

Superconductivity in Two Dimensional atomic quasicrystal

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Acknowledgements



Sangmin Lee



Lucy Nathwani



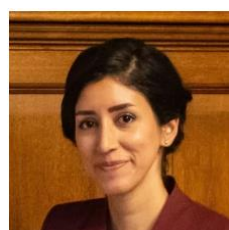
Matthias Conrad
(University of Marburg)



Paul Steinhardt
(Princeton)



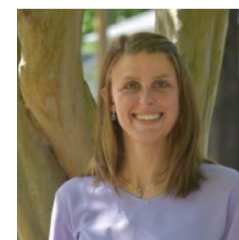
Alex Cui



Talieh Ghiasi
(now at Delft)



Jonathan Zauberman



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(now at University Marburg)

**INTELLECTUAL
VENTURES®**



Quasi-periodic & Quasicrystals

VOLUME 53, NUMBER 20

PHYSICAL REVIEW LETTERS

12 NOVEMBER 1984

Metallic Phase with Long-Range Orientational Order and No Translational Symmetry

D. Shechtman and I. Blech

Department of Materials Engineering, Israel Institute of Technology—Technion, 3200 Haifa, Israel

and

D. Gratias

Centre d'Etudes de Chimie Métallurgique, Centre National de la Recherche Scientifique, F-94400 Vitry, France

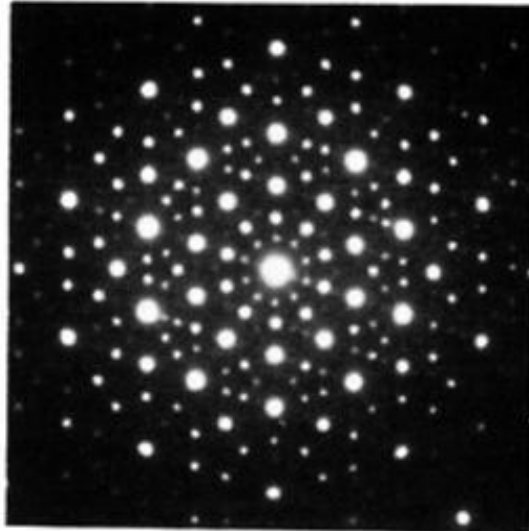
and

J. W. Cahn

Center for Materials Science, National Bureau of Standards, Gaithersburg, Maryland 20760

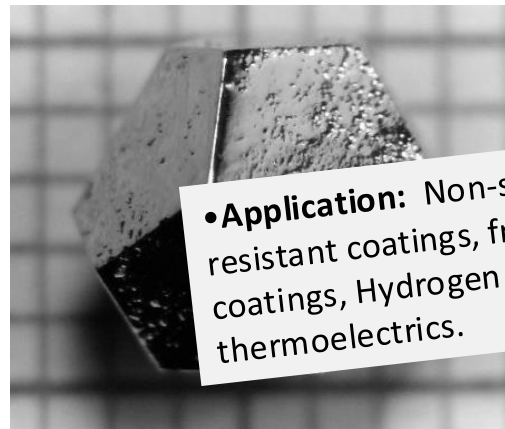
(Received 9 October 1984)

Al–Mn (at 14%) alloy



10-fold symmetry!!

Single grain of icosahedral Ho-Mg-Zn quasicrystal



•**Application:** Non-stick coating, wear-resistant coatings, friction-reducing coatings, Hydrogen storage, photonics, thermoelectrics.

Fisher et al., Phys. Rev. B59, 308 (1999)

VOLUME 53, NUMBER 26

PHYSICAL REVIEW LETTERS

24 DECEMBER 1984

Quasicrystals: A New Class of Ordered Structures

Dov Levine

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

Paul Joseph Steinhardt

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104,

and IBM Thomas J. Watson Research Laboratory,

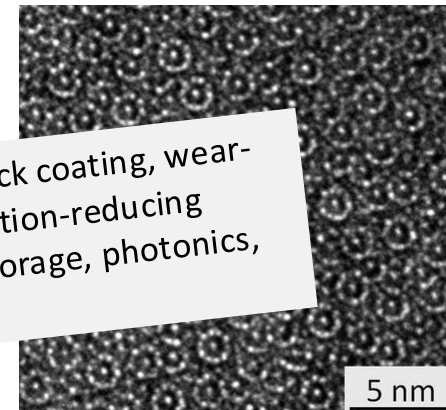
Yorktown Heights, New York 10598

(Received 2 November 1984)

A quasicrystal is the natural extension of the notion of a crystal to structures with *quasi-periodic*, rather than periodic, translational order. We classify two- and three-dimensional quasicrystals by their symmetry under rotation and show that many disallowed crystal symmetries are allowed quasicrystal symmetries. We analytically compute the diffraction pattern of an ideal quasicrystal and show that the recently observed electron-diffraction pattern of an Al-Mn alloy is closely related to that of an icosahedral quasicrystal.

