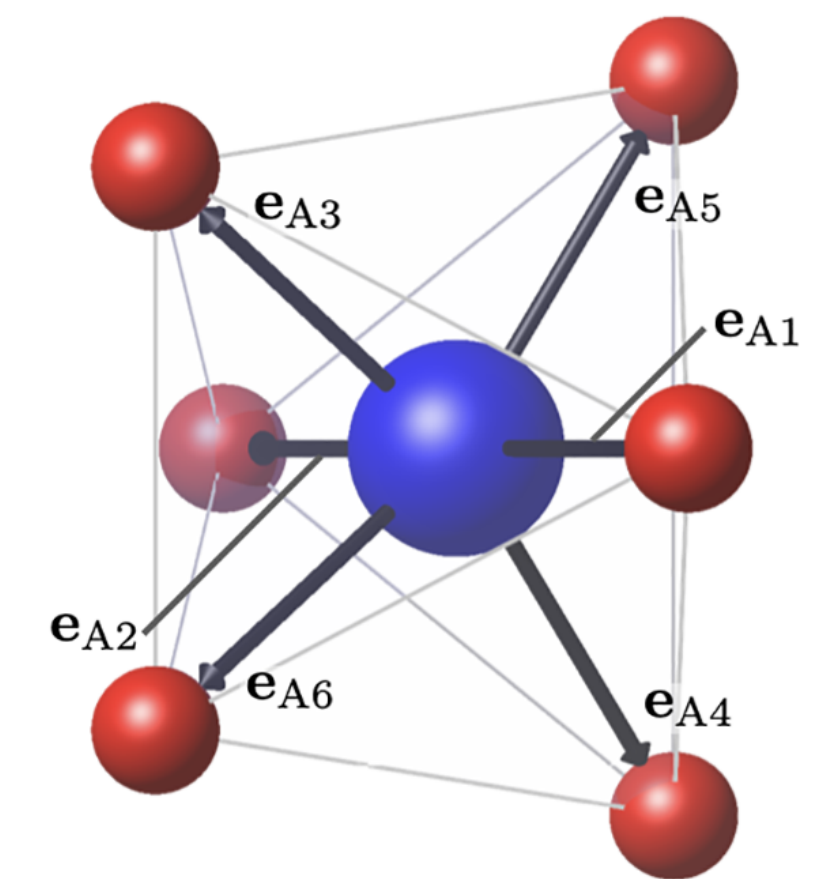
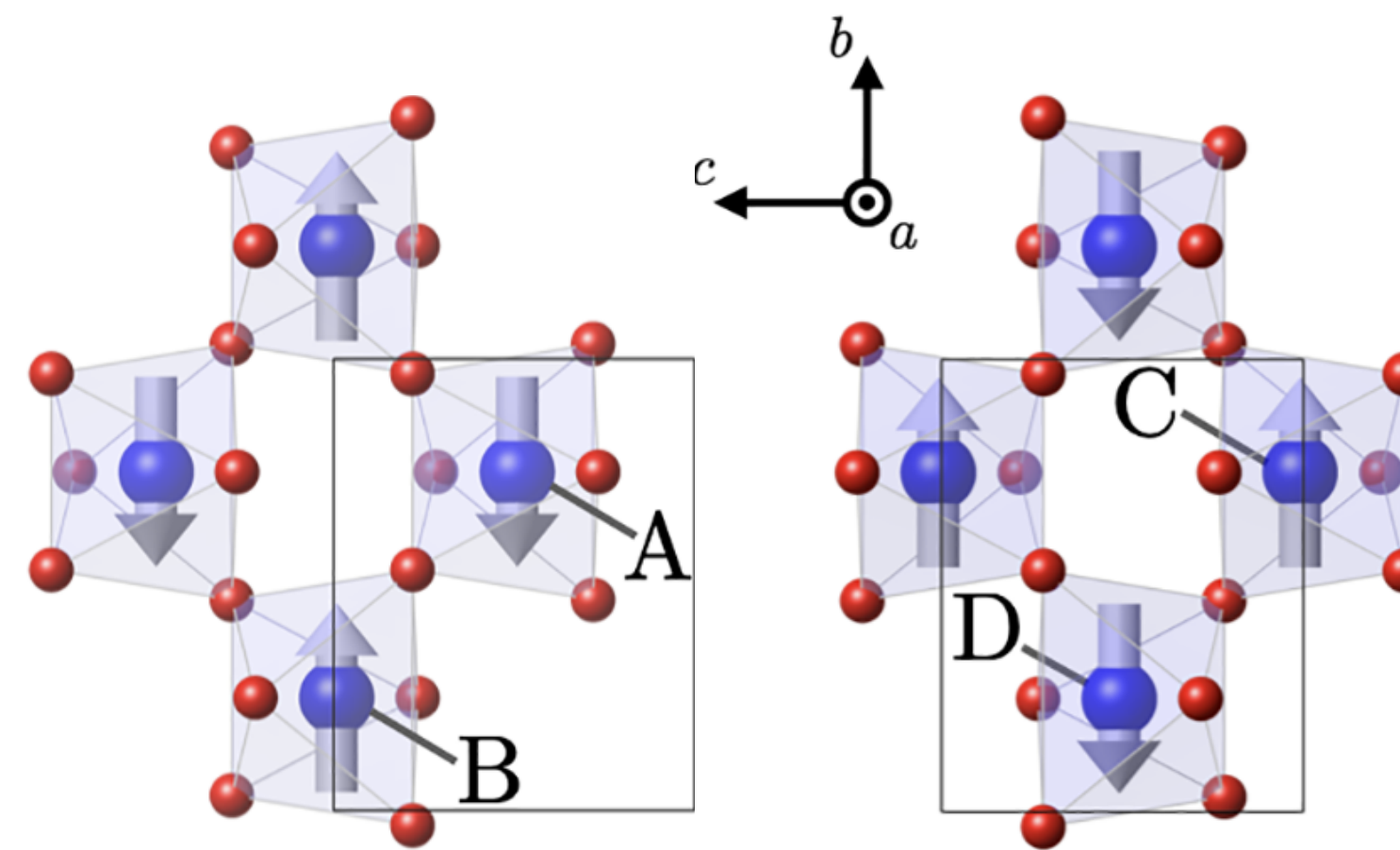
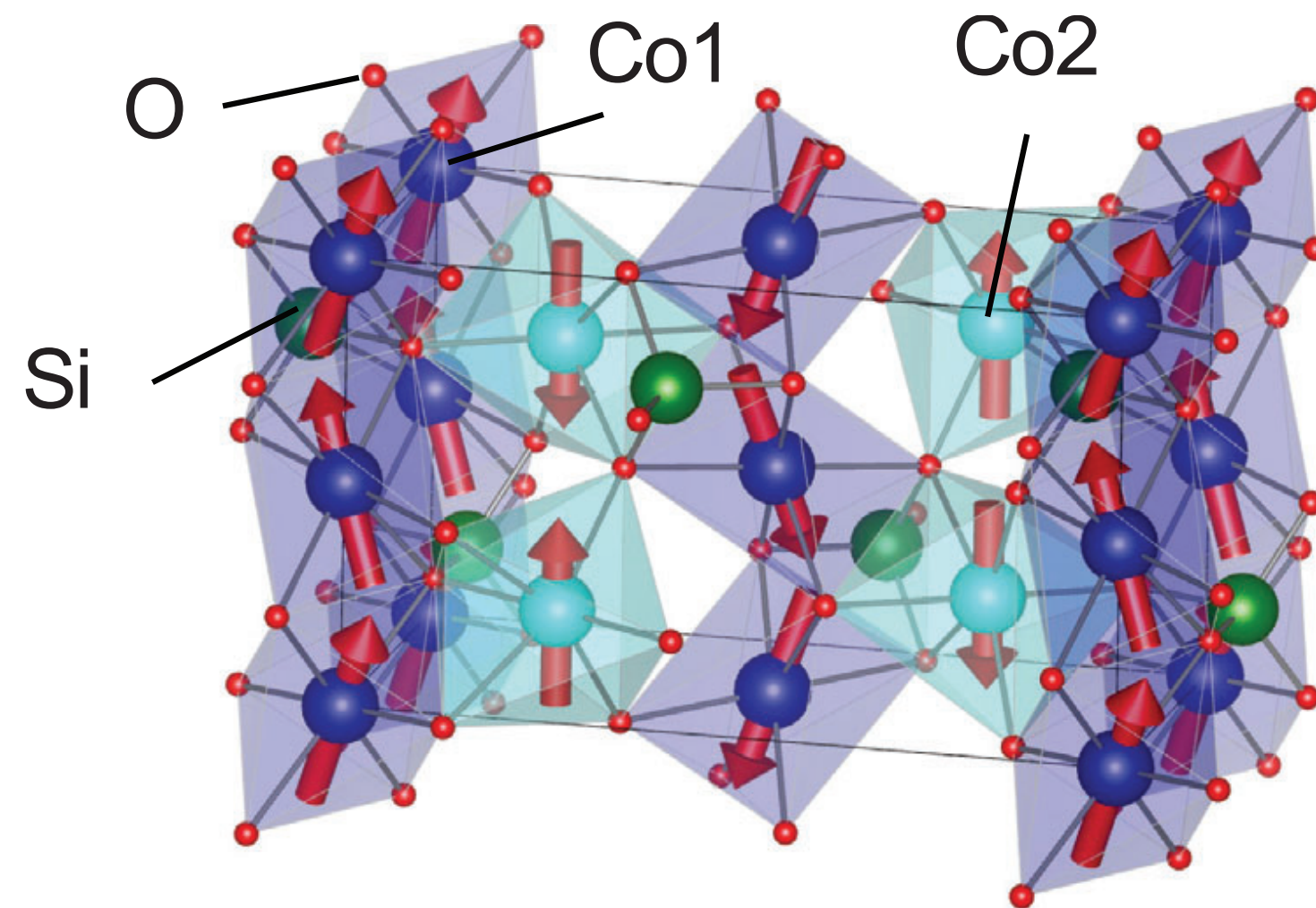
◎ two Co sites in the unit cell

