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Applications of negative energy excitations in magnetic systems

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Collaborators

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Rembert Duine

Artim Bassant

Mexx Regout

Long term motivation
Magnonics

- Devices and circuits; spin waves (magnons)
- Spin current ⇔ electric currents
- For information transport and processing
- Electrically insulating materials ⇒ no Joule heating

- Hurdle; magnons are not conserved and have a finite lifetime

Outline

1. Magnetization dynamics
2. Negative energy modes
3. Spin wave amplification via the bosonic Klein paradox
4. Spontaneous entangled pair production
5. Spin wave injection via an inverted magnet
6. Outlook

Magnetization dynamics

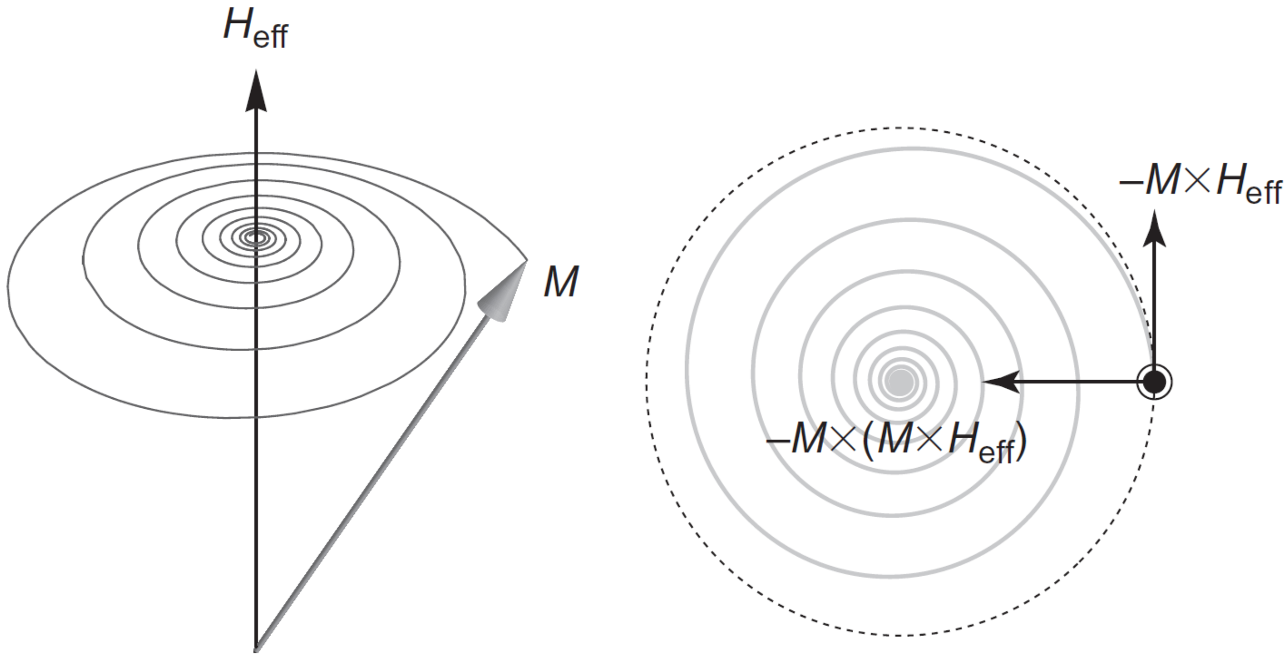
- Landau Lifshitz Gilbert (LLG) equation

$$\frac{\partial \mathbf{n}}{\partial t} = -\gamma \mathbf{n} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t}$$

- Effective magnetic field

$$H_{\text{eff}} \equiv -\delta \mathcal{F} / \delta \mathbf{n}$$

$$\mathcal{F} = \int dV \left\{ A(\nabla_i \mathbf{n})^2 - \mu_0 H_{\text{e}} M_s n_z \right\}$$

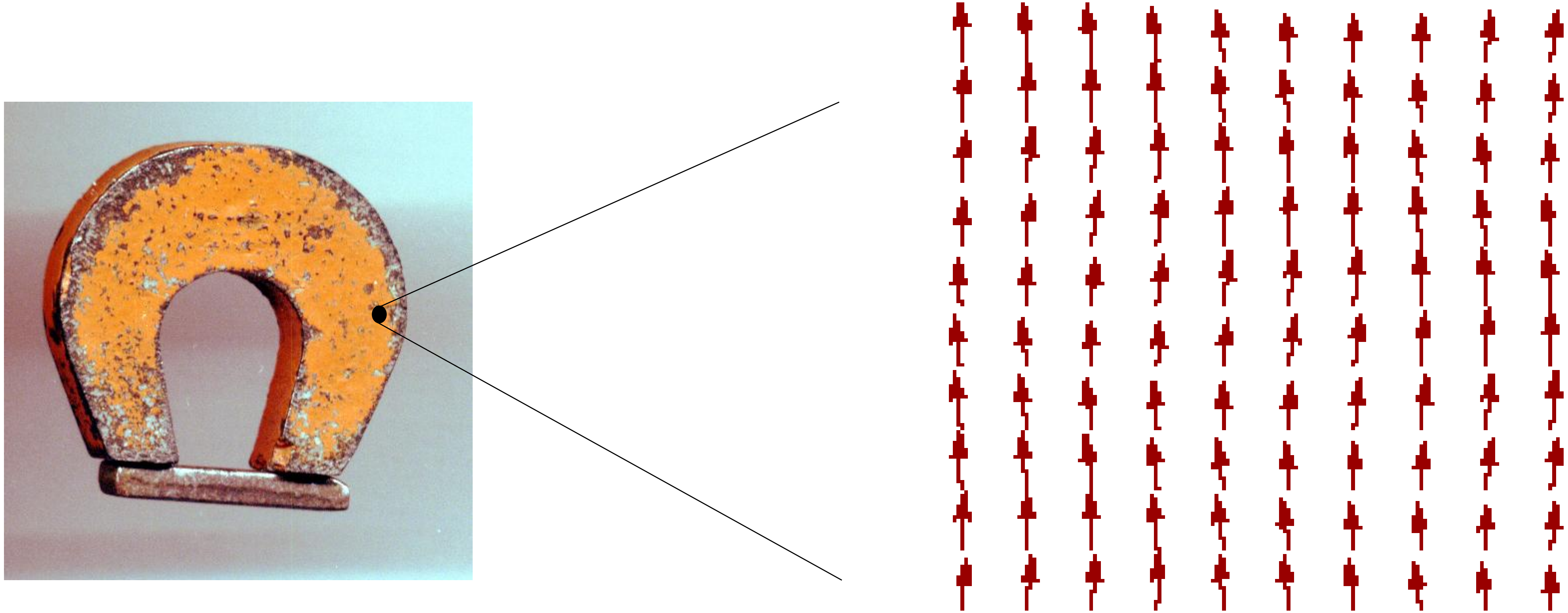


Kim, Spin-Torque-Oscillators

Spin waves

- Landau Lifshitz Gilbert (LLG) equation

$$\frac{\partial \mathbf{n}}{\partial t} = -\gamma \mathbf{n} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t}$$

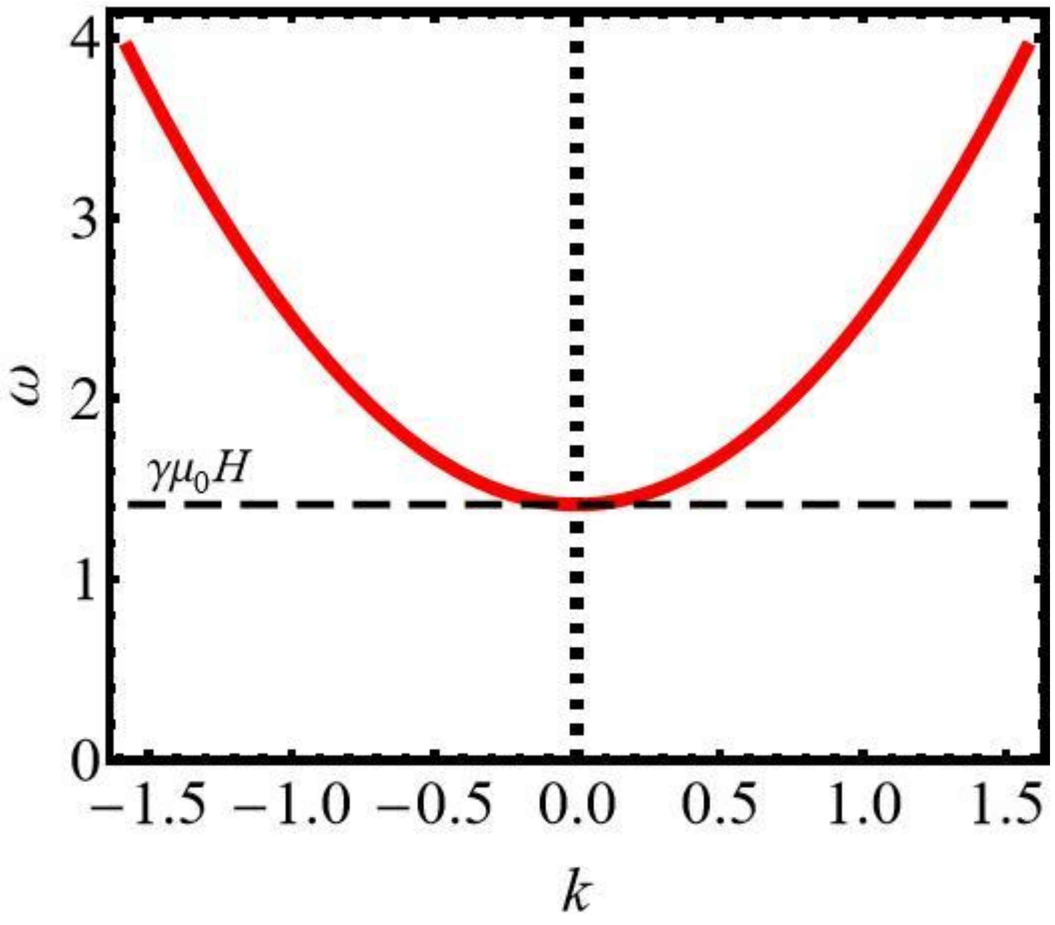


Jonathan Flynn, University of Southampton

Spin waves

- Landau Lifshitz Gilbert (LLG) equation
$$\frac{\partial \mathbf{n}}{\partial t} = -\gamma \mathbf{n} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t}$$

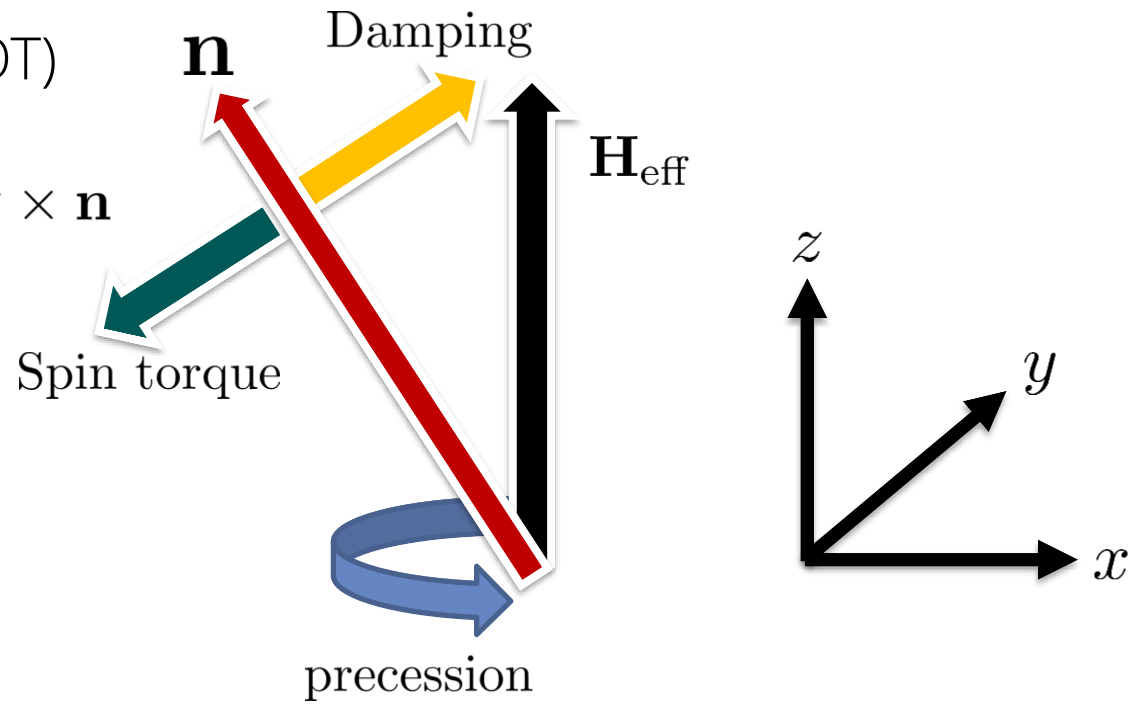
- Spin wave dispersion
$\omega(\mathbf{k}) = 2\gamma \bar{A} \mathbf{k}^2 / |\mathbf{M}| + \gamma \mu_0 |\mathbf{H}|.$