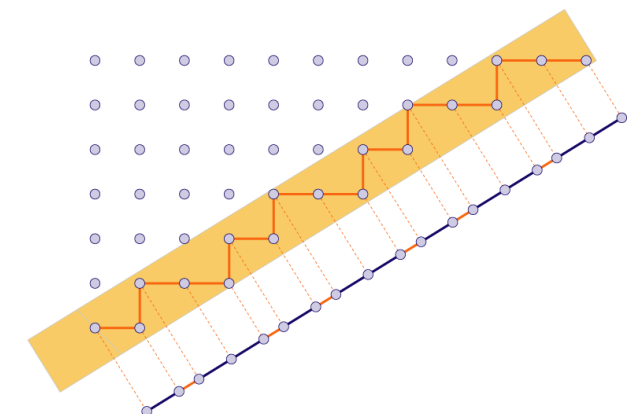
PACS numbers: 61.50.Em, 61.55.Hg, 64.70.Ew

Al₇₁Ni₂₄Fe₅ quasicrystal from meteorite



Bindi, et al., Science Report (2015)

Projection from **Higher Dimensional Periodic Crystals**
to **Lower Dimensional Quasicrystals**



Superconducting Quasicrystals

ARTICLE

DOI: 10.1038/s41467-017-02667-x

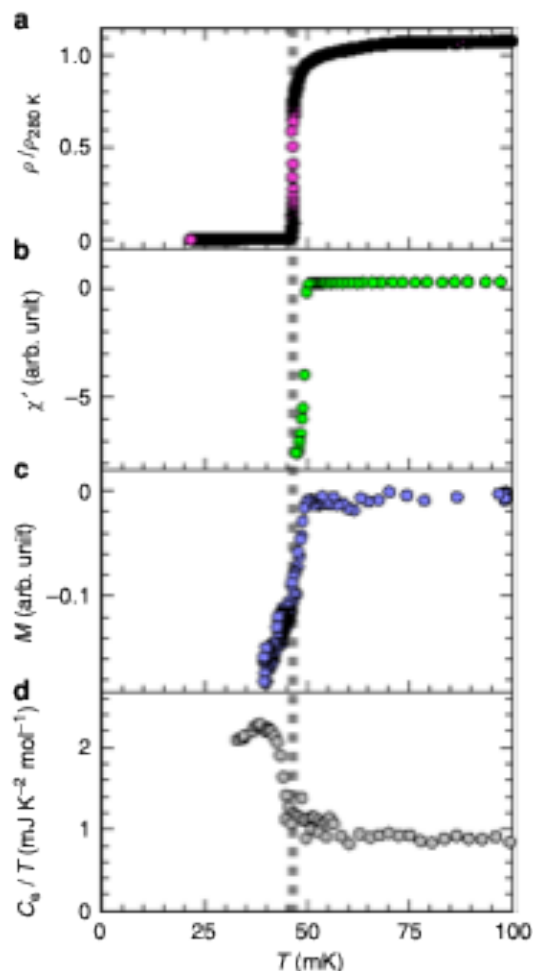
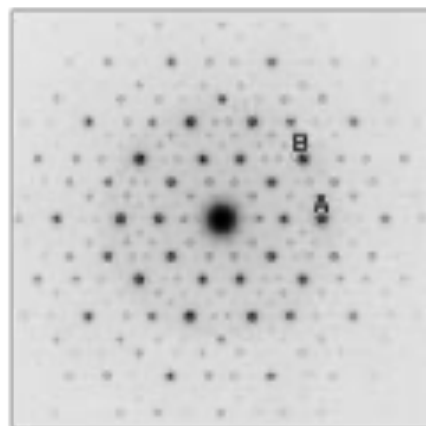
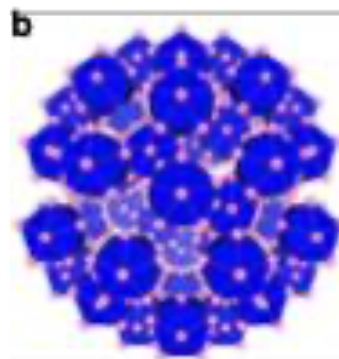
OPEN

NATURE COMMUNICATIONS | (2018)9:154

Discovery of superconductivity in quasicrystal

K. Kamiya^{1,5}, T. Takeuchi², N. Kabeya³, N. Wada¹, T. Ishimasa⁴, A. Ochiai³, K. Deguchi¹, K. Imura¹ & N.K. Sato¹

Al-Zn-Mg QC₅



PHYSICAL REVIEW B **95**, 024509 (2017)

Superconductivity on a quasiperiodic lattice: Extended-to-localized crossover of Cooper pairs

Shiro Sakai,¹ Nayuta Takemori,¹ Akihisa Koga,² and Ryotaro Arita¹

¹Center For Emergent Matter Science, RIKEN, Wako, Saitama 351-0198, Japan

²Department of Physics, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan

(Received 12 August 2016; revised manuscript received 20 November 2016; published 18 January 2017)

PHYSICAL REVIEW B **100**, 014510 (2019)

Conventional superconductivity in quasicrystals

Ronaldo N. Araújo and Eric C. Andrade

Instituto de Física de São Carlos, Universidade de São Paulo, C.P. 369, São Carlos, SP, 13560-970, Brazil

PHYSICAL REVIEW RESEARCH **3**, 023195 (2021)

- Quasiperiodic/fractal gap structures
- Modified critical fluctuation
- Unusual collective modes

Letter

Gapless superconductivity and its real-space topology in quasicrystals

Kazuma Saito¹, Masahiro Hori,^{1,2} Ryo Okugawa¹, K. Tanaka² and Takami Tohyama¹

¹Department of Applied Physics, Tokyo University of Science, Katsushika, Tokyo 125-8585, Japan

²Department of Physics and Engineering Physics, and Centre for Quantum Topology and Its Applications (quanTA), University of Saskatchewan, 116 Science Place, Saskatoon, Saskatchewan, Canada S7N 5E2

(Received 1 October 2024; revised 19 February 2025; accepted 13 June 2025; published 25 June 2025)