- Co1: local P preserved, 1D zigzag chain of edge-sharing CoO_6 , non-collinear AFM
- Co2: **local P broken**, 2D square-like lattice of corner-sharing CoO_6 , **collinear AFM**

➔ spin model for ME-active Co2 sites, assuming the d - p hybridization mechanism

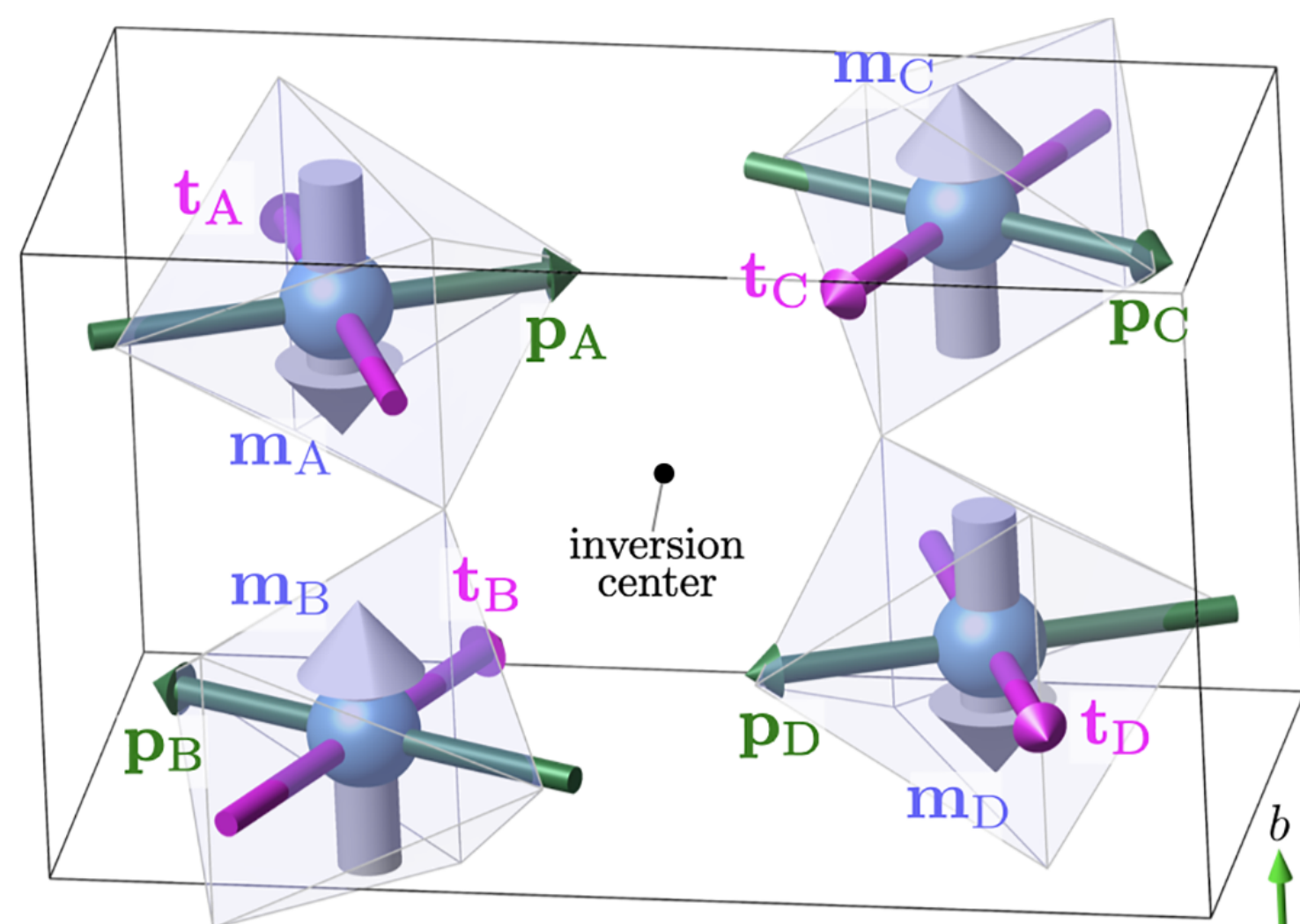
$$\mathcal{H} = J \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}'} - A \sum_{\mathbf{r}} (S_{\mathbf{r}}^b)^2 - \mathbf{E} \cdot \sum_{\mathbf{r}} \mathbf{p}_{\mathbf{r}} \quad \mathbf{p}_{\mathbf{r}} = \sum_{\mu} \lambda_{\alpha(\mathbf{r})\mu} (\mathbf{S}_{\mathbf{r}} \cdot \hat{\mathbf{e}}_{\alpha(\mathbf{r})\mu})^2 \hat{\mathbf{e}}_{\alpha(\mathbf{r})\mu} \quad J = 1, A = 0.1, \lambda_{\alpha\mu} = 1$$

T. Arima *et al.*, J. Phys. Soc. Jpn. 76, 073702 (2007)



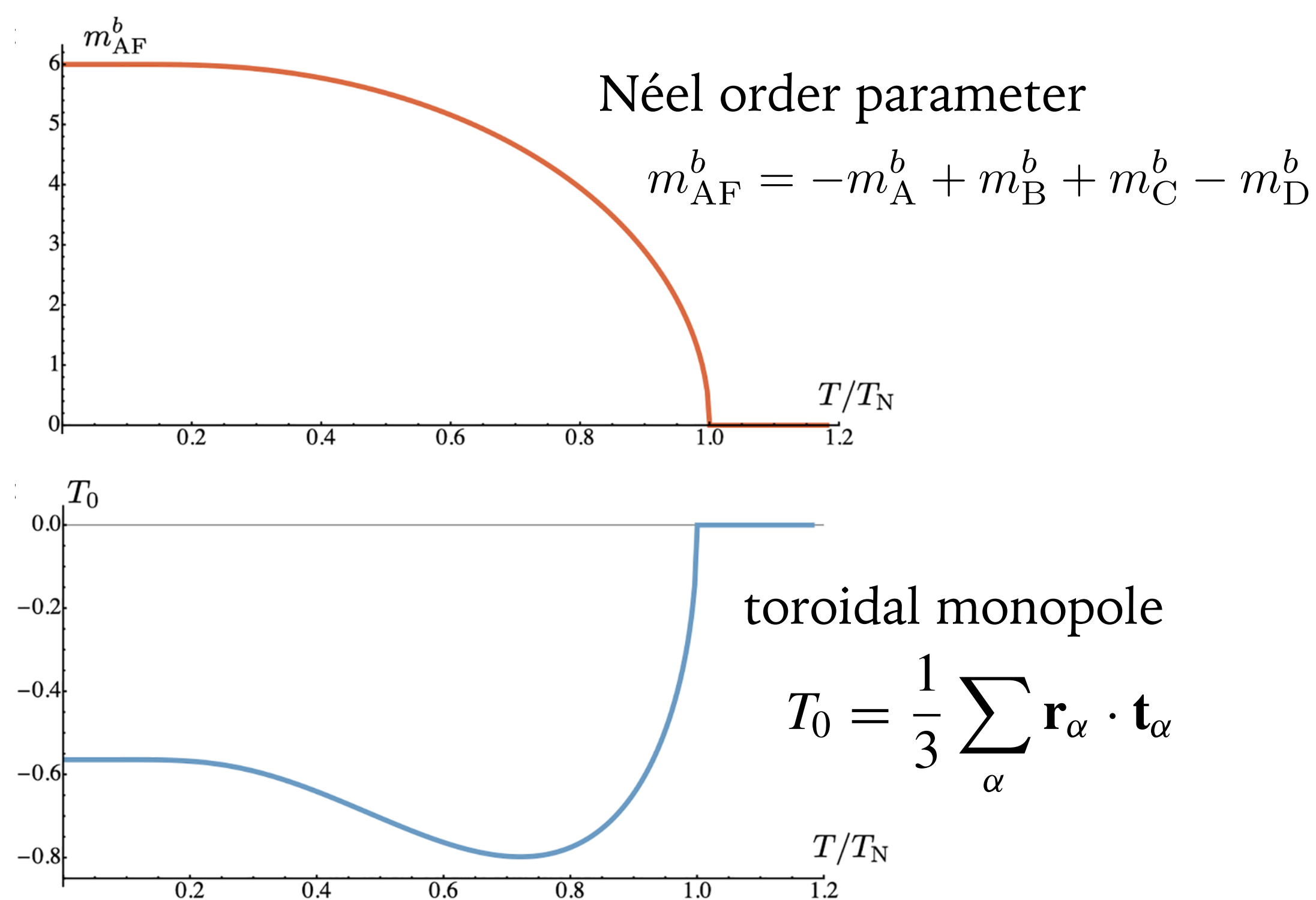
Magnetic toroidal monopole

- For $\mathbf{E}=0$, a collinear AFM order sets in below T_N : each Co2 site acquires a toroidal moment, and their spatial arrangement yields a finite scalar toroidal monopole T_0 .