Current driven magnetization dynamics

- LLG equation with spin-orbit torque (SOT)

$$\frac{\partial \mathbf{n}}{\partial t} = -\gamma \mathbf{n} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{n} \times \frac{\partial \mathbf{n}}{\partial t} + J_s \mathbf{n} \times \hat{z} \times \mathbf{n}$$



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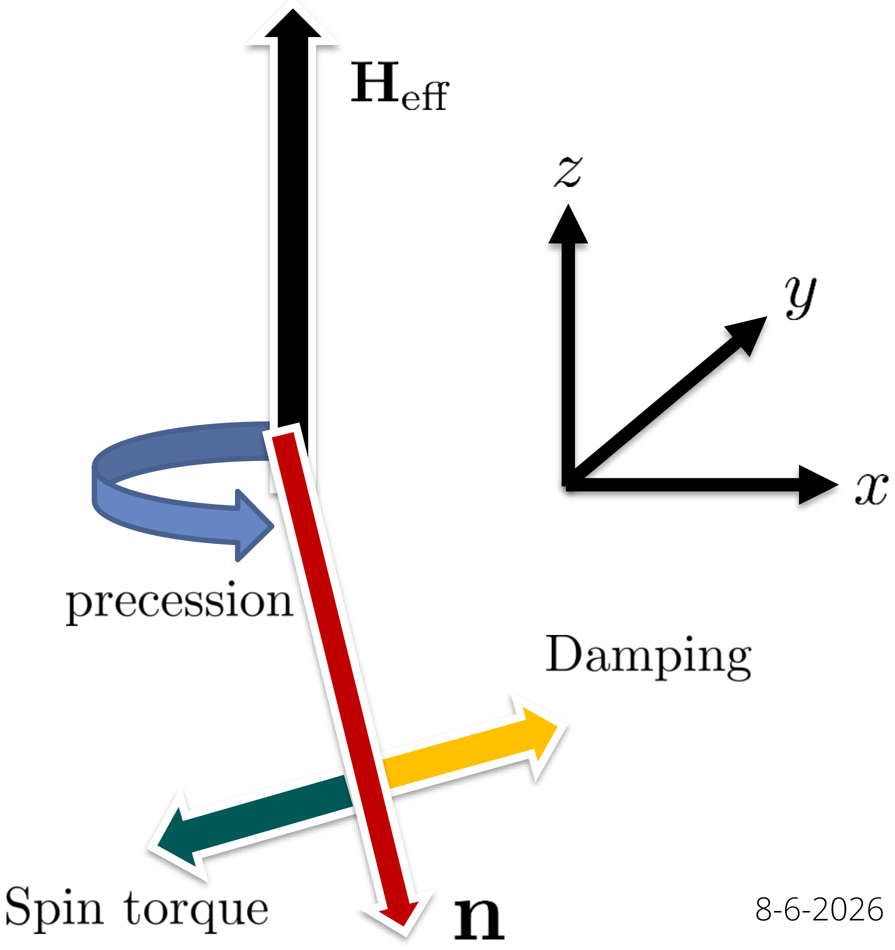
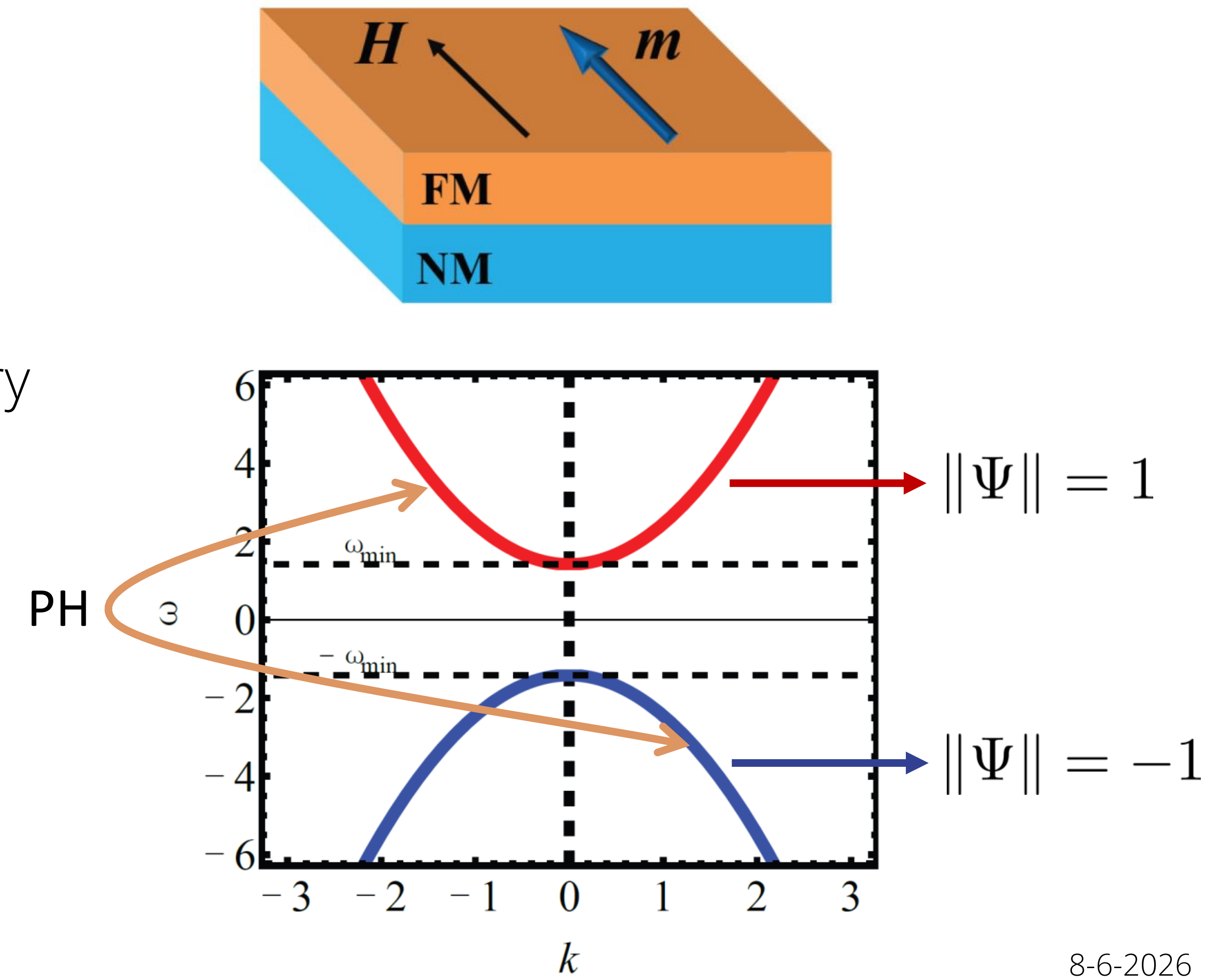
- Experimental realization of invert magnetization
- **B. Özyilmaz et al., Phys. Rev. Lett. 91, 067203 (2003),**
- **H. Kurebayashi et al., arXiv:2601.08738 (2026),**
- **E. Karadza et al., arXiv:2601.09569 (2026),**
- **H. Wang et al., arXiv:2601.15231 (2026).**



Magnons

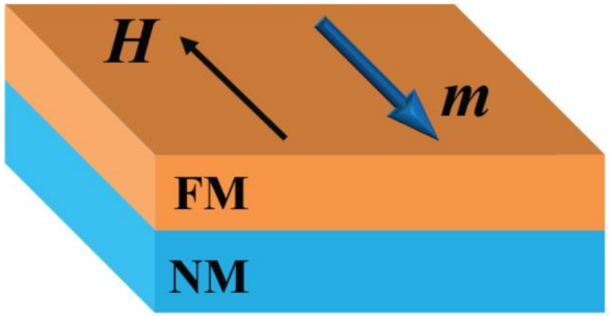
- Energy of excitation
  $E = \|\Psi\|\omega$

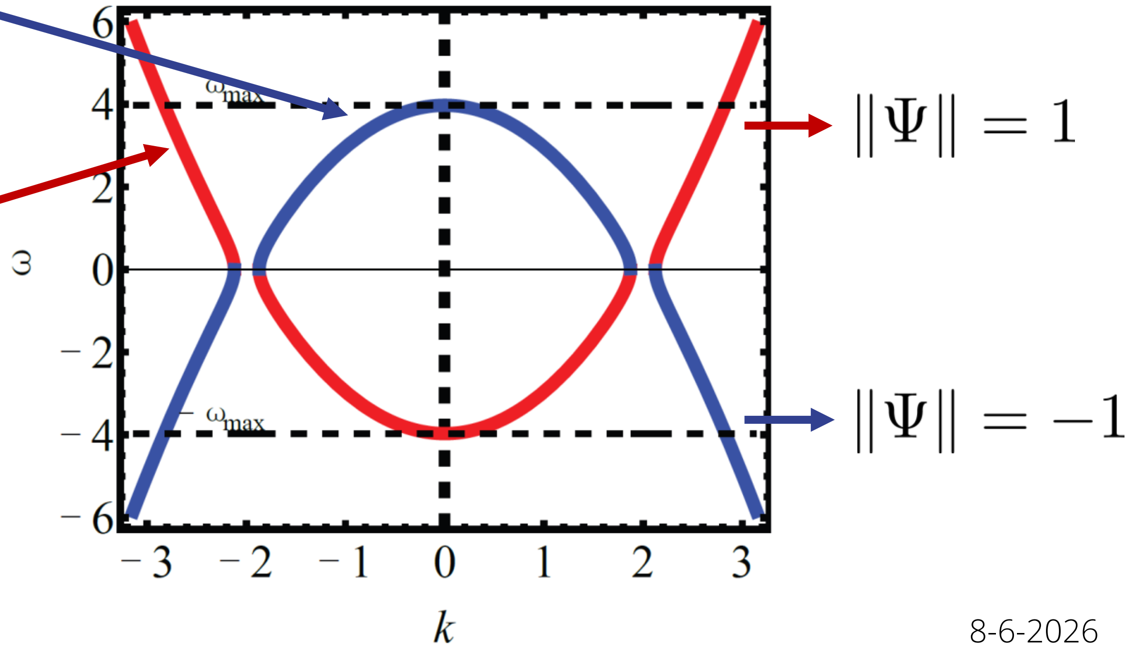
- Both bands describe the same physics, due particle hole symmetry