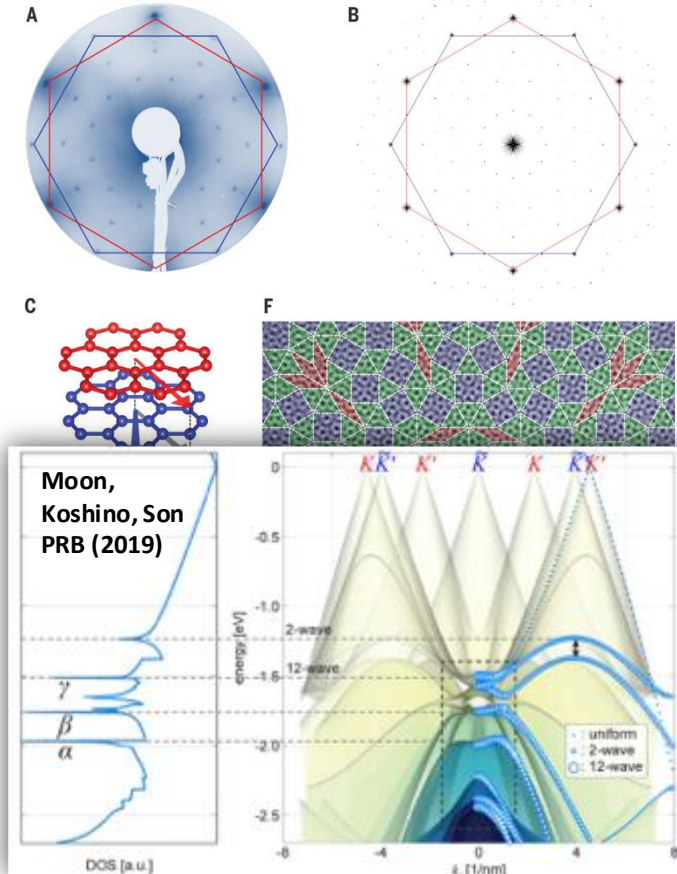
Van der Waals Interlayer Quasi Crystals

QUASICRYSTALS

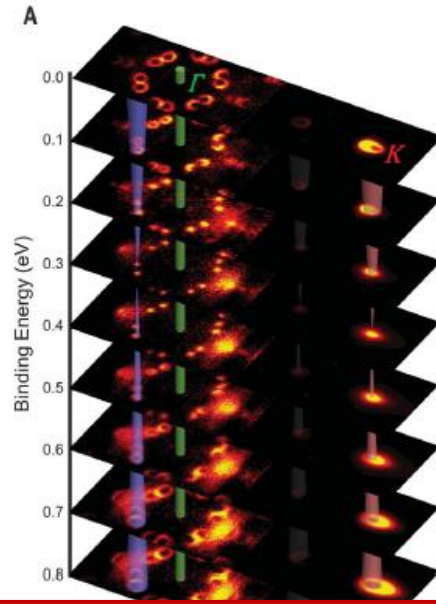
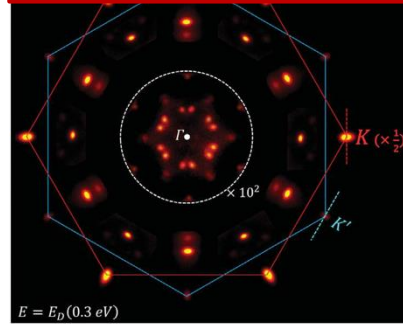
Ahn et al., Science 361, 782–786 (2018) 24 August 2018

Dirac electrons in a dodecagonal graphene quasicrystal

Sung Joon Ahn^{1*}, Pilkyung Moon^{2,3*}, Tae-Hoon Kim^{4*}, Hyun-Woo Kim¹, Ha-Chul Shin¹, Eun Hye Kim¹, Hyun Woo Cha⁴, Se-Jong Kahng⁵, Philip Kim⁶, Mikito Koshino⁷, Young-Woo Son^{8,†}, Cheol-Woong Yang^{4,†}, Joung Rea Ahn^{1,9,†}



Appearance of higher order Umklapp features



- Weak interlayer interaction/ High energy QC features
- Long wavelength moire approximation

Article

494 | Nature | Vol 625 | 18 January 2024

Tuning commensurability in twisted van der Waals bilayers

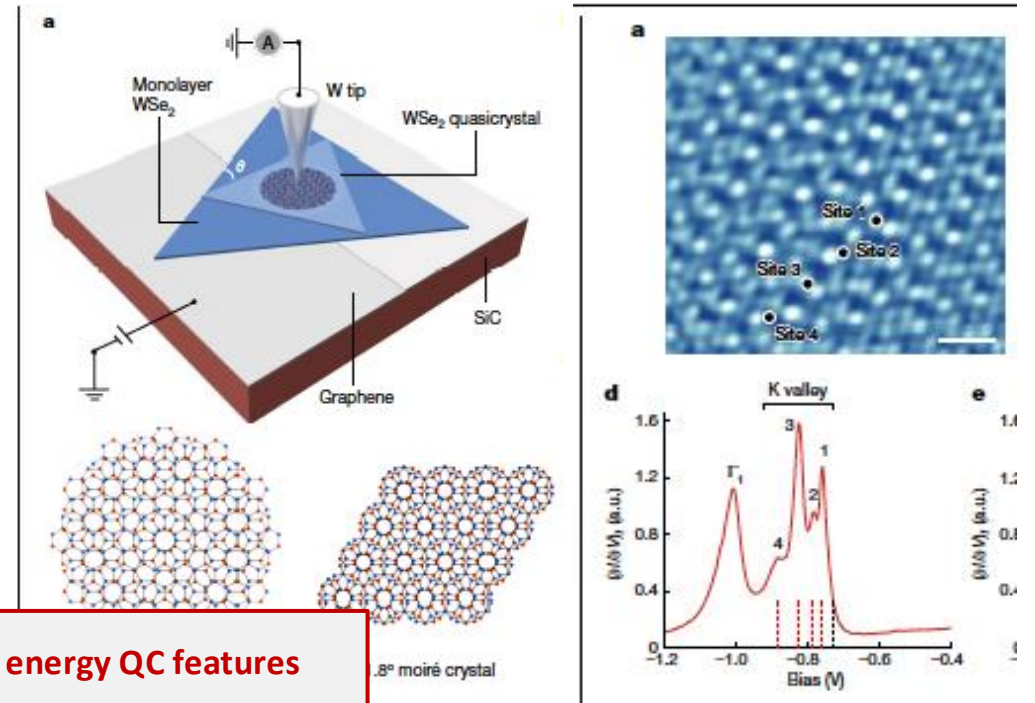
<https://doi.org/10.1038/s41586-023-06904-w>