local toroidal moment: $\mathbf{t}_\alpha = \mathbf{p}_\alpha \times \mathbf{m}_\alpha$

K. Kimura and T. Kimura, Phys. Rev. Lett. 132, 036901 (2024)
T. Hayashida *et al.*, Adv. Mater. 37, 2414876 (2025)

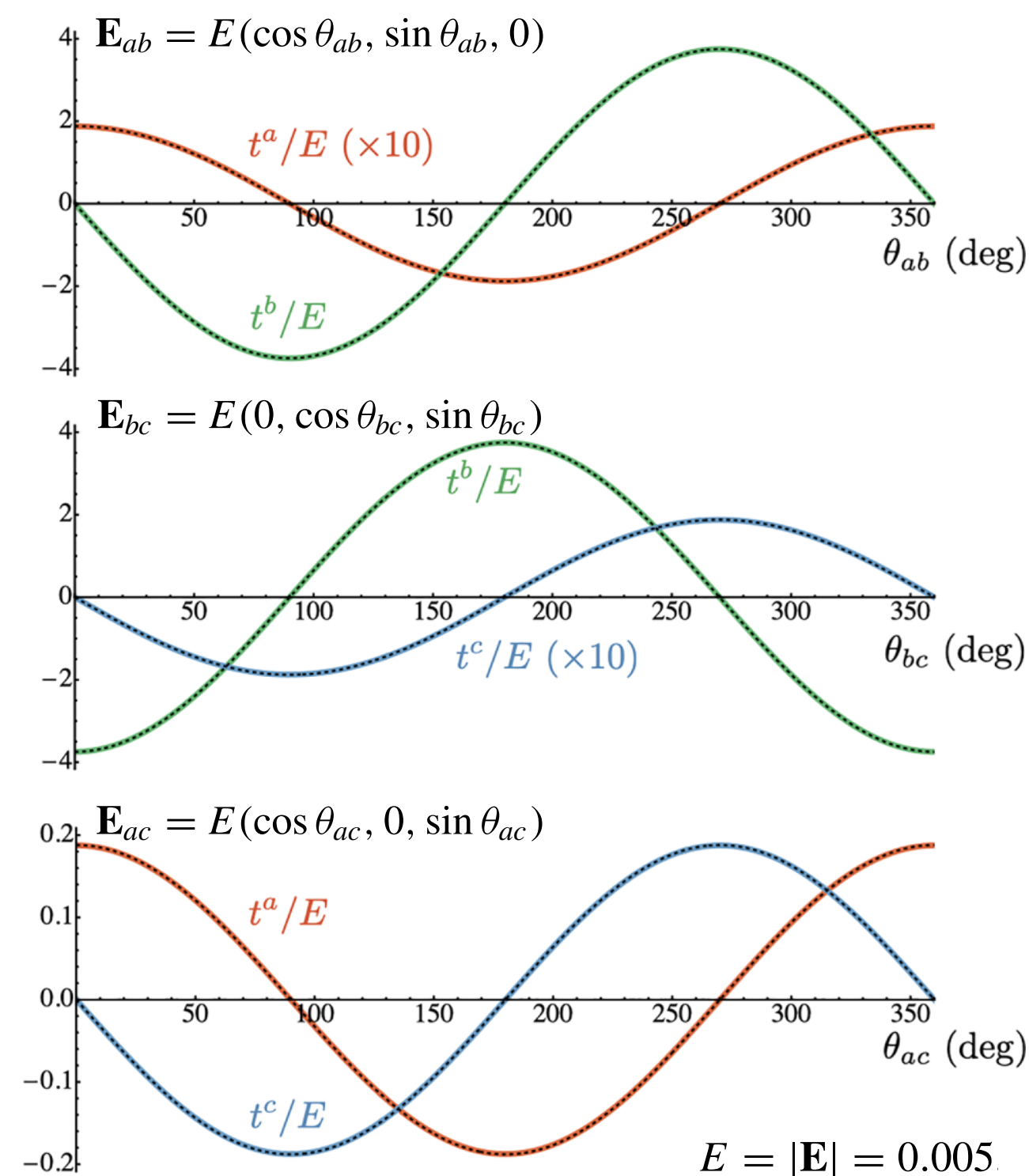


Magnetic toroidal monopole originates from the collinear AFM at local P symmetry broken sites.

E-induced magnetic toroidal moment

© $T_0 + \mathbf{E} \rightarrow$ magnetic toroidal moment: $t^\mu = X_{\mu\nu}^{tE} E^\nu$

H. Schmid, Ferroelectrics 252, 41 (2001); J. Phys.: Condens. Matter 20, 434201 (2008).



linear ME tensor: diagonal

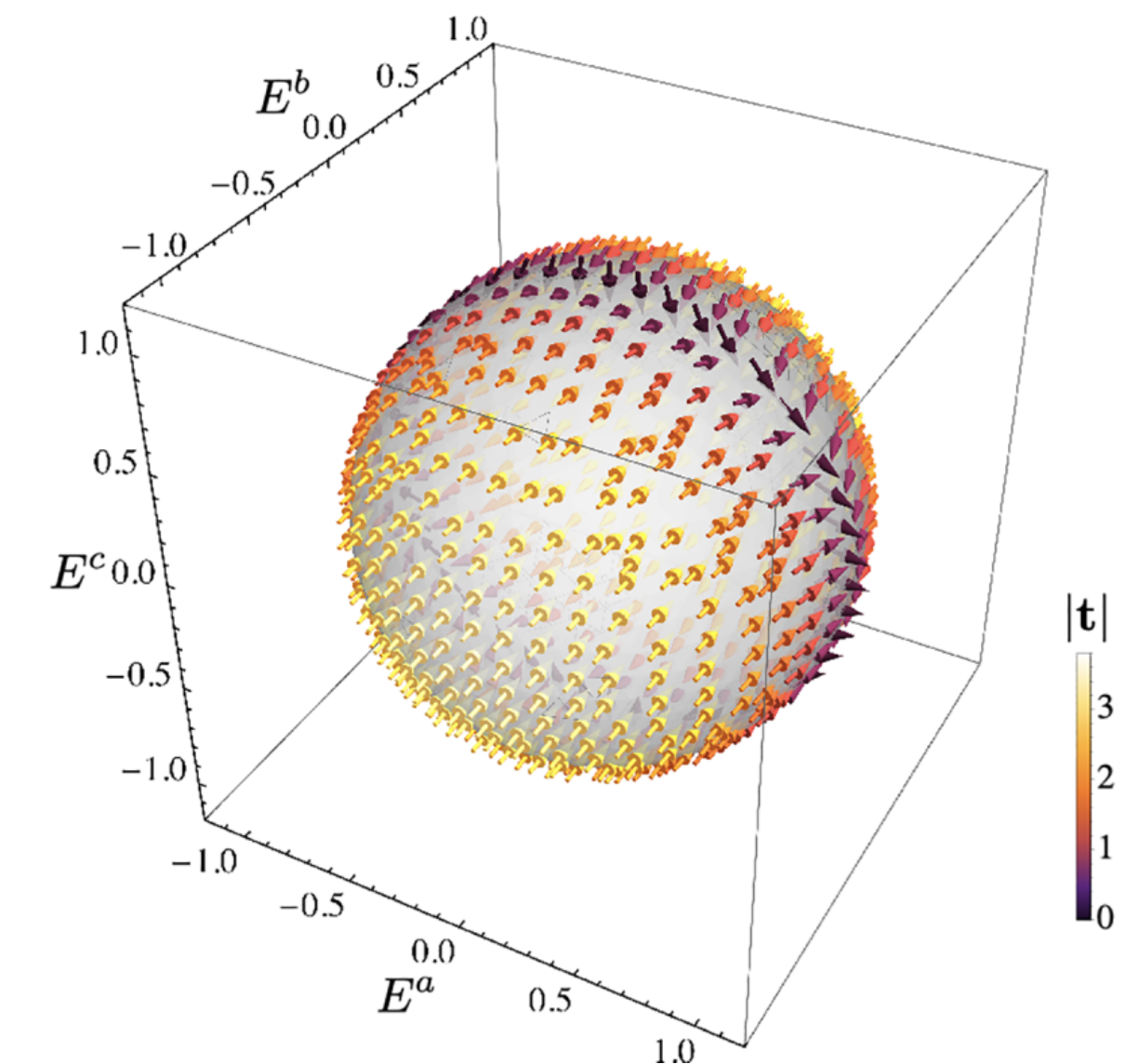
$$X_{\mu\nu}^{tE} = \begin{bmatrix} X_{aa} & 0 & 0 \\ 0 & X_{bb} & 0 \\ 0 & 0 & X_{cc} \end{bmatrix}$$

$$X_{aa} = X_{cc} = 0.188$$

$$X_{bb} = -3.748$$

X_{aa} and X_{cc} become identical due to symmetry and Onsager reciprocity.