Magnons vs antimagnons

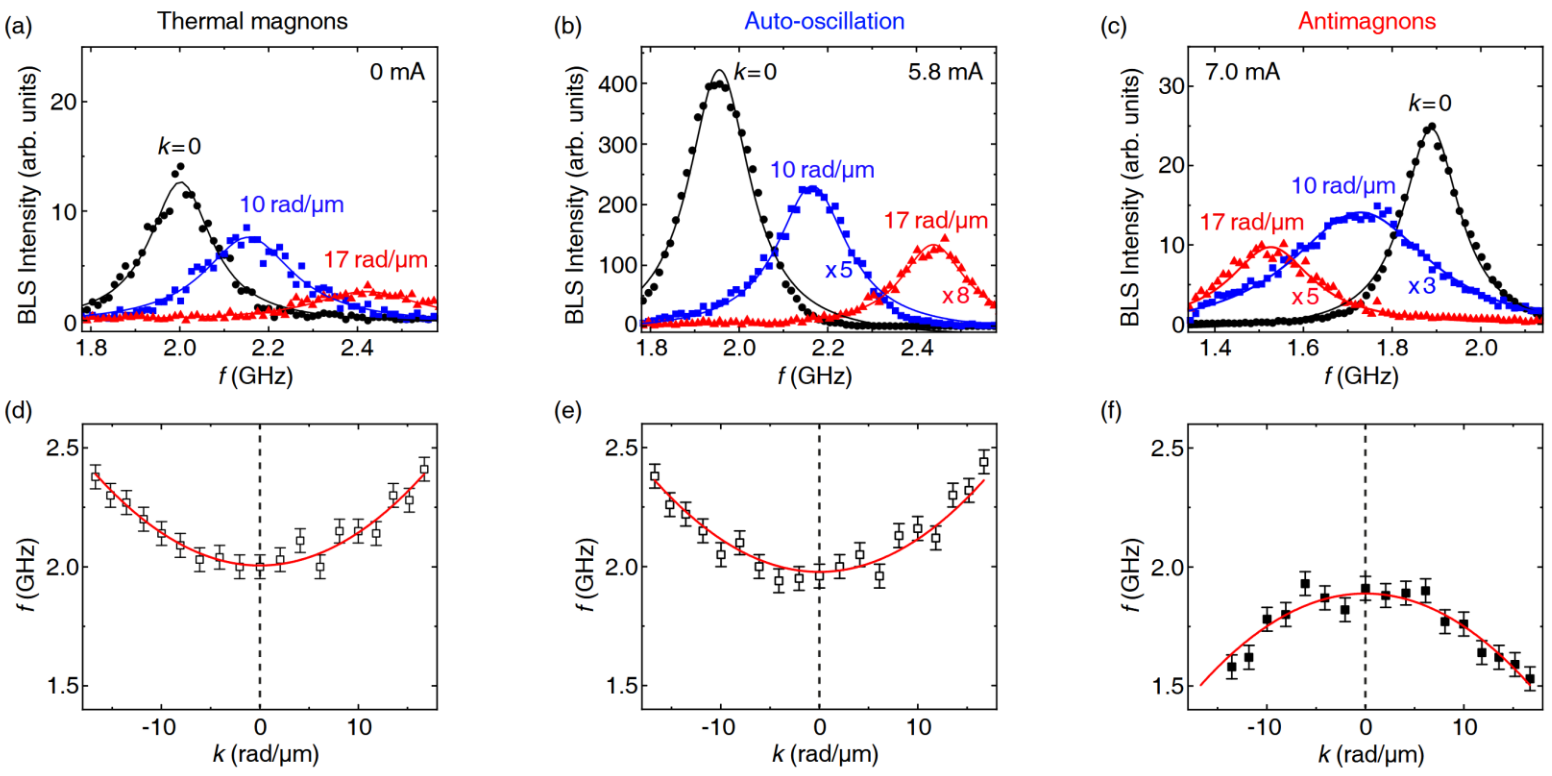
- Energy of excitation $E = \|\Psi\|\omega$
- Antimagnon: norm and frequency have opposite sign
- Negative energy excitation
- Opposite handedness (spin) w.r.t. magnons





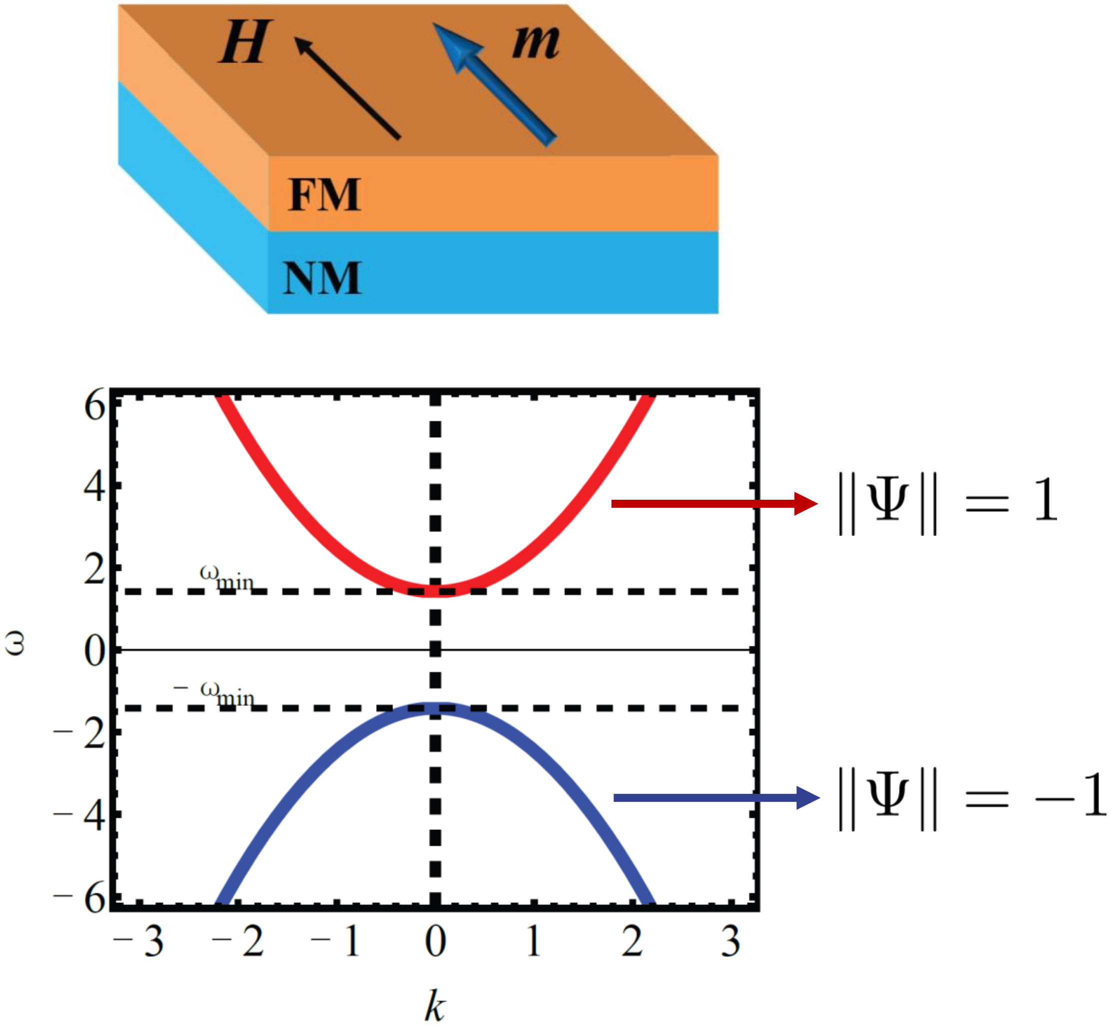
JSH, H. Y. Yuan and R. A. Duine, *AIP Advances* 14, 025303 (2024)

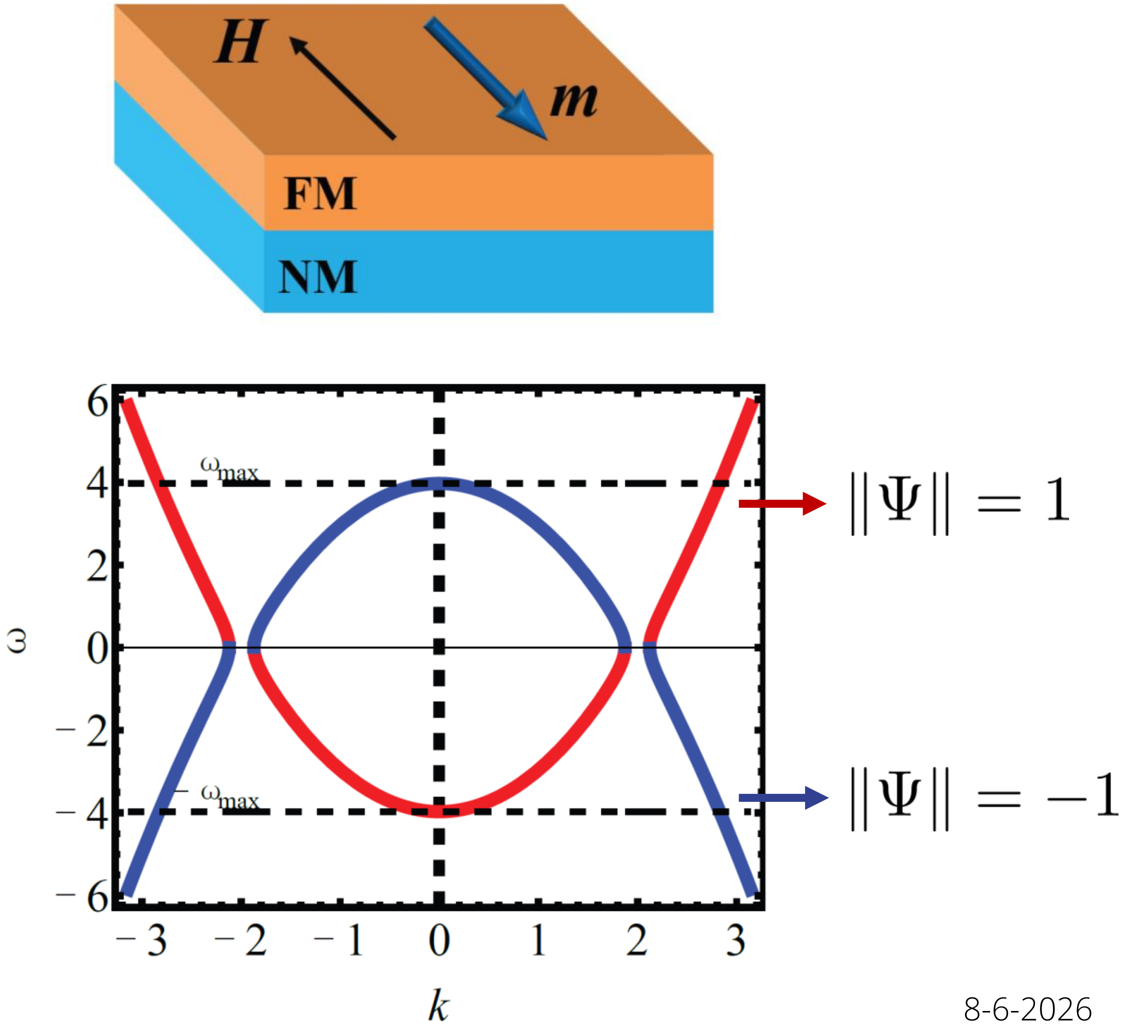
The inverted state has been realized experimentally



H. Wang et al., arXiv:2601.15231 (2026).

Magnons vs antimagnons





JSH, H. Y. Yuan and R. A. Duine*, AIP Advances* 14, 025303 (2024)