Received: 26 March 2023

Accepted: 28 November 2023

Published online: 17 January 2024

Yanxing Li^{1,2}, Fan Zhang^{1,2}, Viet-Anh Ha^{1,2}, Yu-Chuan Lin^{3,4}, Chengye Dong³, Qiang Gao¹, Zhida Liu¹, Xiaohui Liu¹, Sae Hee Ryu⁵, Hyunsue Kim¹, Chris Jozwiak⁶, Aaron Bostwick⁶, Kenji Watanabe⁶, Takashi Taniguchi⁶, Bishoy Kousa¹, Xiaoqin Li¹, Eli Rotenberg⁶, Eslam Khalaf¹, Joshua A. Robinson³, Feliciano Giustino^{1,2} & Chih-Kang Shih^{1,2,3}



2D Atomic Quasi Crystal

$$\frac{3+2\sqrt{3}}{4} = 1.616025404 \dots$$

Angew. Chem. Int. Ed. **1998**, *37*, No. 10

A Dodecagonal Quasicrystalline Chalcogenide**

Matthias Conrad, Frank Krumeich, and Bernd Harbrecht*

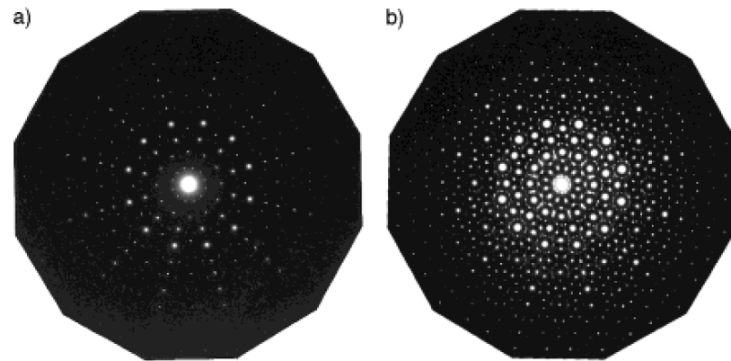
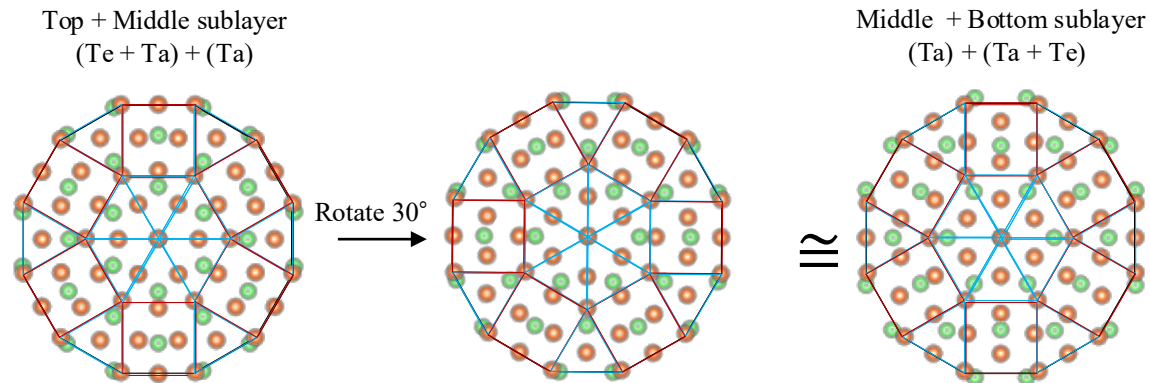
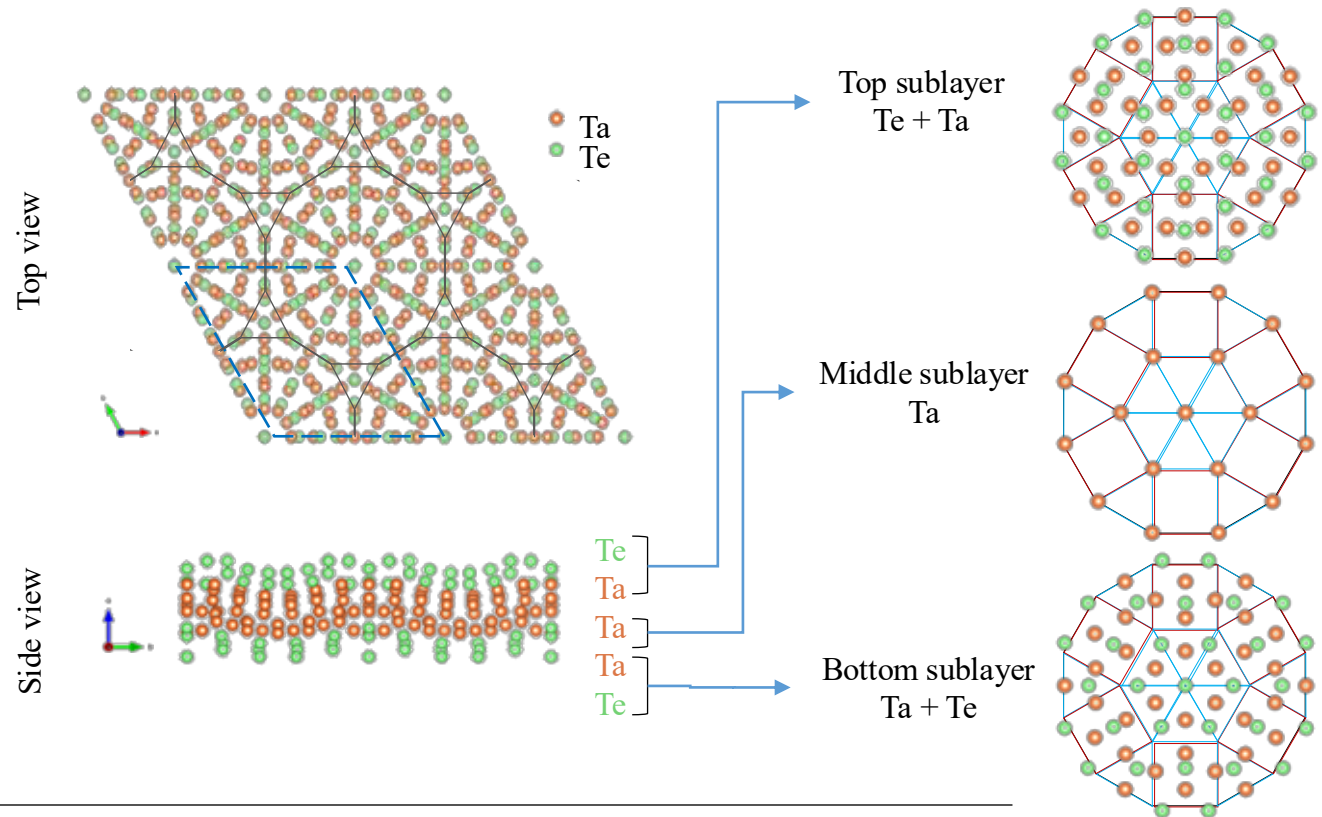
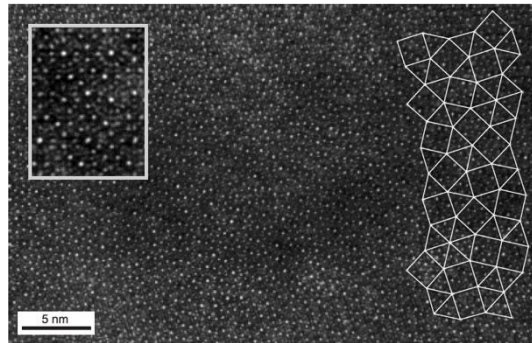