E-direction dependence of the induced toroidal-moment response

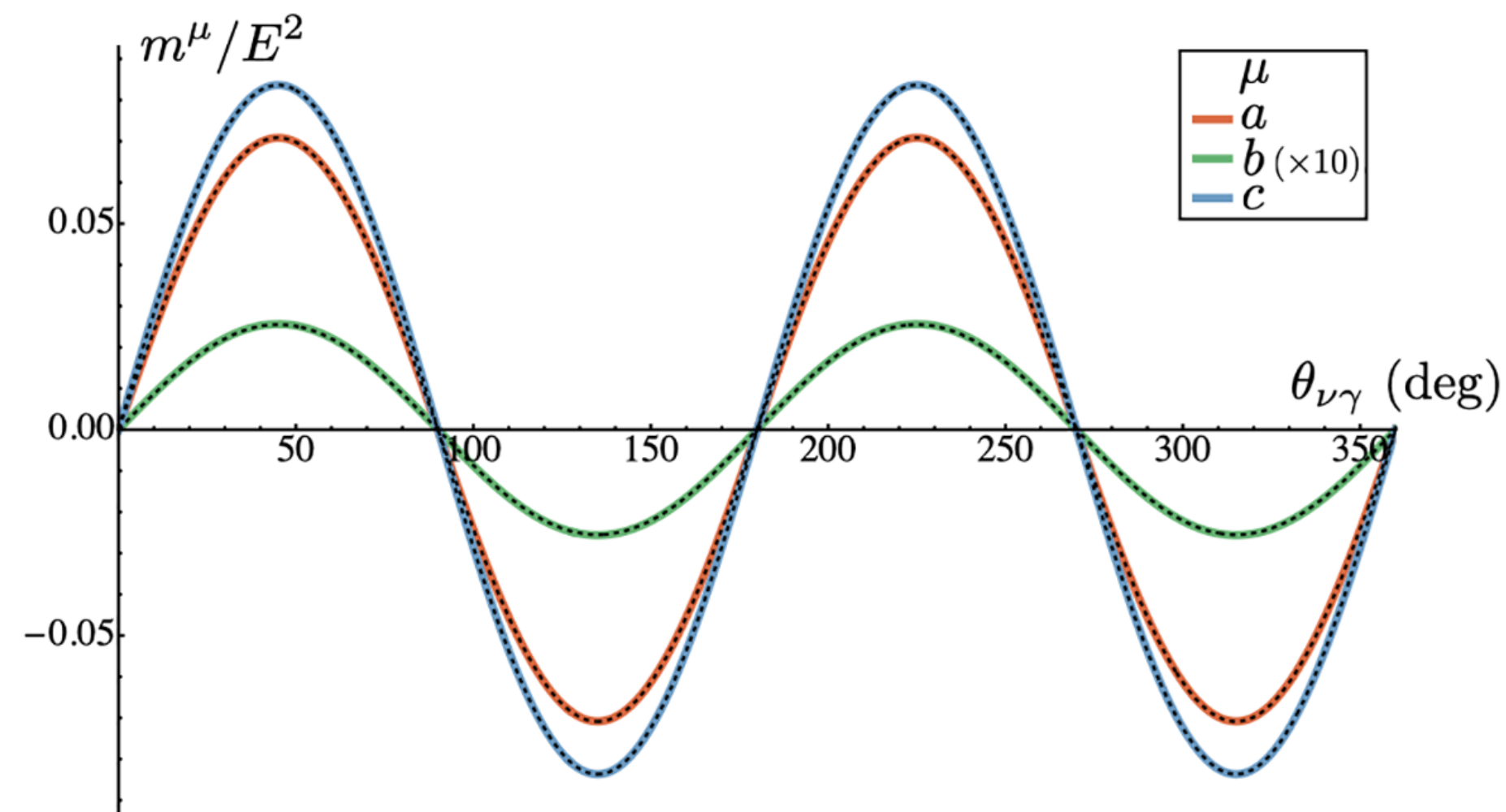


The induced toroidal response is highly anisotropic and strongest along the Néel-vector direction, providing the microscopic understanding of NDD observed in experiments.

Nonlinear ME response via E-induced $\mathbf{t}$

◎ E-induced $\mathbf{t} \rightarrow$ asymmetric ME tensor $-(\mathbf{E}) \rightarrow$ uniform magnetization $\mathbf{m} \perp \mathbf{E}, \mathbf{t}$

N. A. Spaldin, M. Fiebig, and M. Mostovoy, J. Phys.: Condens. Matter 20, 434203 (2008)



$$m^\mu = X_{\mu\nu\gamma}^{mEE} E^\nu E^\gamma$$

only $\mu \neq \nu \neq \gamma \neq \mu$ components are nonzero

$$X_{abc}^{mEE} = X_{acb}^{mEE} = 0.1417$$

$$X_{bca}^{mEE} = X_{bac}^{mEE} = 0.0051 \rightarrow m^a \sim m^c \gg m^b$$

$$X_{cab}^{mEE} = X_{cba}^{mEE} = 0.1673$$

E-induced toroidal moment results in a transverse even-order *E*-induced magnetization.

This antisymmetric toroidal-type ME effect awaits experimental verification.

NB) magnetic point group *mmm*: T and PT broken $\rightarrow$ S-type altermagnet, which allows transverse even-order E-induced $\mathbf{M}$

S.-W. Cheong and F.-T. Huang, npj Quantum Mater. 10, 38 (2025).

Similar responses are found in other AFM olivines, such as Fe_2GeS_4 , Fe_2SiO_4 , Mn_2GeO_4 , Mn_2SiS_4 , ...

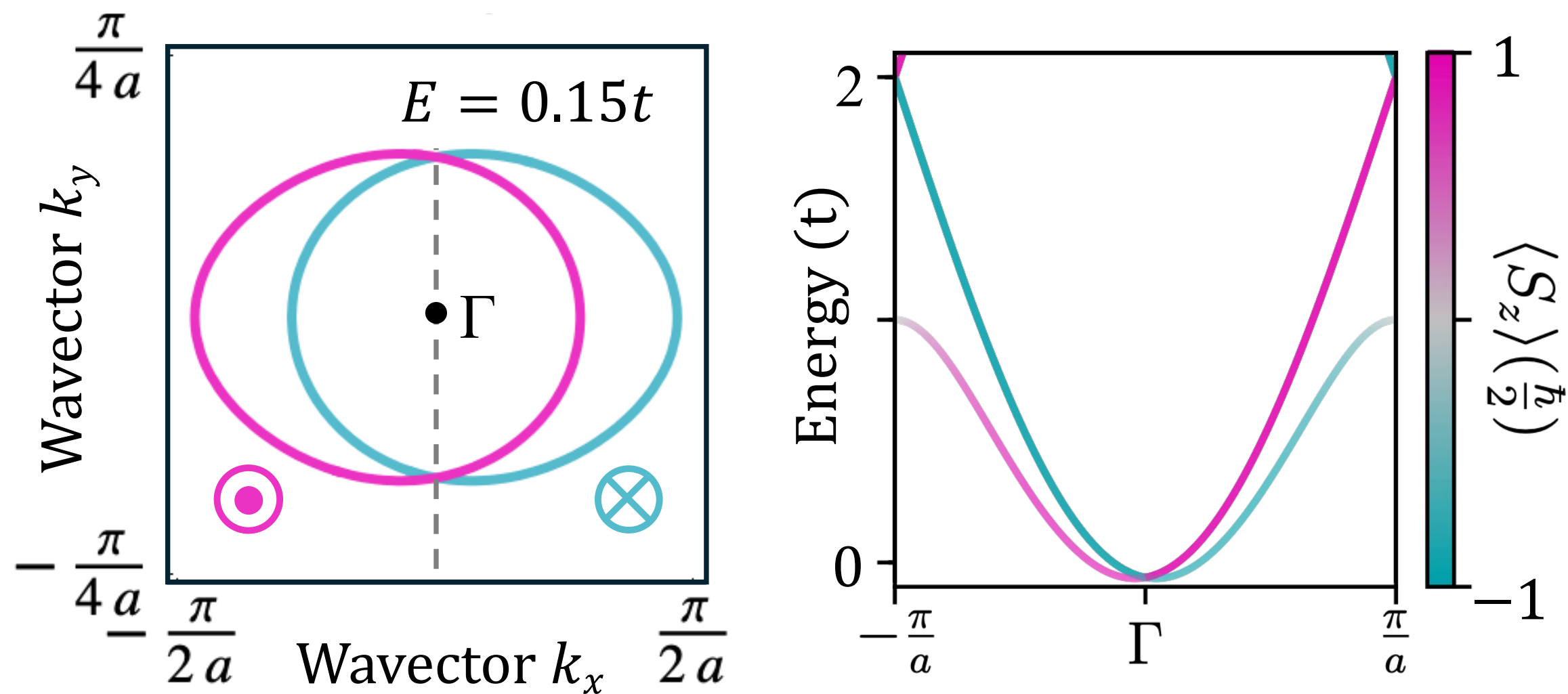
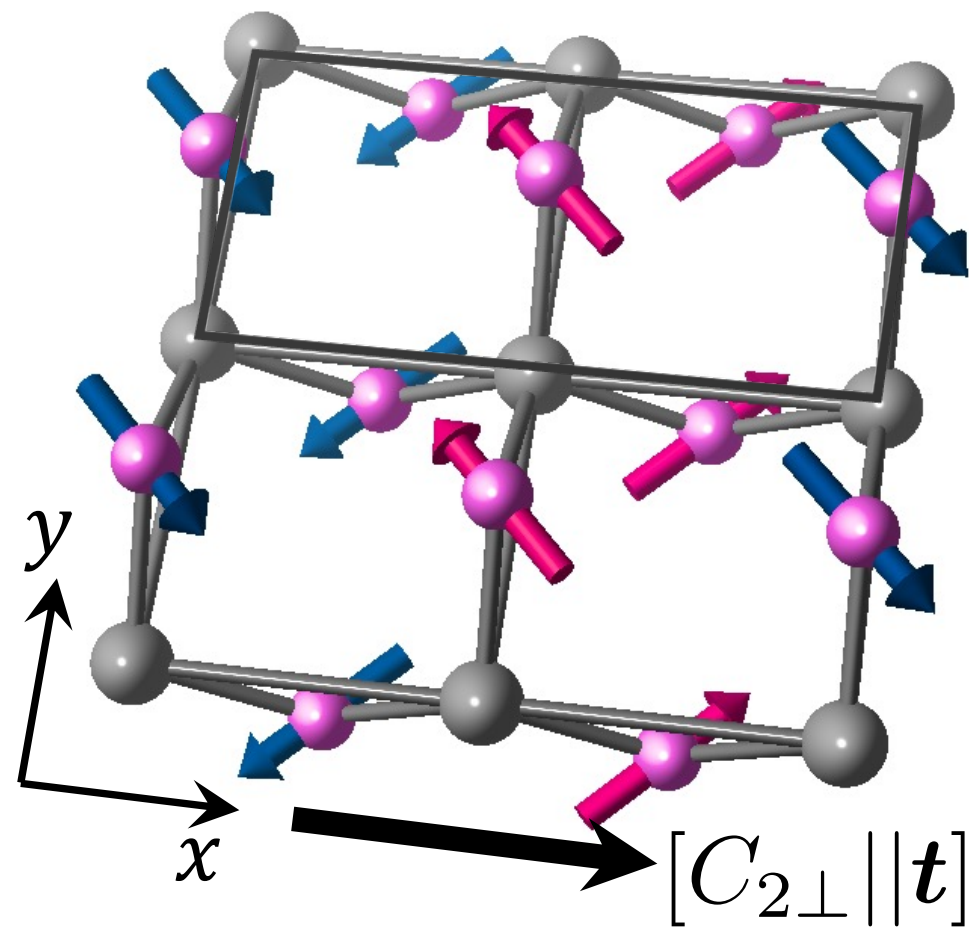
commensurate spin-spiral metal and unconventional AHE