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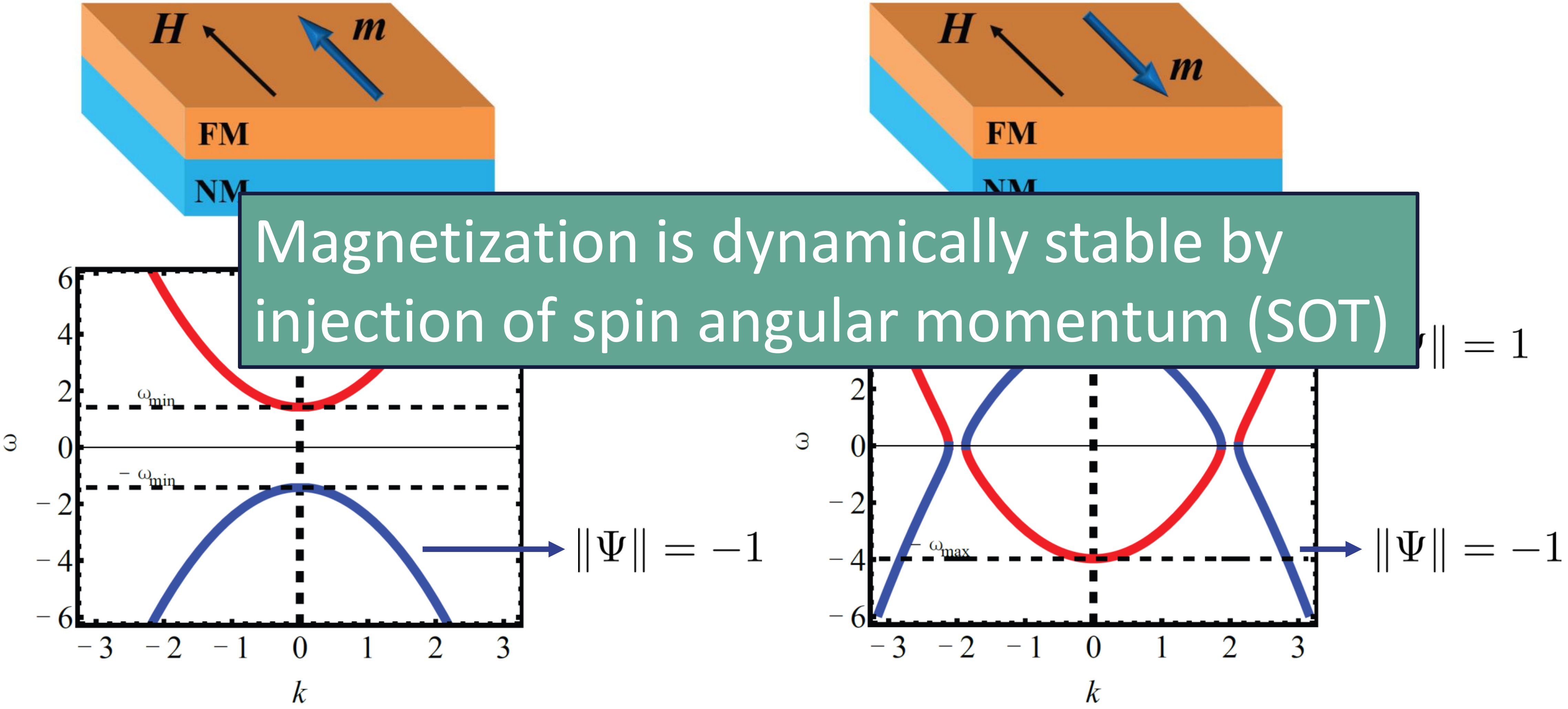


Magnetization is dynamically stable by injection of spin angular momentum (SOT)

JSH, H. Y. Yuan and R. A. Duine, *AIP Advances* 14, 025303 (2024)

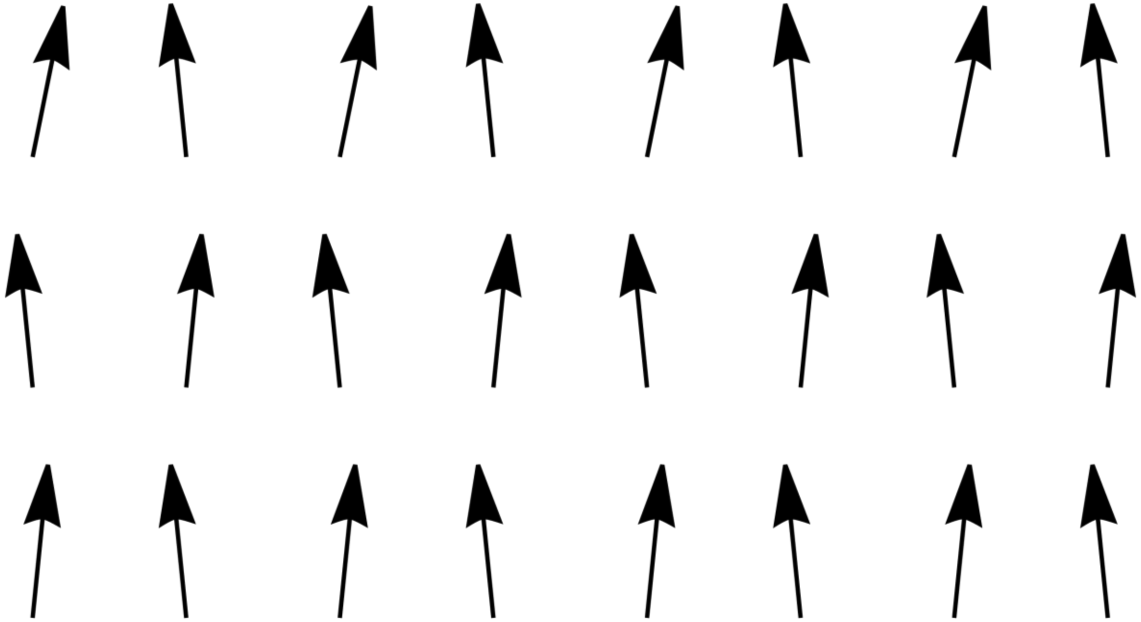
Spin waves and the Bogoliubov-de Gennes equation

- Linearize LLG+SOT using the complex field
  $\Psi = (1/\sqrt{2})\,(\hat{e}_1 + \mathrm{i}\hat{e}_2)\cdot\mathbf{n}$

- Bogoliubov-de Gennes equation

$$\frac{\mathrm{i}(1+\mathrm{i}\alpha\sigma_z)\partial_t}{\gamma\mu_0 M_s}\begin{pmatrix}\Psi\\ \Psi^*\end{pmatrix} = (\mathcal{L}_\pm + \mathrm{i}I_s)\begin{pmatrix}\Psi\\ \Psi^*\end{pmatrix}$$

$$\mathcal{L}_\pm = \left(\Delta \pm h - \Lambda^2\nabla^2\right)\sigma_z + \mathrm{i}\Delta\sigma_y$$



Bogoliubov-de Gennes equation

- Bogoliubov-de Gennes equation

$$\frac{\mathrm{i}(1+\mathrm{i}\alpha\sigma_z)\partial_t}{\gamma\mu_0 M_s}\begin{pmatrix}\Psi\\ \Psi^*\end{pmatrix} = (\mathcal{L}_\pm + \mathrm{i}I_s)\begin{pmatrix}\Psi\\ \Psi^*\end{pmatrix},$$

$$\mathcal{L}_\pm = \left(\Delta \pm h - \Lambda^2\nabla^2\right)\sigma_z + \mathrm{i}\Delta\sigma_y$$

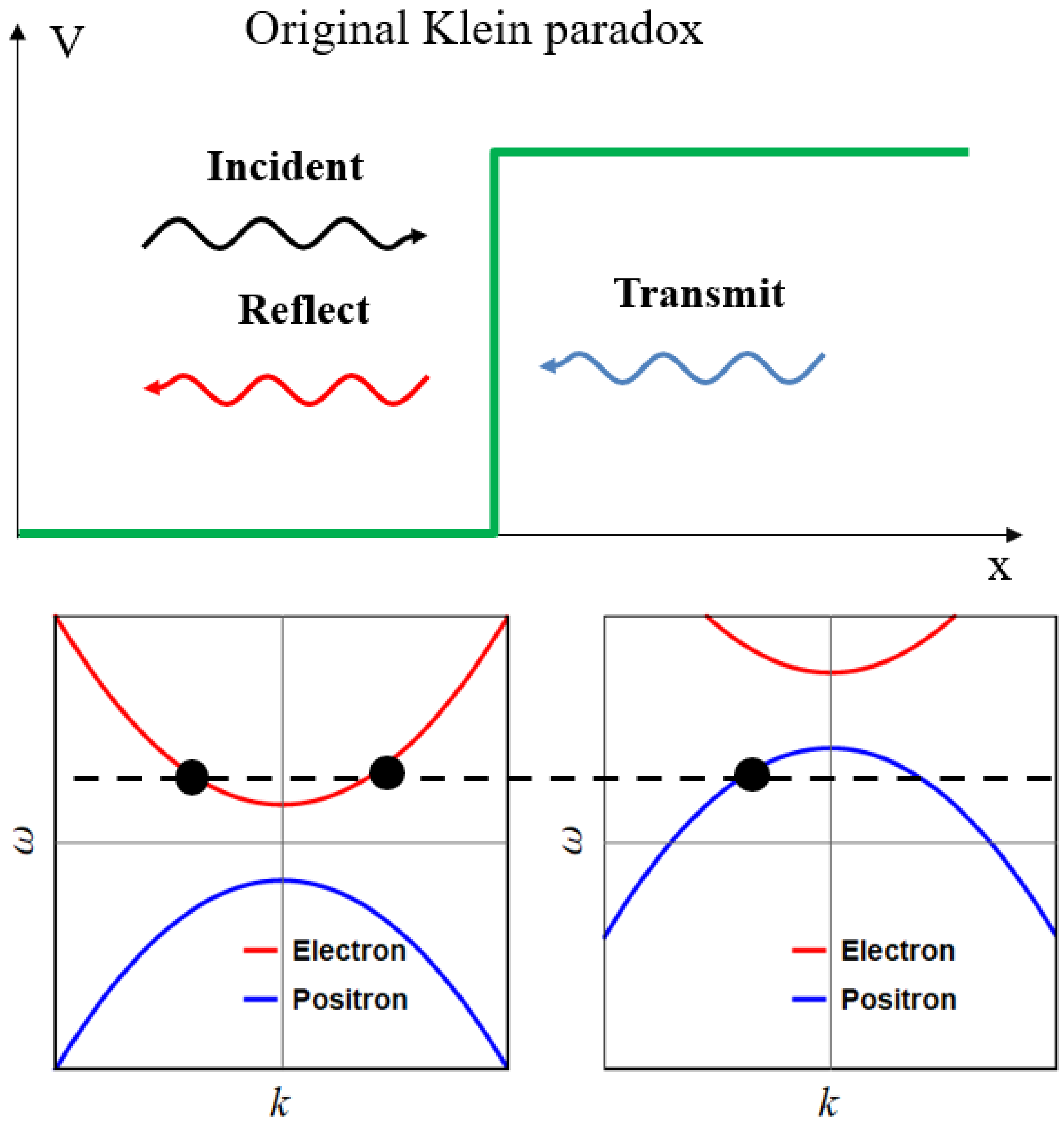
- Symmetry relations
  - Pseudo-Hermitian $\sigma_z \mathcal{L}_\pm^\dagger \sigma_z = \mathcal{L}_\pm \implies \|\Psi\|$
  - Particle-hole symmetry $\omega \to -\omega *\implies \|\Psi\| \to -\|\Psi\|$
- Energy of excitation $E = \|\Psi\|\omega$