Figure 2. Reflection-poor (a) and reflection-rich (b) electron diffractograms of dd-Ta_{1.6}Te.



van der Waals 2D Atomic Quasi Crystals

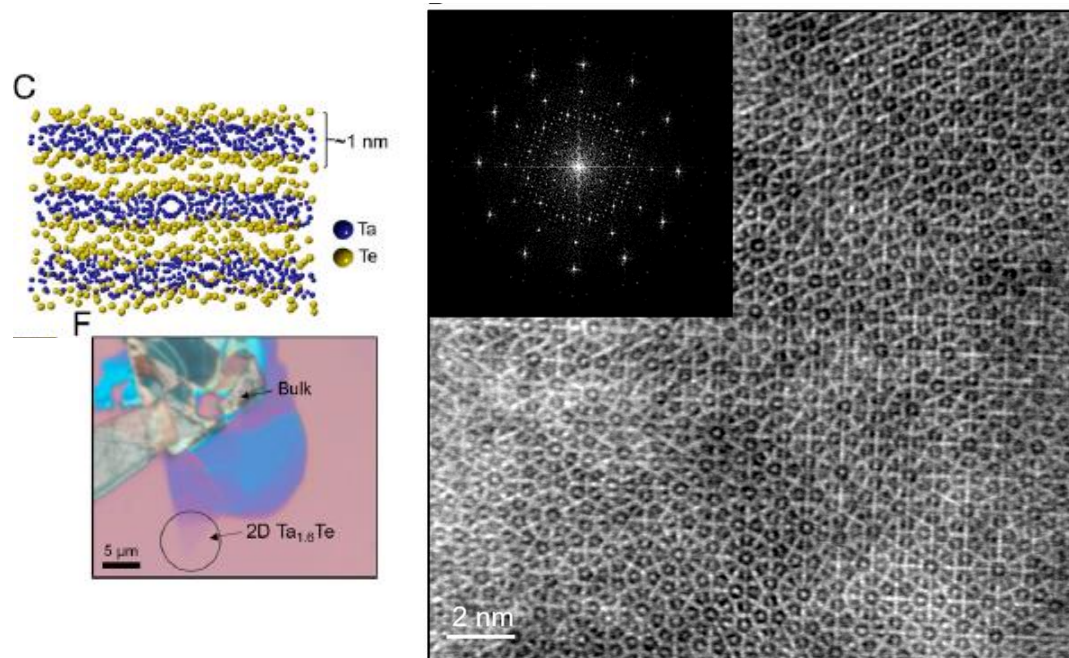
Nature Communications | (2024)15:1529

Layer-dependent topological phase in a two-dimensional quasicrystal and approximant