S. Okumura, M. M. Moritz, and Y. Motome, preprint (arXiv:2512.14071)

R. Yamada, M. T. Birch, P. R. Baral *et al.*, Nature **646**, 837 (2025)

Why spiral magnets?

- Coplanar spiral magnetic textures can activate odd-parity multipoles, including magnetic toroidal dipoles, within the broader multipole framework.
- Among them, a commensurate spin spiral, which is P-odd but T-even with spatial translation by magnetic period, can generate odd-parity spin splitting in a metal: *p-wave magnet*.

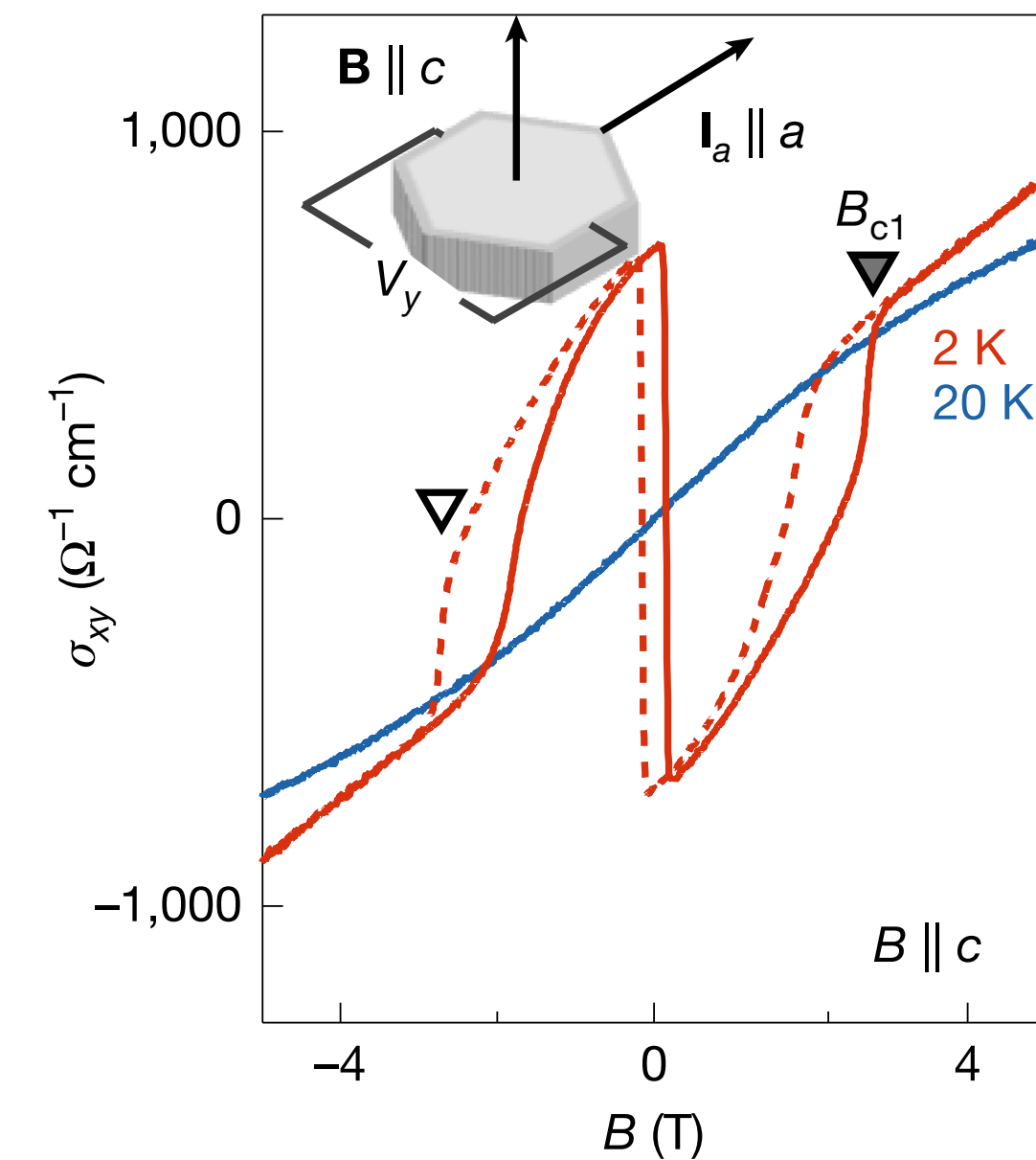
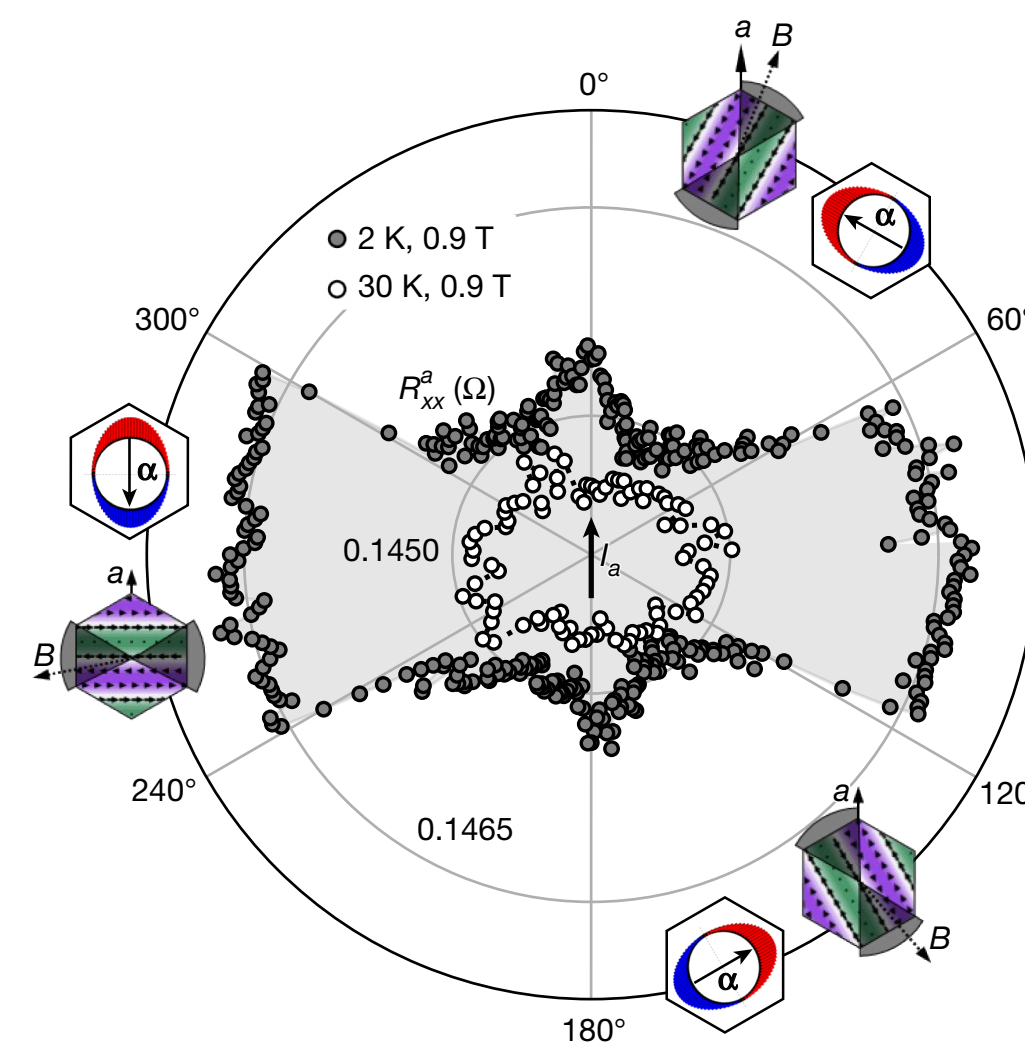
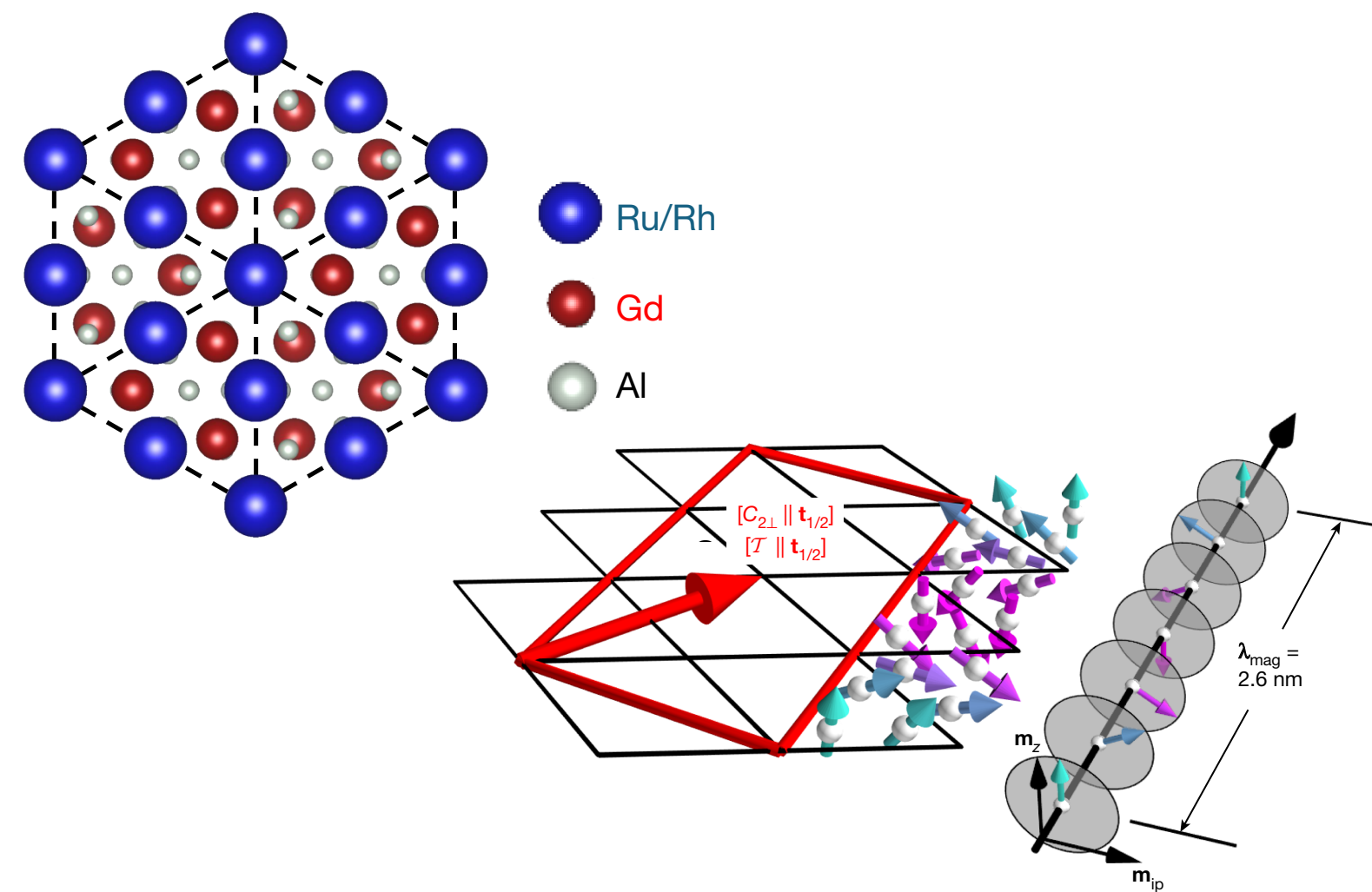