The Fermionic and Bosonic Klein Paradox

- Potential step at least twice the rest mass

- Fermionic
- **Paradoxical non-zero transmission**

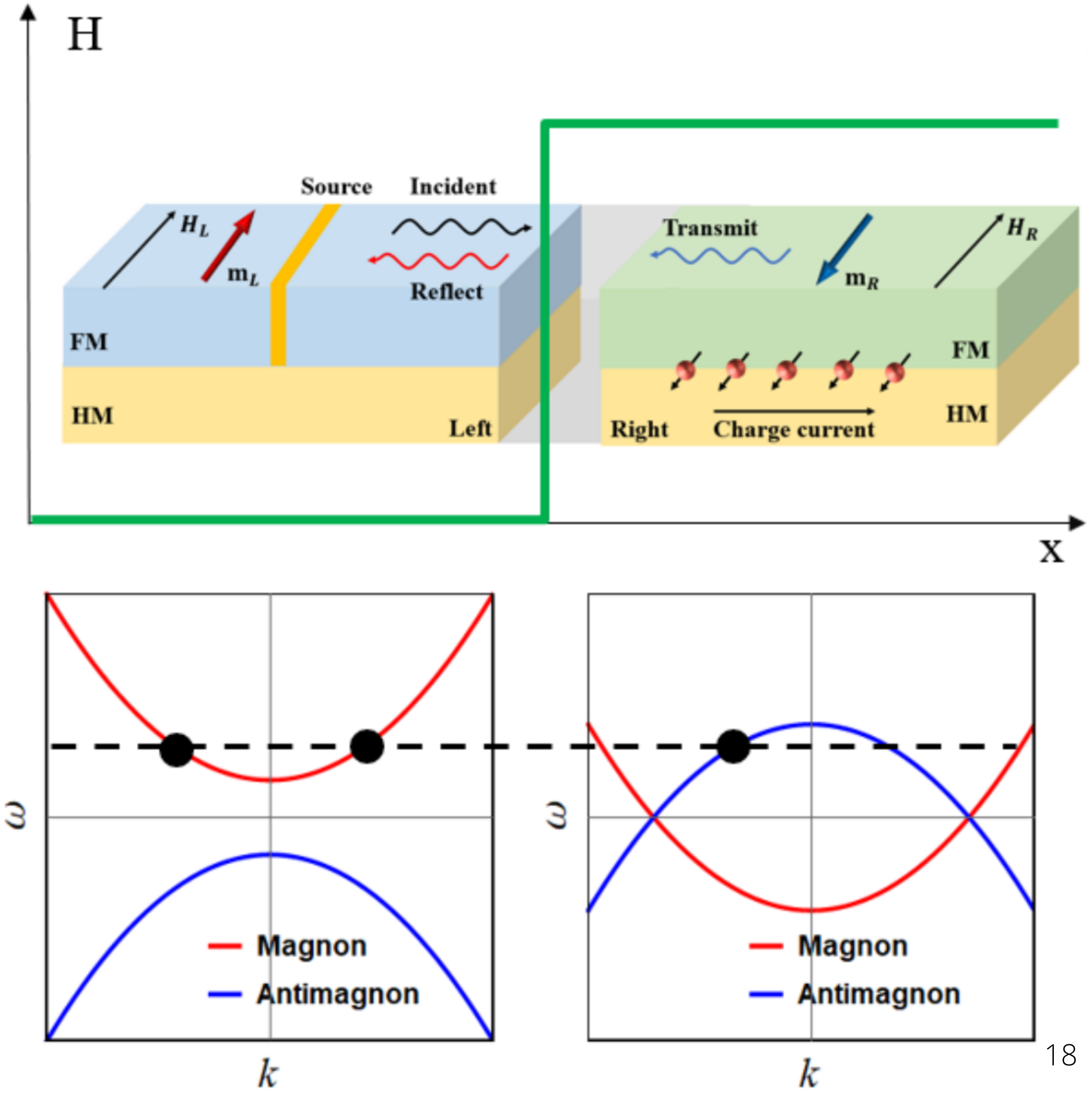
- Bosonic
- **Superradiant reflection coefficient**



Magnonic Klein paradox

Superradiant reflection coefficient

- Interfacial coupling (RKKY)
- Coupling between magnons and anti-magnons if $h_R - h_L > 0$
- Spin conservation at the interface gives $|R|^2 - |T|^2 = 1$
- **Since antimagnons carry negative spin current**
- Superradiant reflection coefficient $|R|^2 > 1, \text{ if } h_L < \omega < h_R$



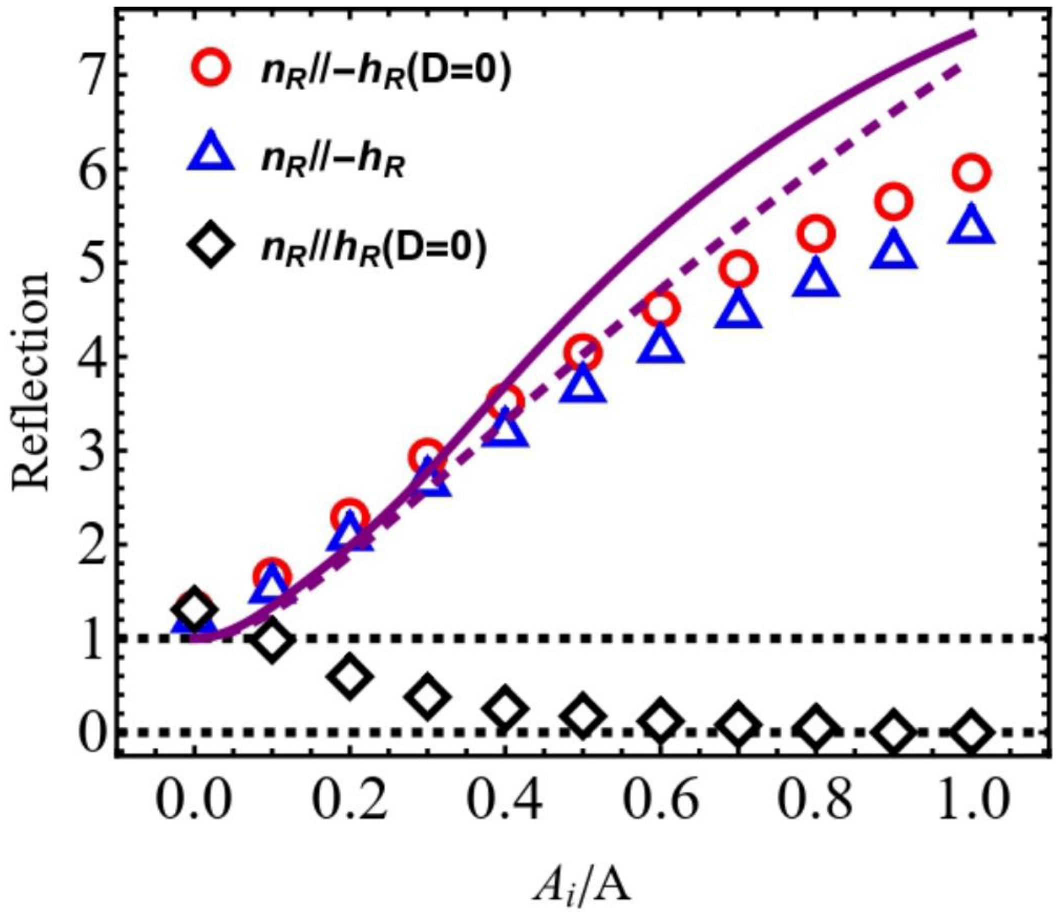
JSH, H. Y. Yuan, R. A. Duine, **Phys. Rev. Applied 18, 064026**

Spin-wave amplifier via the Bosonic Klein paradox

- When magnons are coupled to antimagnons

- The reflected spin current is

$$|R|^2 = \frac{h_R - h_L + \Lambda_c^{-2}(\omega - h_L)(h_R - \omega) + 2\sqrt{\omega - h_L}\sqrt{h_R - \omega}}{h_R - h_L + \Lambda_c^{-2}(\omega - h_L)(h_R - \omega) - 2\sqrt{\omega - h_L}\sqrt{h_R - \omega}}$$



JSH, H. Y. Yuan, R. A. Duine, **Phys. Rev. Applied 18, 064026**

Quantum regime; spontaneous pair production

- Pair production

$$\langle 0, in|\hat{b}^{\dagger}_{L,M}\hat{b}_{L,M}|0, in\rangle = \langle 0, in|\hat{b}^{\dagger}_{L,AM}\hat{b}_{L,AM}|0, in\rangle = |T|^2$$

- Magnon and antimagnon current via Landauer-Buttiker

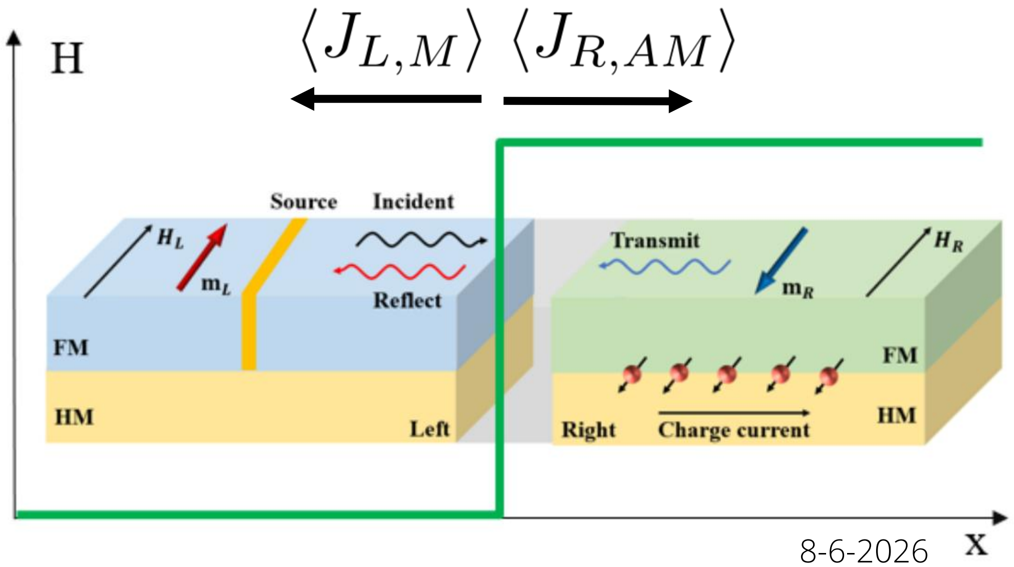
$$\langle \hat{J}_{L,M}\rangle = \langle \hat{J}_{R,AM}\rangle = -\int \mathrm{d}\omega |T(\omega)|^2 (1 + f_L(\omega) + f_R(\omega))$$

- Distribution functions

$$f_L = n_B(\omega) \quad f_R = n_B(-\omega + \Delta\mu)$$

- Pair production distribution independent in both average current and current noise

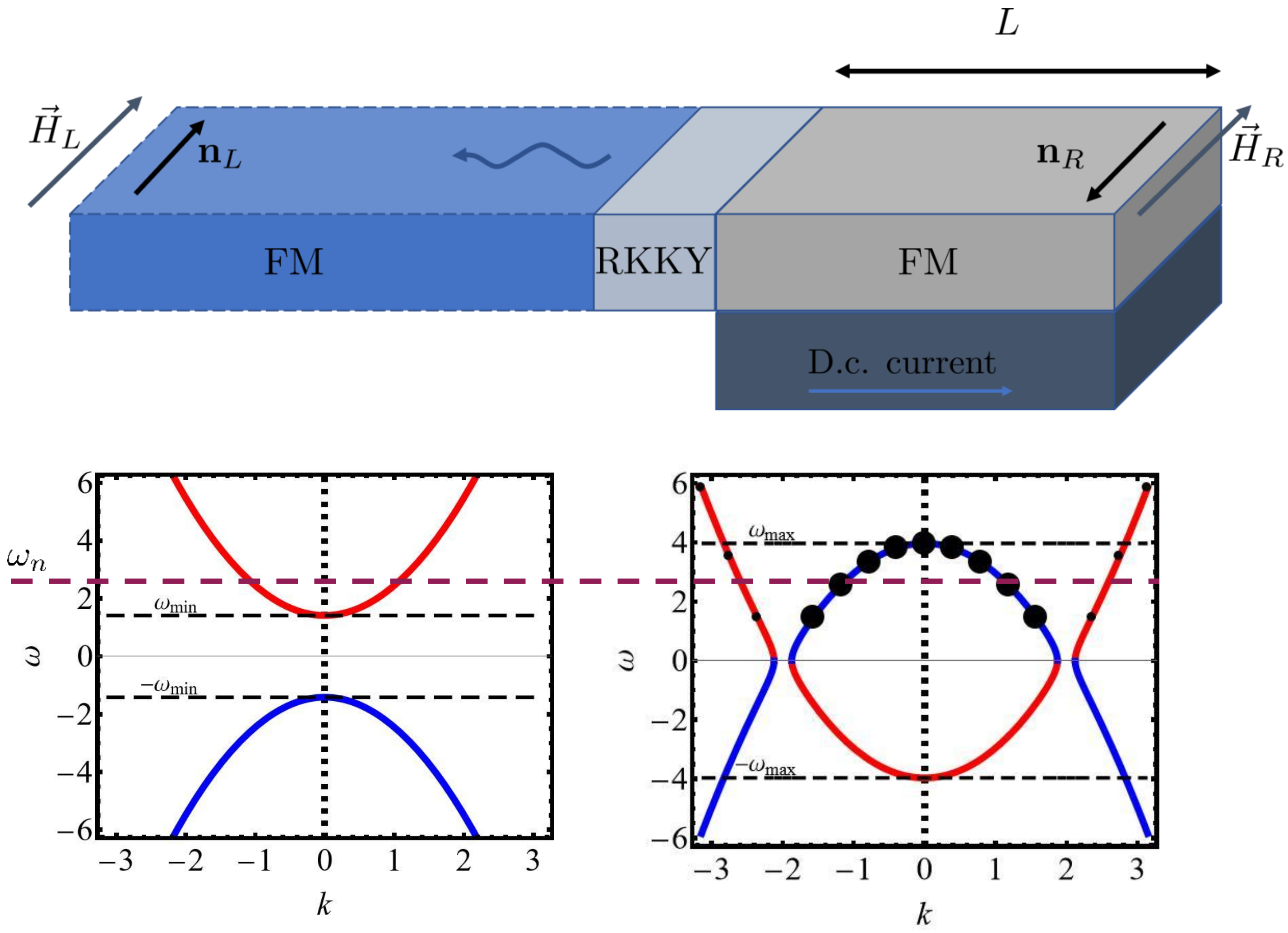
- Detectable at milli-Kelvin



A. L. Bassant, M. E. Y. Regout, **JSH**, R. A. Duine, **Phys. Rev. B 110, 094441 (2024)**

Single-mode spin-wave laser driven by spin-orbit torque

- Finite region with negative energy states coupled to continuum of spin waves
- Inject coherent single mode spin waves via a DC-current
- Tunable via for instance external magnetic field