Jeffrey D. Cain^{a,b,c,1}, Amin Azizi^{a,c,1}, Matthias Conrad^d, Sinéad M. Griffin^{b,e}, and Alex Zettl^{a,b,c,2}

^aDepartment of Physics, University of California, Berkeley, CA 94720; ^bMaterials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720; ^cKavli Energy NanoSciences Institute, University of California, Berkeley and Lawrence Berkeley National Laboratory, Berkeley, CA 94720; ^dFachbereich Chemie, Philipps-Universität Marburg, 35032 Marburg, Germany; and ^eThe Molecular Foundry, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

Edited by Angel Rubio, Max Planck Institute for the Structure and Dynamics of Matter, Hamburg, Germany, and approved August 24, 2020 (received for review July 17, 2020)



- Can be cleaved down to monolayer!
- Quasicrystals & Approximants

Article

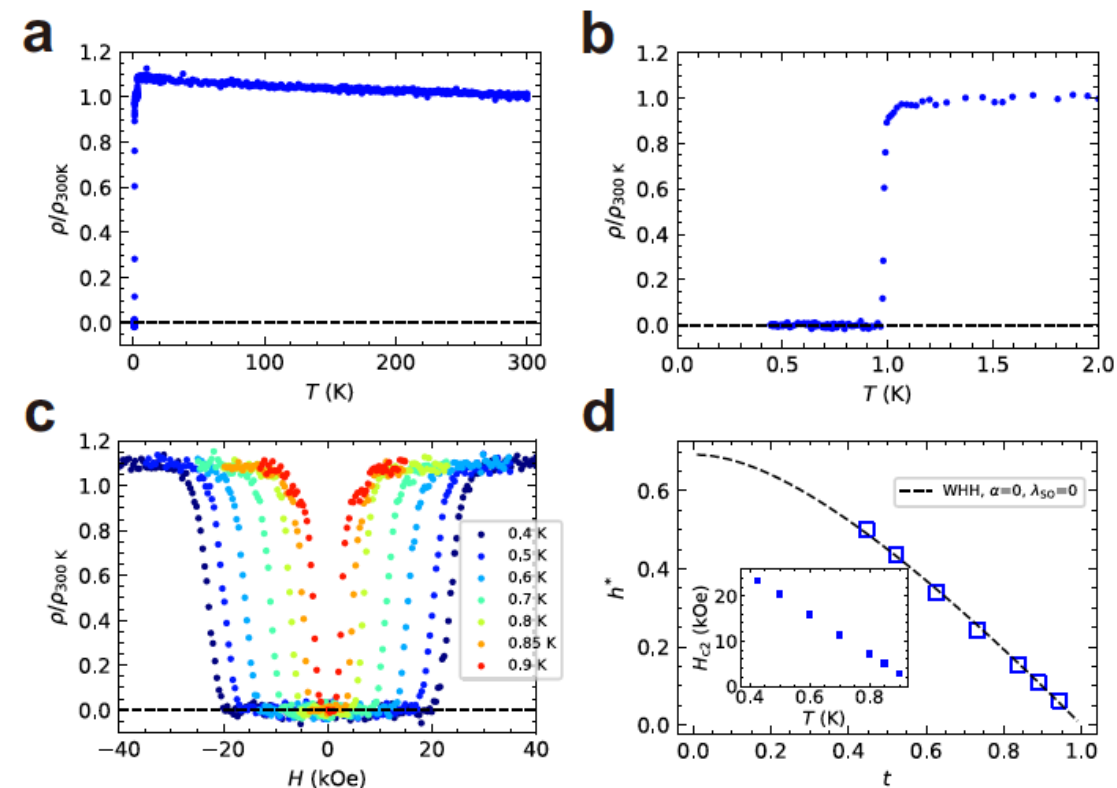
Superconductivity in a van der Waals layered quasicrystal

Measurement was done on polycrystals

Received: 29 August 2023

Accepted: 8 February 2024

Yuki Tokumoto¹, Kotaro Hamano¹, Sunao Nakagawa¹, Yasushi Kamimura¹, Shintaro Suzuki², Ryuji Tamura³ & Keiichi Edagawa¹