A. B. Hellenes *et al.*, arXiv:2309.01607

Spiral magnets are toroidal-adjacent odd-parity systems — *unconventional responses?*

p -wave magnets

- Several candidates have been reported: CeNiAsO [A. B. Hellenes *et al.*, arXiv:2309.01607; H. Zhou *et al.*, arXiv:2509.07351], **NiI₂** [Q. Song *et al.*, Nature 642, 64 (2025)], and **Gd(Ru_{1- δ} Rh _{δ})₄Al₁₂** [R. Yamada, M. T. Birch, P. R. Baral *et al.*, Nature 646, 837 (2025)].
- An unconventional AHE was reported in Gd(Ru_{1- δ} Rh _{δ})₄Al₁₂.



R. Yamada, M. T. Birch, P. R. Baral *et al.*, Nature 646, 837 (2025)

giant AHE with peculiar field dependence — *microscopic origin?*

Model

◎ spin-charge coupled model with a commensurate spin spiral

cf. B. Brekke *et al.*, Phys. Rev. Lett. 133, 236703 (2024)

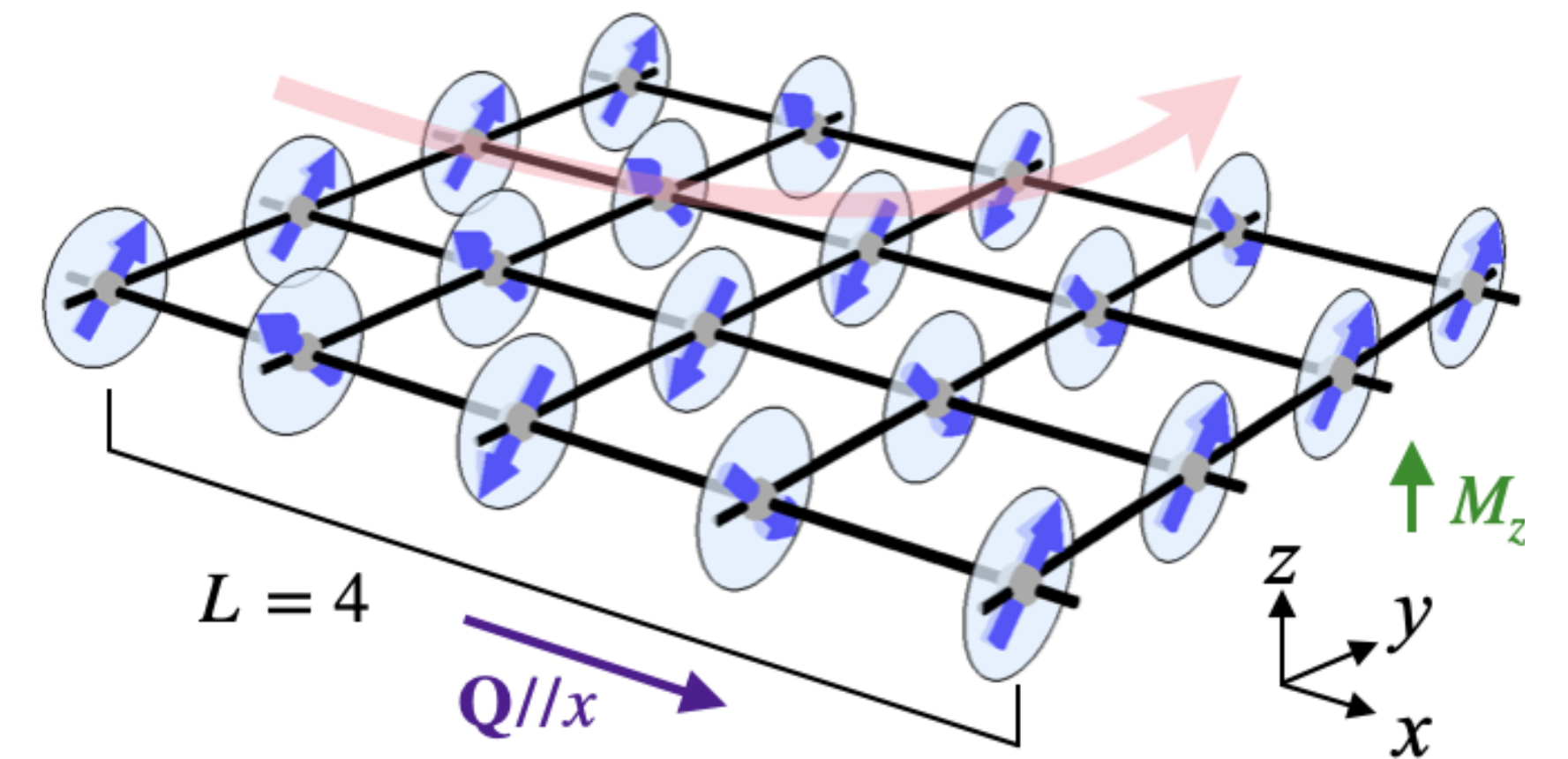
$$\mathcal{H} = -t \sum_{\langle lj \rangle s} c_{ls}^\dagger c_{js} + \text{h.c.} - J \sum_{lss'} c_{ls}^\dagger \boldsymbol{\sigma}_{ss'} c_{ls'} \cdot \mathbf{S}_l$$