JSH, H. Y. Yuan, R. A. Duine, **arXiv:2408.14857**

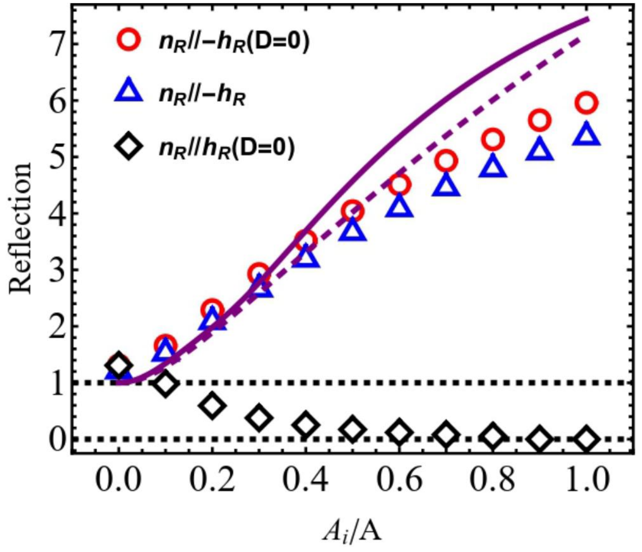
Take home message

- Antiparticle like excitations occur around an inverted magnetic ground state
- Coupling magnonic excitations to these “antimagnonic” excitations can be used to make amplifiers and lasers for magnons
- Quantum mechanically; spontaneous pair production

Outlook

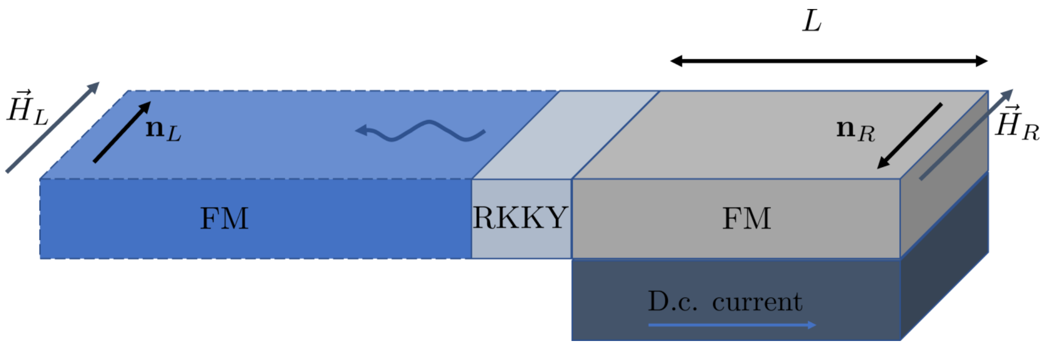
Magnon antimagnon coupling can be used to:

- **Amplify spin waves of an interface**
- **Coherently inject single-mode spin waves using DC currents**
- **Study the quantum mechanical properties; pair production**

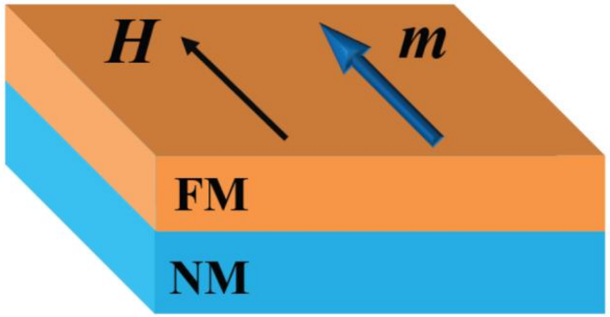


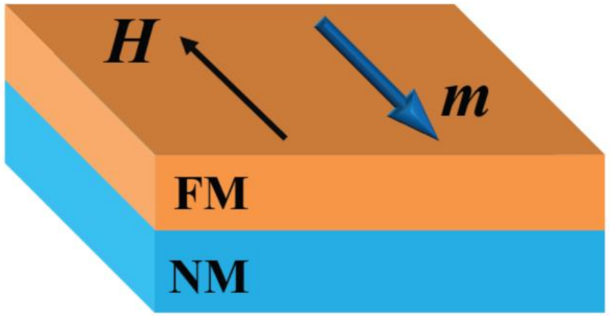
Future research:

- **Explore spin-wave laser using other types of driving**
- **Antiferromagnets**



Quantization and Statistics





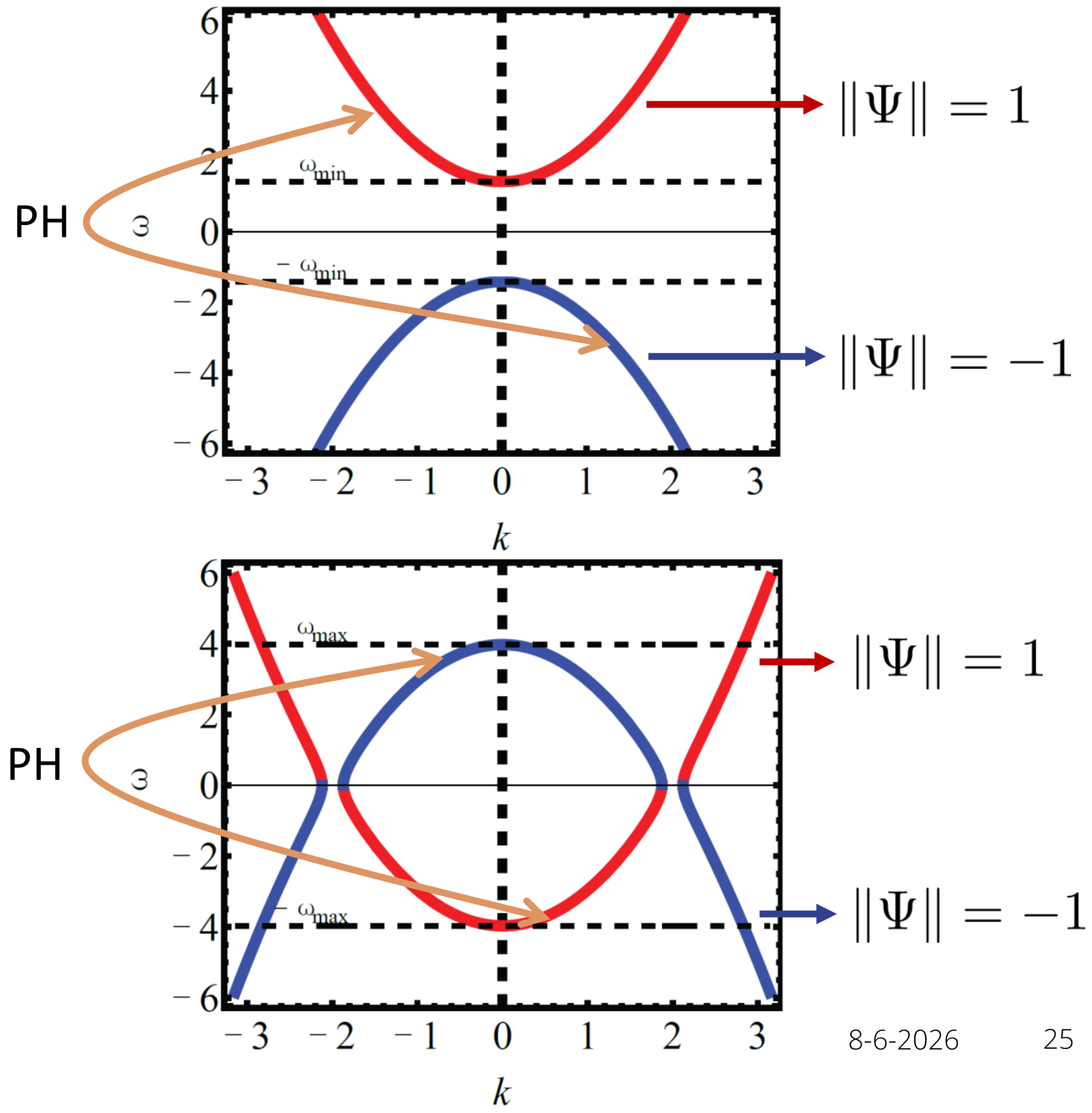
- Distribution function of excitations in the normal and inverted magnet