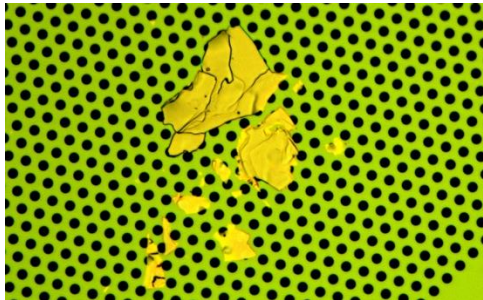
Opening Up 25-Year-Old Samples

Flux grown single crystals of
Ta-Te Quasicrystals and Approximants

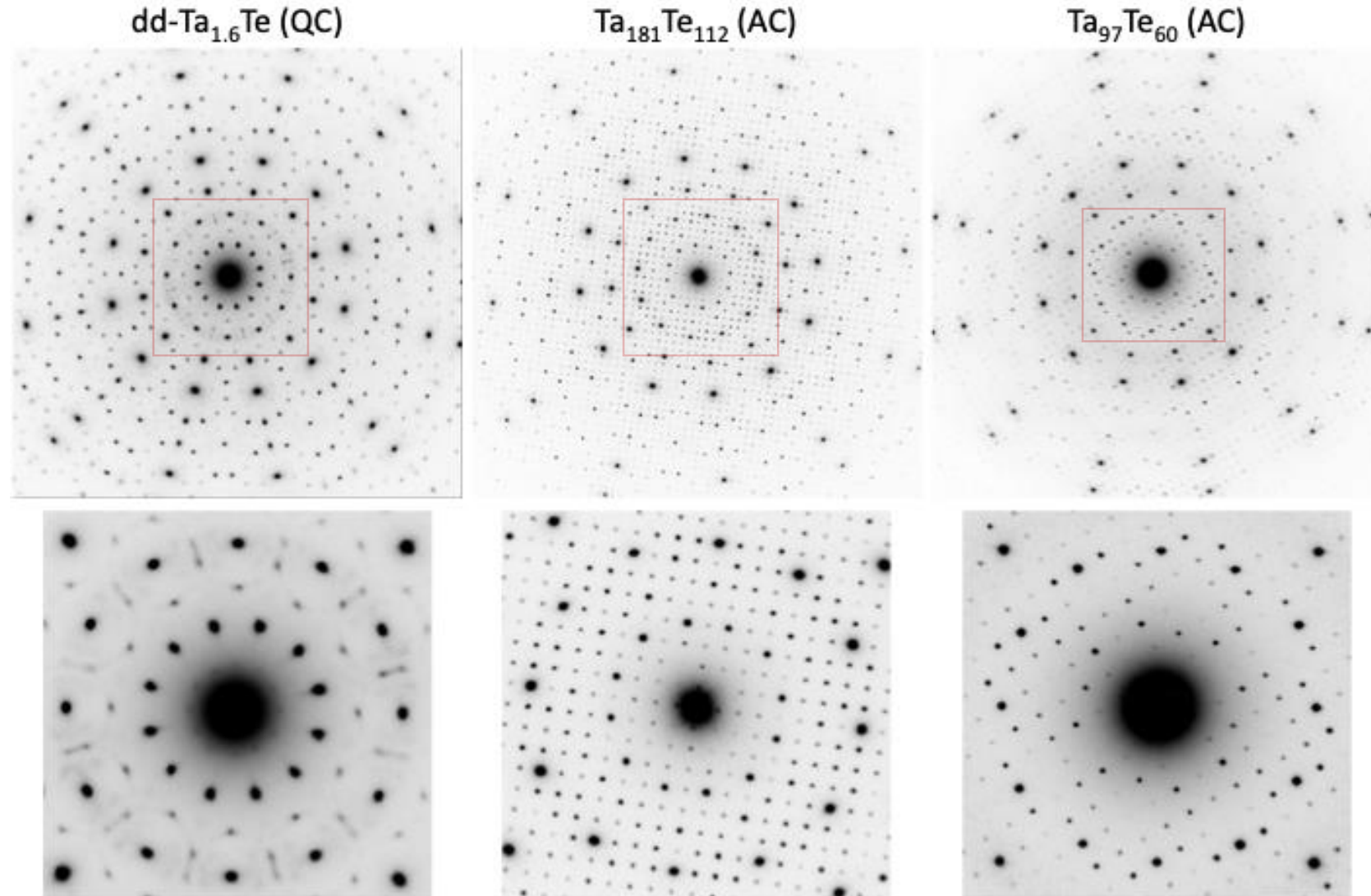


Samples are air-sensitive!