$$+ i\lambda \sum_{\langle lj \rangle ss'} \mathbf{g}_{lj} \cdot c_{ls}^\dagger \boldsymbol{\sigma}_{ss'} c_{js'} + \text{h.c.}$$

antisymmetric SOC from P breaking

$$\mathbf{g}_{lj} = -\mathbf{g}_{jl}$$

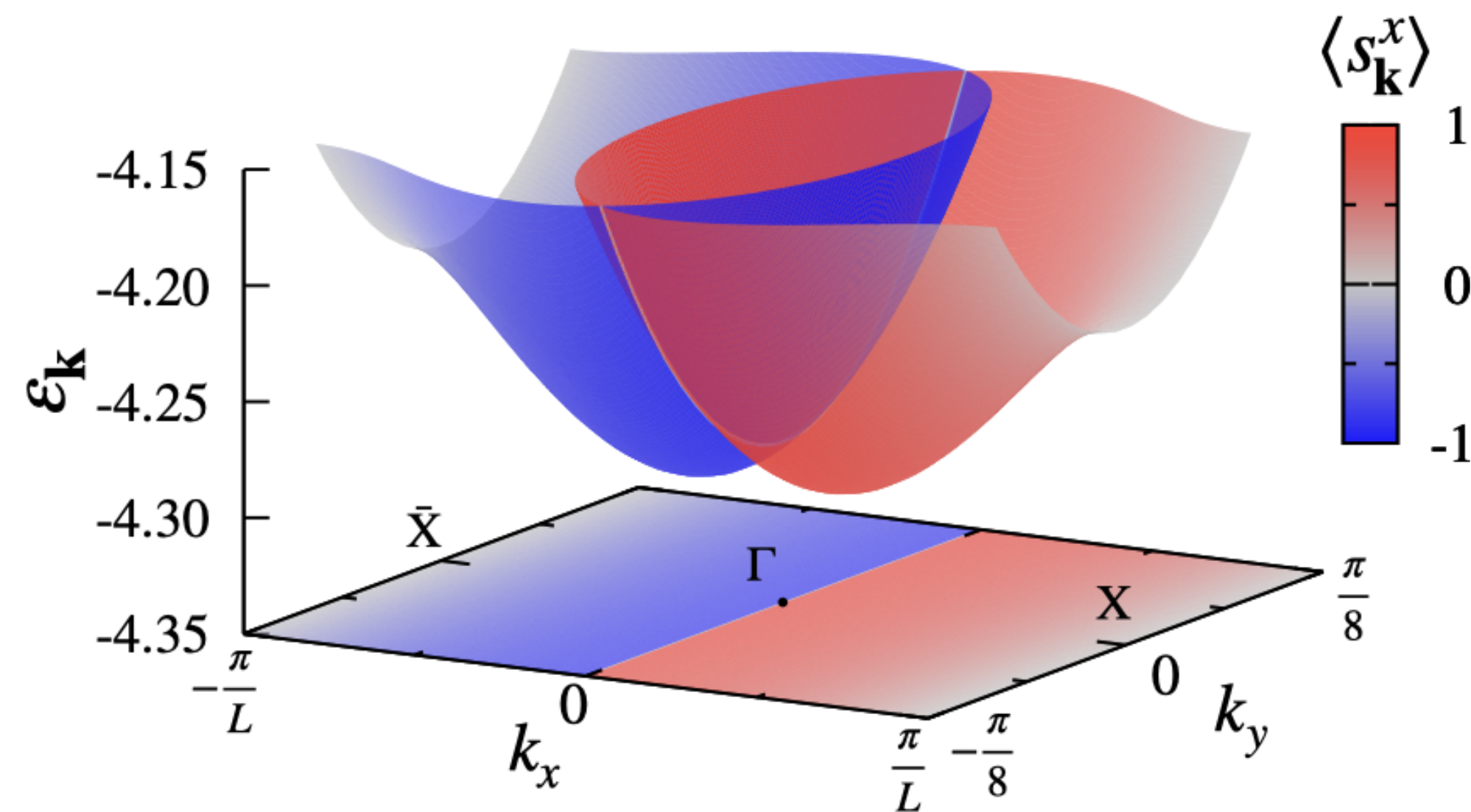
$$\mathbf{S}_l = \mathbf{S}_l^{\alpha\beta;\mu} \propto \hat{\mathbf{e}}_\alpha \cos \mathcal{Q}_l + \hat{\mathbf{e}}_\beta \sin \mathcal{Q}_l + m \hat{\mathbf{e}}_\mu \quad \mathcal{Q}_l = \mathbf{Q} \cdot \mathbf{r}_l + \varphi$$



yz spiral running along x
pitch $L = 4$, phase $\varphi = \pi/4$

Band structure under spin spiral

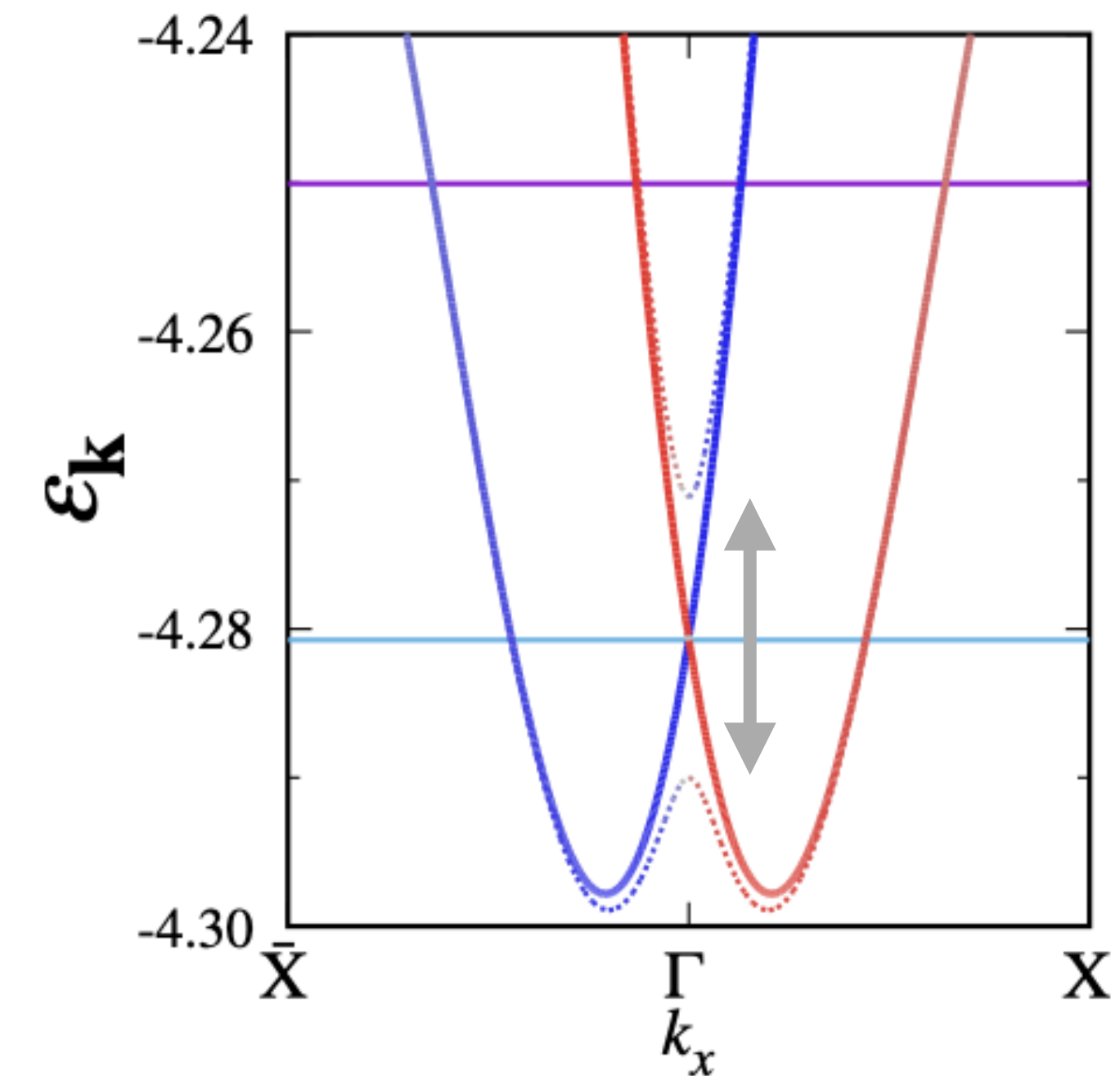
© p -wave type spin splitting perpendicular to the spiral plane, forming spin-nodal lines



spin nodal line at $k_x = 0$ is protected by symmetries:

$$C_\pi^x \text{ T: } \langle \mathbf{s}_k^x \rangle = -\langle \mathbf{s}_{-\mathbf{k}}^x \rangle; C_Q^x \text{ t}_x: \langle \mathbf{s}_k^y \rangle = \langle \mathbf{s}_k^z \rangle = 0$$