$$f_{\text{normal}}(\omega, \beta) = \frac{1}{e^{\beta\omega} - 1}$$

$$f_{\text{inv}}(\omega, \beta) = \frac{1}{e^{\beta(\omega+\Delta\mu)} - 1}$$

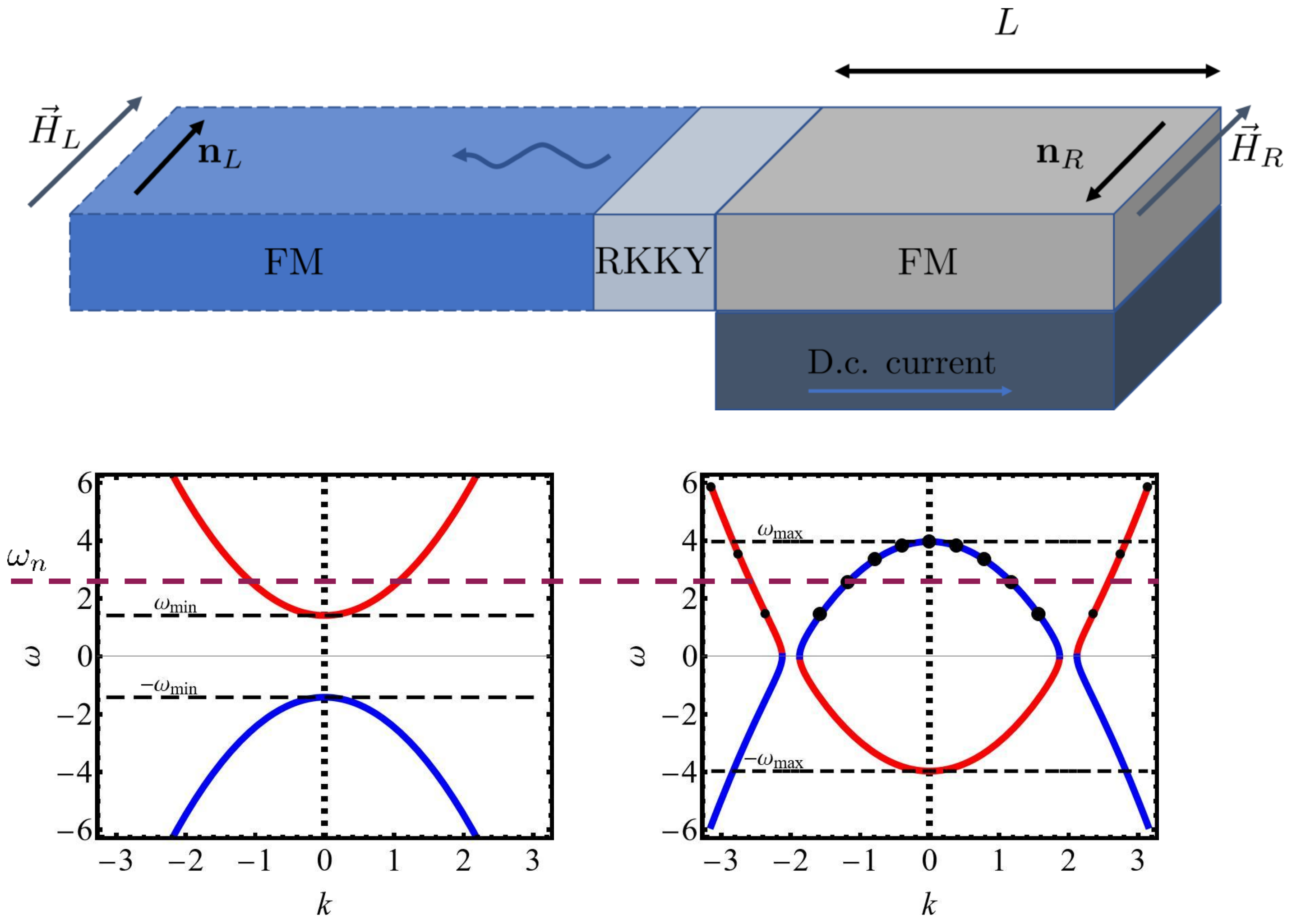
Quantization

- Positive norm $[a_k, a^\dagger_{k'}] = \delta_{k,k'}$
- Negative norm $[b_k, b^\dagger_{k'}] = -\delta_{k,k'}$
- PH-symmetry $b_k = a^\dagger_{-k}$
- Using both branches useful for energy conserving processes, like scattering



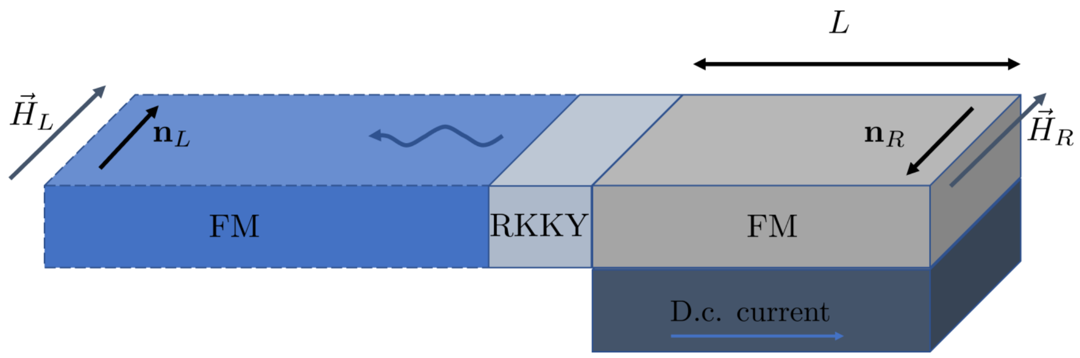
The setup

- Finite size inverted magnet coupled to continuum of spin waves
- Mode coalescence of spin waves with discrete negative-energy facilitates the lasing onset
- **Like exceptional point formation**



JSH, H. Y. Yuan, R. A. Duine, **arXiv:2408.14857**

The setup



- Discrete modes in the right magnet for weak coupling

$$\Psi_R = \sum_m A_m(t) e^{\mathrm{i}\phi_m(t)} e^{-\mathrm{i}\omega_m t} \sqrt{\frac{2-\delta_{m,0}}{L}} \cos[k_m x]$$

- The mode expansion gives effective laser equations

$$\partial_t \phi_n \simeq -\alpha(\partial_T A_n)(1 + 3A_n^2/4L) + 2\Lambda_c \Lambda / L$$

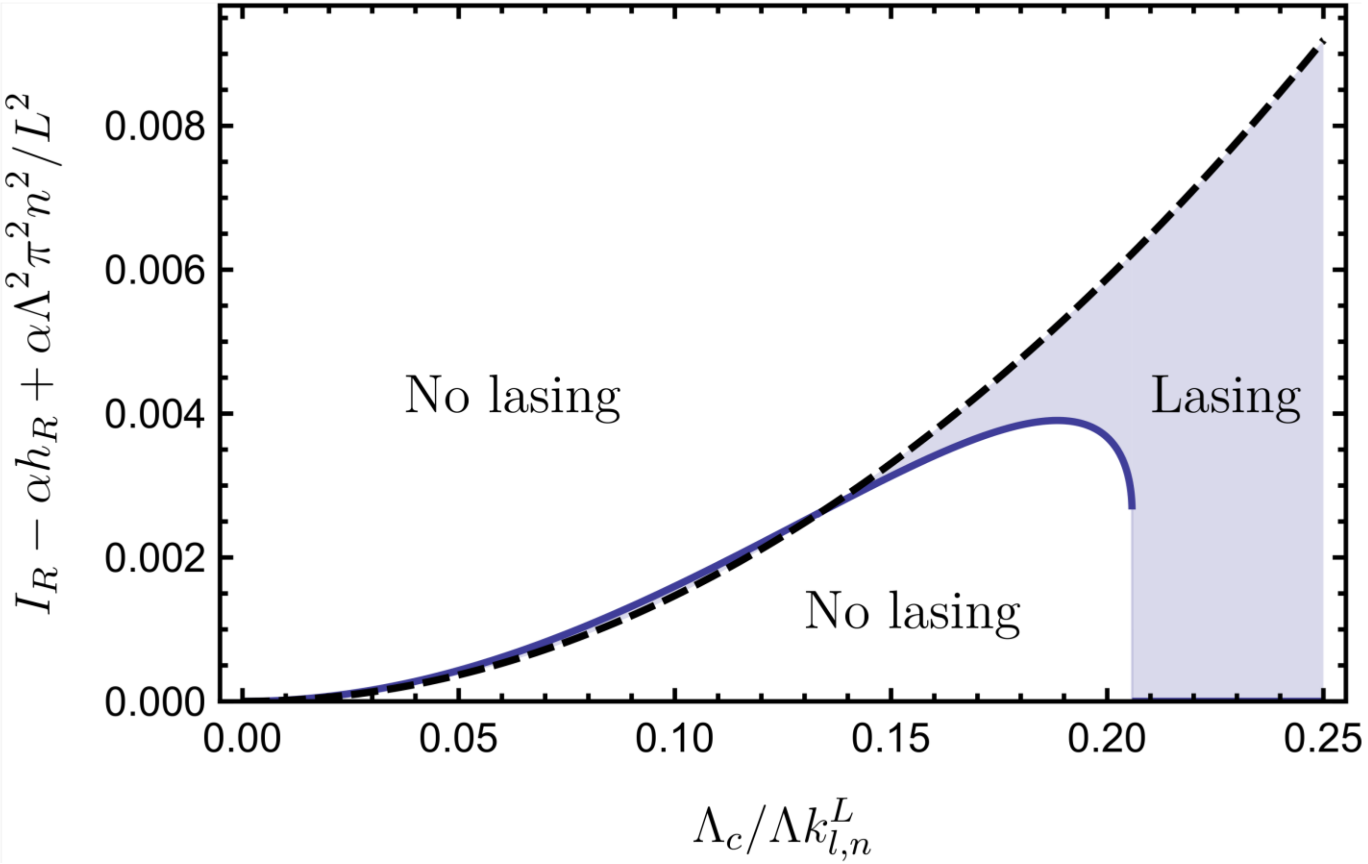
$$\partial_t A_n \simeq -(2\Lambda_c^2/k_l^L L)(1 - A_n^2/L)A_n - (\alpha\omega_n - \partial_t \phi_n + I_R)(1 - 3A_n^2/4L)A_n$$

- **Lasing onset due to mode coalescence**
- **Stabilized via SOT (driving) and non-linear interactions**

JSH, H. Y. Yuan, R. A. Duine, **arXiv:2408.14857**

Stability analyses for single mode spin-wave laser

- Other modes vanish, due to non-linear interactions
- Defines parameter region for single-mode spin wave laser
- Critical coupling strength $\Lambda^2_{c,\text{critical}} \gtrsim 3\alpha\Lambda^2 k_n^2 |k_{l,0}^L| L/2.$



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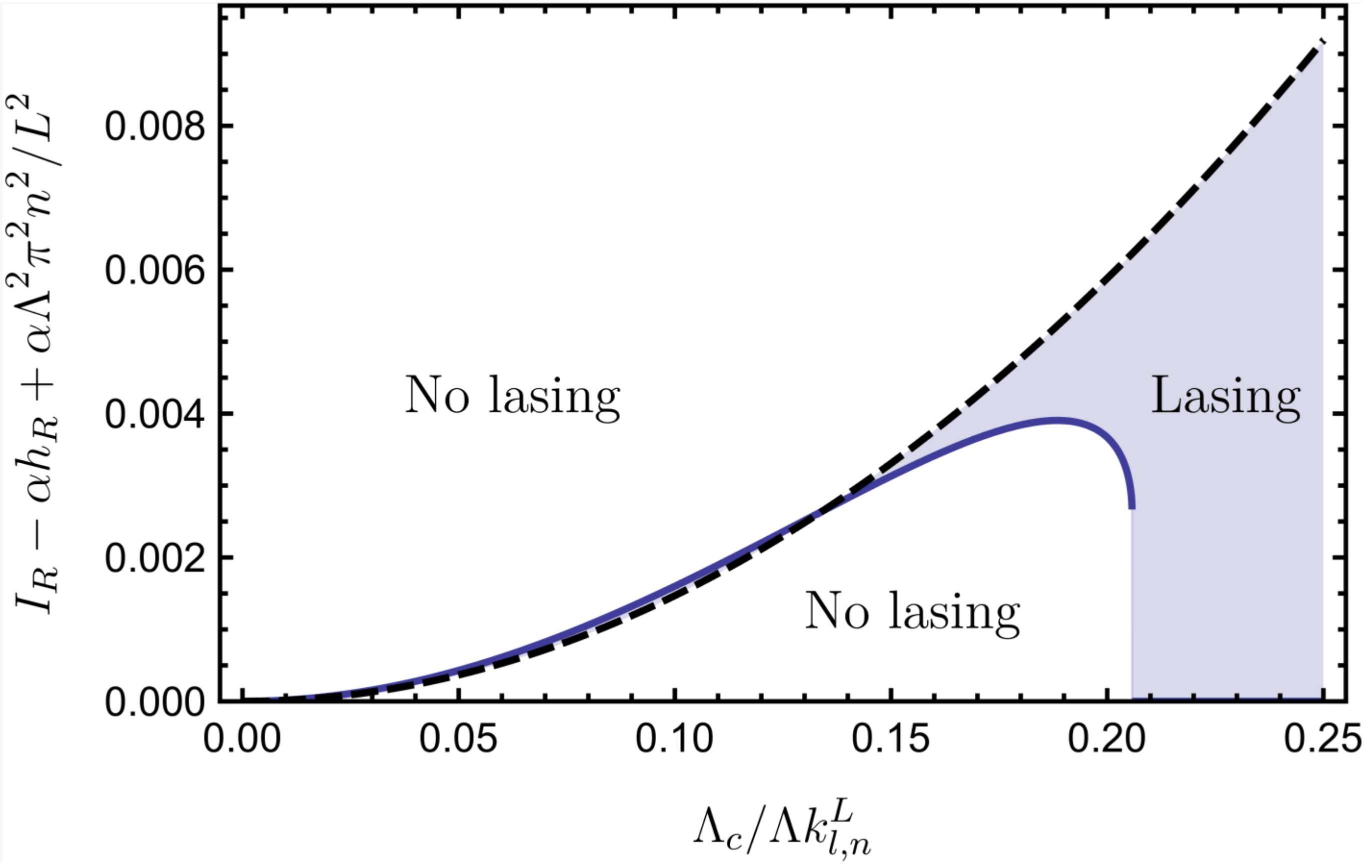
Stability analyses for single mode spin-wave laser