T: time-rev, C_θ^μ : θ -rot around μ , t_μ : 1-site translation along μ

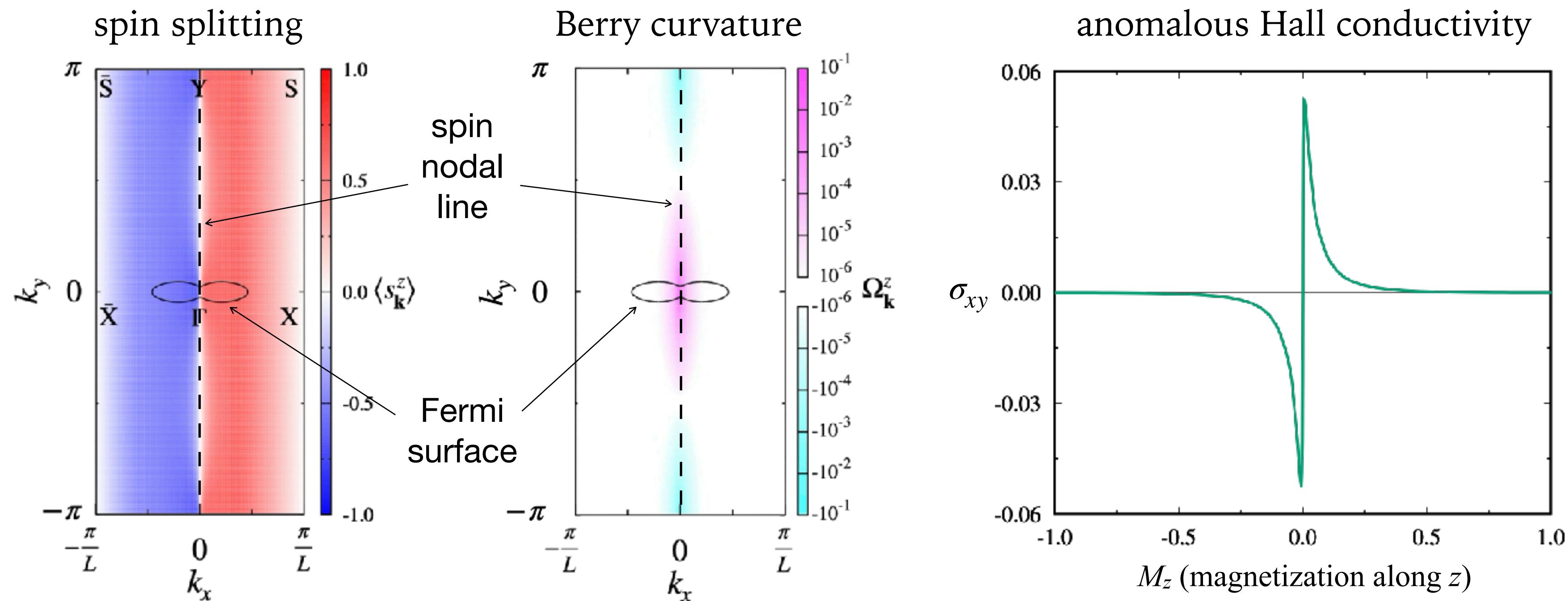


SOC + net $\mathbf{M} \perp$ the helical plane
 $\rightarrow$ gap out the spin-nodal lines

topological properties?

Berry curvature and AHE

- Berry curvature concentrates near the gapped nodal lines, contributing to unconventional AHE.



The spiral-induced AHE exhibits distinct $\mathbf{M}$ dependence from the conventional AHE and THE.

It appears universally for general electron filling, J , λ , and period of spin spiral.

Spiral-induced AHE

- ◎ low-energy effective model: $H_{\mathbf{k}}^{\text{eff}} = t(k_x^2 + k_y^2)\sigma^0 - pk_x\sigma^\rho + \tilde{\lambda}k_y\sigma^\nu - \tilde{m}\sigma^\mu \equiv d_{\mathbf{k}}^0\sigma^0 + \mathbf{d}_{\mathbf{k}} \cdot \boldsymbol{\sigma}$

➡ Berry curvature: $\Omega_{\mathbf{k}\pm}^z = \mp \epsilon^{\rho\nu\mu} d_{\mathbf{k}}^\mu \partial_{k_x} d_{\mathbf{k}}^\rho \partial_{k_y} d_{\mathbf{k}}^\nu / (2|\mathbf{d}_{\mathbf{k}}|^3)$

cf. J. Mitscherling *et al.*, arXiv:2603.09736

The spiral-induced AHE requires finite x, y, z components of the spin polarization, SOC, and $\mathbf{M}$.

helical plane $\alpha\beta$		xy			yz			zx		
spin polarization		$\langle s_{\mathbf{k}}^z \rangle$			$\langle s_{\mathbf{k}}^x \rangle$			$\langle s_{\mathbf{k}}^y \rangle$		
SOC	ν	$\mu \quad \mathbf{M}$								
		x	y	z	x	y	z	x	y	z
	x	-	$\pm$	-	-	-	-	-	-	$\mp$
	y	$\mp$	-	-	-	-	$\pm$	-	-	-
	z	-	-	-	-	$\mp$	-	$\pm$	-	-

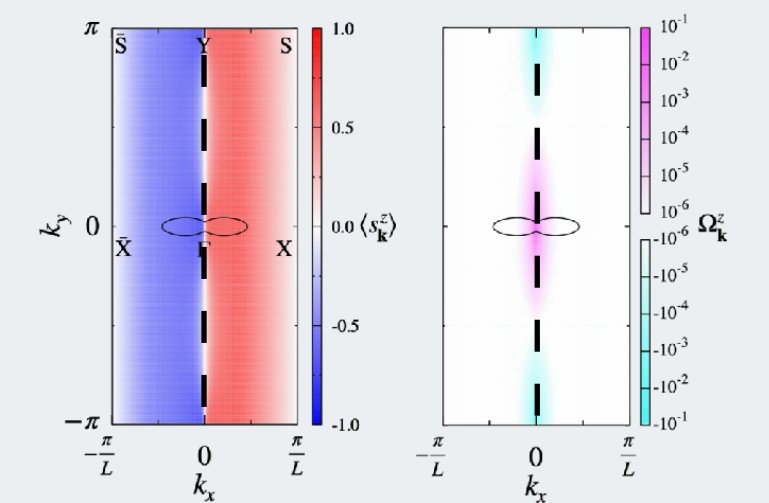
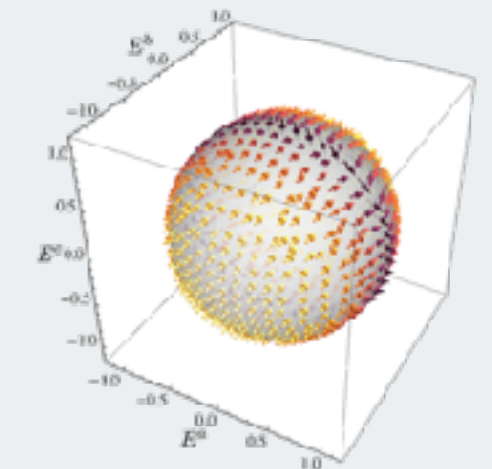
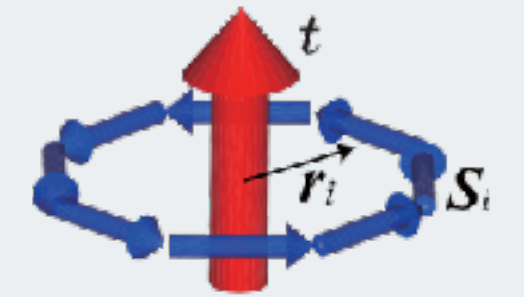
- The sign of σ_{xy} depends not only on the signs of SOC and $\mathbf{M}$, but also on the spiral chirality, as p changes its sign under $Q \rightarrow -Q$.

The spiral-induced AHE is highly sensitive to the spiral-plane orientation and the chirality, providing a robust and universal hallmark of commensurate spin spirals.

Summary and outlook

■ take-home messages

- Magnetic toroidics provides a compact language bridging symmetry breaking and unconventional responses in compensated magnets.
 - The canonical toroidal metal shows how odd-parity magnetic order in a metal directly produces transport and ME responses.
- The same concept extends naturally to
 - ✓ AFM olivine (S-type altermagnets) → nonlinear ME responses
Y. Kato, T. Hayashida, K. Matsumoto, T. Kimura, and Y. Motome, Phys. Rev. B **113**, 094411 (2026)
 - ✓ spin-spiral magnets (*p*-wave magnets) → unconventional spin-spiral AHE
S. Okumura, M. M. Moritz, and Y. Motome, preprint (arXiv:2512.14071)
R. Yamada, M. T. Birch, P. R. Baral *et al.*, Nature **646**, 837 (2025)



□ outlook

- impacts of electron correlation and fluctuations beyond mean-field pictures
- connection to unconventional superconductivity

Acknowledgement

Collaborators

Theory:

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Shun Okumura (UTokyo/QPEC/RIKEN), Moritz M. Hirschmann (RIKEN)

Satoru Hayami (Hokkaido Univ.), Hiroaki Kusunose (Meiji Univ.)

Experimental:

Rinsuke Yamada (UTokyo), Max T. Birch (RIKEN), Priya R. Baral (UTokyo), Max Hirschberger (UTokyo), ...

Takeshi Hayashida (Radboud Univ.), Koei Matsumoto (UTokyo), Tsuyoshi Kimura (UTokyo)

Fundings



No. JP22K03509, JP25H00392, JP25H01247. ...