- Highly tuneable setup
- No non-linear frequency shift
- Single-mode spin wave emission
- Realization of a black hole laser



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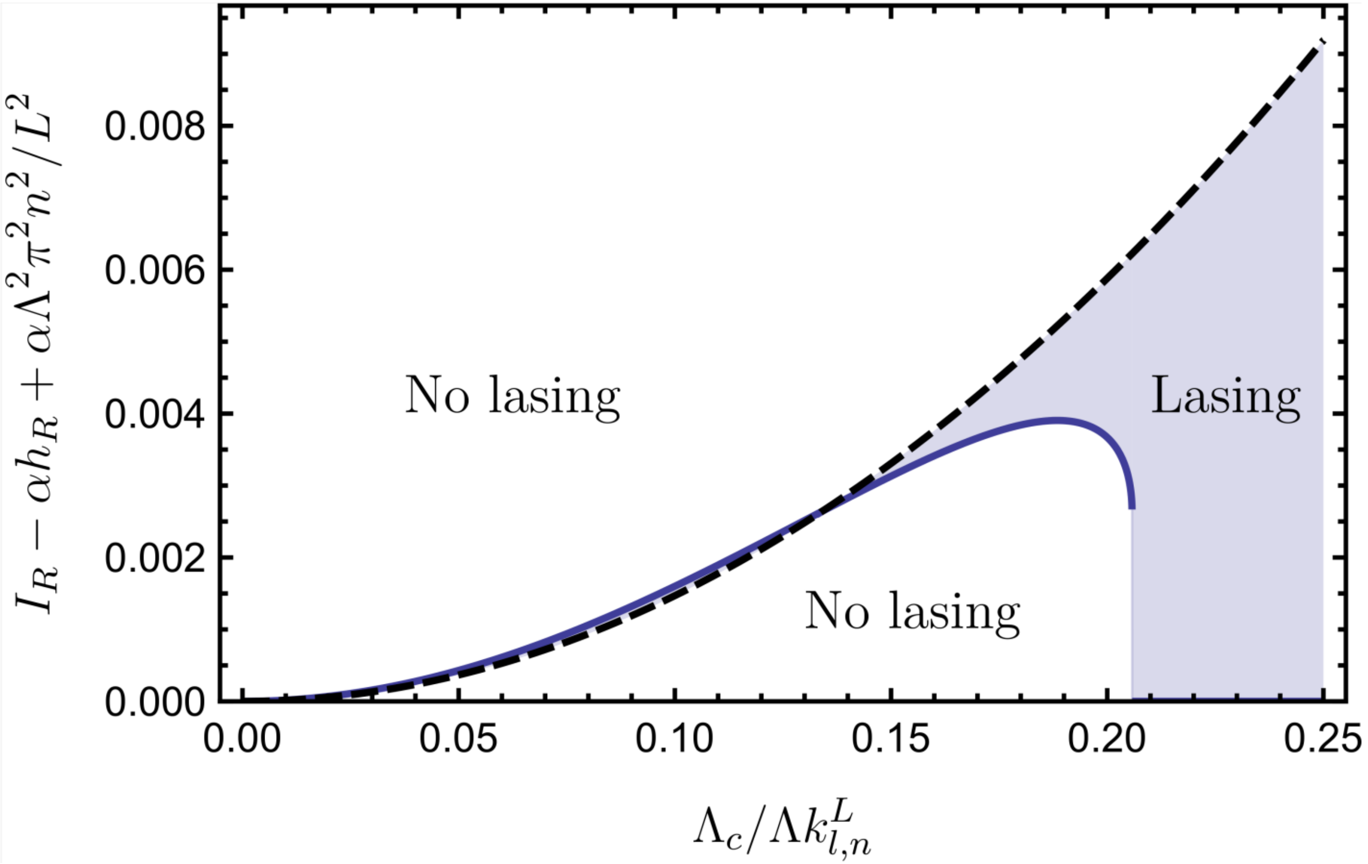


Towards the quantum regime; normal scattering

- Scattering of wavefunctions in the normal case; relating in and out states

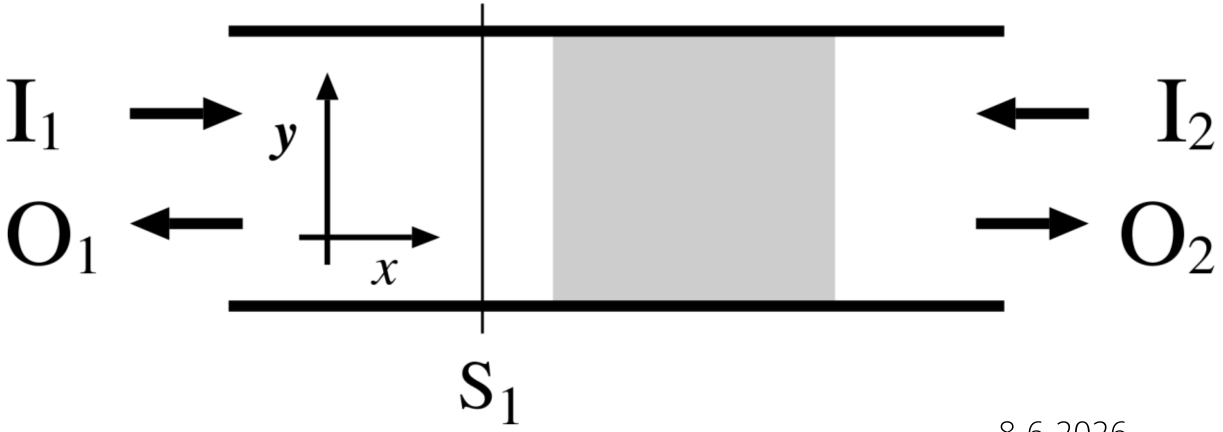
$$\begin{pmatrix} O_{L,M} \\ O_{R,M} \end{pmatrix} = \mathcal{S} \begin{pmatrix} I_{L,M} \\ I_{R,M} \end{pmatrix} \quad \mathcal{S} = \begin{pmatrix} R & T \\ T' & R' \end{pmatrix} \quad \mathcal{S}^\dagger \mathcal{S} = \mathbb{1} \implies |R|^2 + |T|^2 = 1$$

- Transformation for creation (annihilation) operators of scattering states

$$\begin{pmatrix} \hat{b}_{L,M}(\omega) \\ \hat{b}_{R,M}(\omega) \end{pmatrix} = \mathcal{S}^* \begin{pmatrix} \hat{a}_{L,M}(\omega) \\ \hat{a}_{R,M}(\omega) \end{pmatrix}$$

- Particle flux from vacuum

$$\langle 0, in | \hat{b}^\dagger_{L,M} \hat{b}_{L,M} | 0, in \rangle = 0$$



M. J. M. de Jong, C. W. J. Beenakker, **Shot noise in mesoscopic systems**

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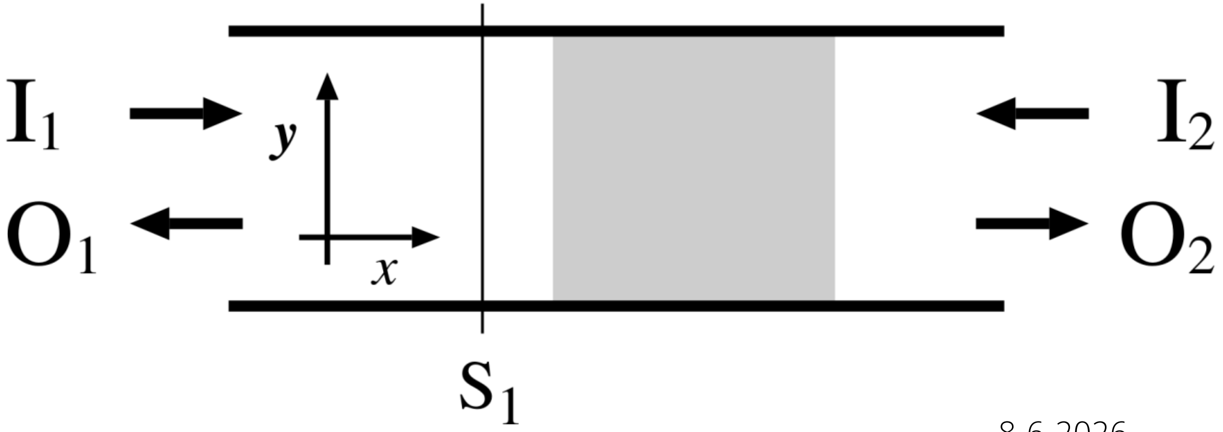
- Current from Landauer-Buttiker

$$\langle \hat{J}_L \rangle = \frac{1}{\hbar} \int \mathrm{d}\omega |T(\omega)|^2 \left[f_L(\omega) - f_R(\omega) \right]$$



M. J. M. de Jong, C. W. J. Beenakker, **Shot noise in mesoscopic systems**

Quantum effects; scattering with an inverted magnetization

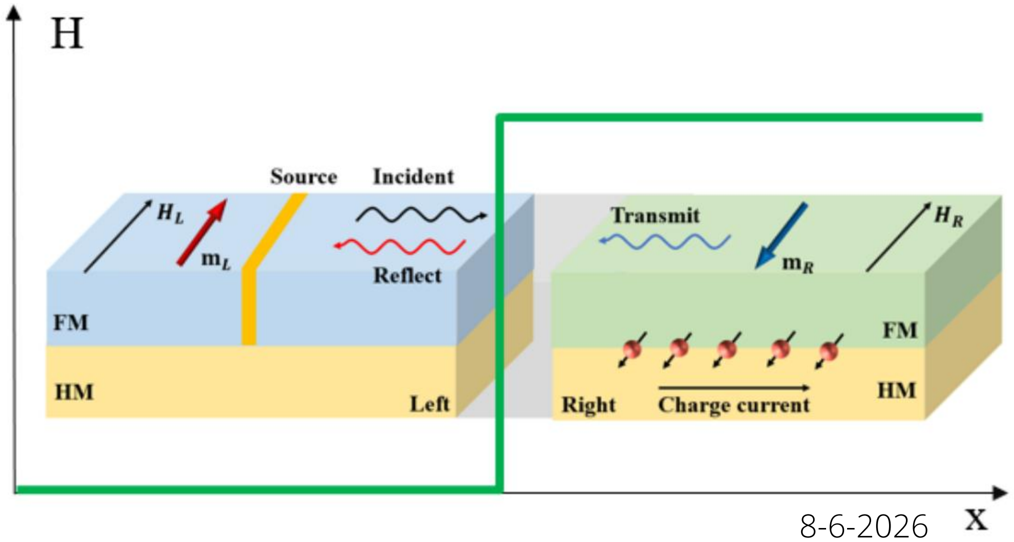
- Scattering of wavefunctions with antimagnons

$$\begin{pmatrix} O_{L,M} \\ O_{R,AM} \end{pmatrix} = \mathcal{S} \begin{pmatrix} I_{L,M} \\ I_{R,AM} \end{pmatrix} \quad \mathcal{S} = \begin{pmatrix} R & T \\ T' & R' \end{pmatrix} \quad \mathcal{S}^\dagger \eta \mathcal{S} = \eta \implies |R|^2 - |T|^2 = 1$$

$$\eta = \mathrm{diag}(1, -1)$$

- Transformation for creation (annihilation) operators of scattering states

$$\begin{pmatrix} b_{L,M}(\omega) \\ b^\dagger_{R,AM}(-\omega) \end{pmatrix} = \eta \mathcal{S}^* \eta \begin{pmatrix} a_{L,M}(\omega) \\ a^\dagger_{R,AM}(-\omega) \end{pmatrix}$$



A. L. Bassant, M. E. Y. Regout, JSH, R. A. Duine, **Phys. Rev. B 110, 094441 Sep (2024)**

Outlook

- Non-linear treatment of continuum spin waves in the left magnet
- Quantum regime
- **Especially with two interfaces**
- Extend to antiferromagnets