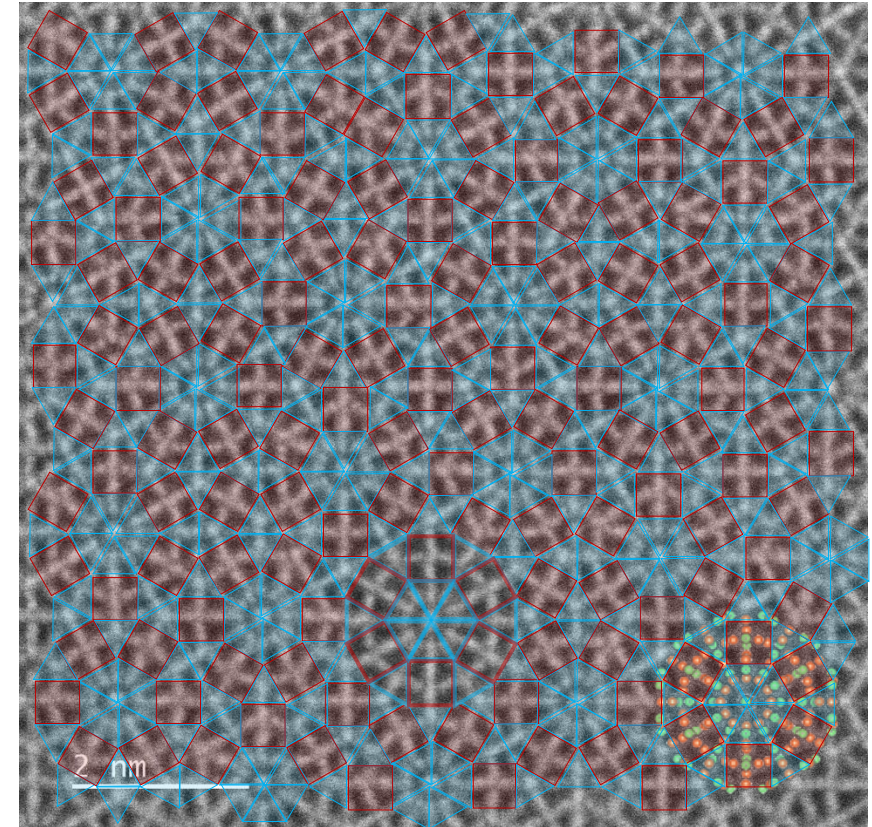
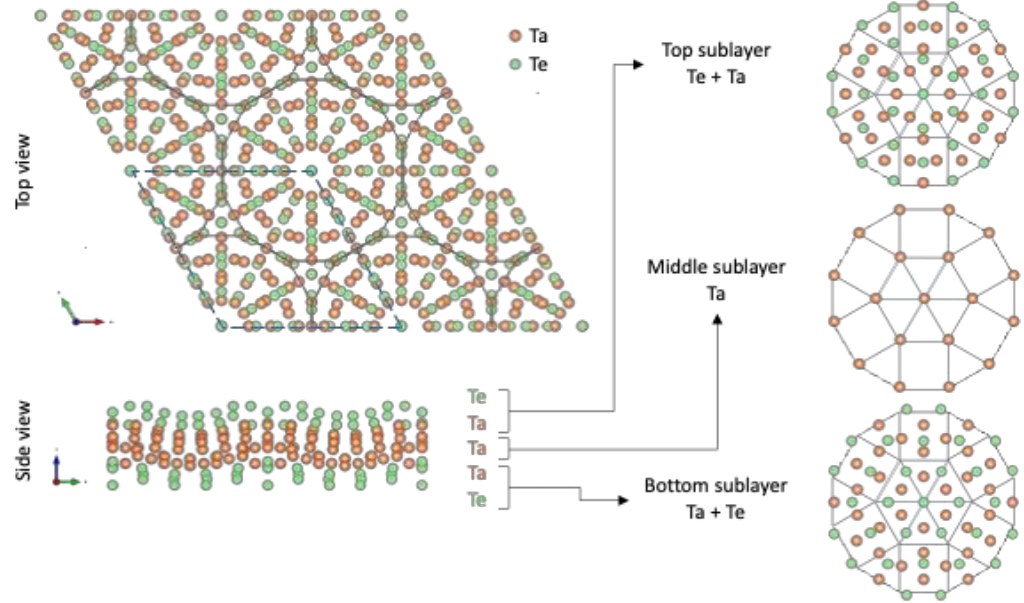
TEM sample preparation



Selective Area Diffraction



Dodecagonal Atomic Clusters: Building Block

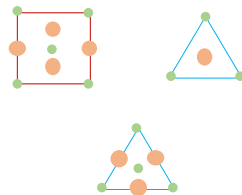


Ta
Te

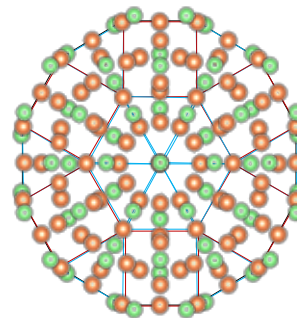
Basic building blocks:
(prototiles)



Decoration with atoms:



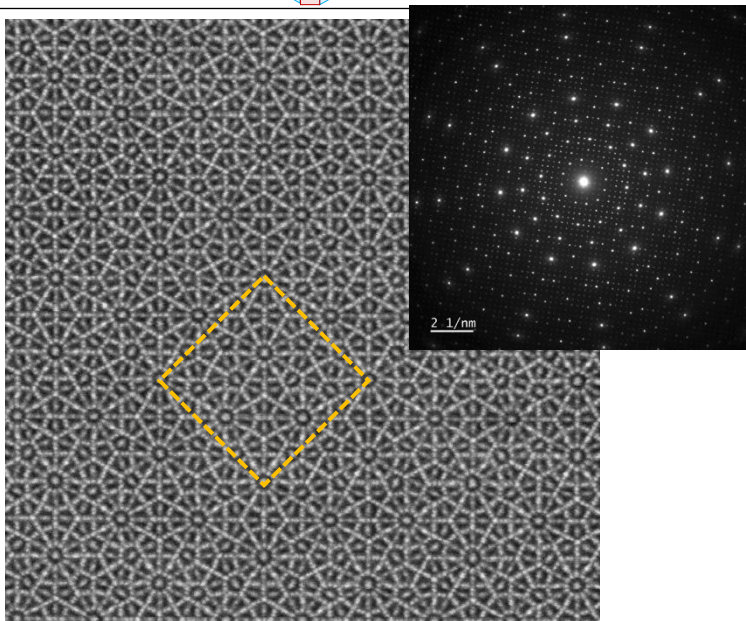
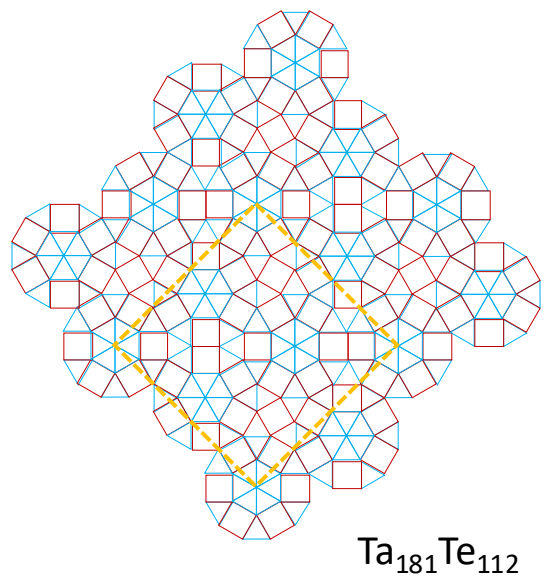
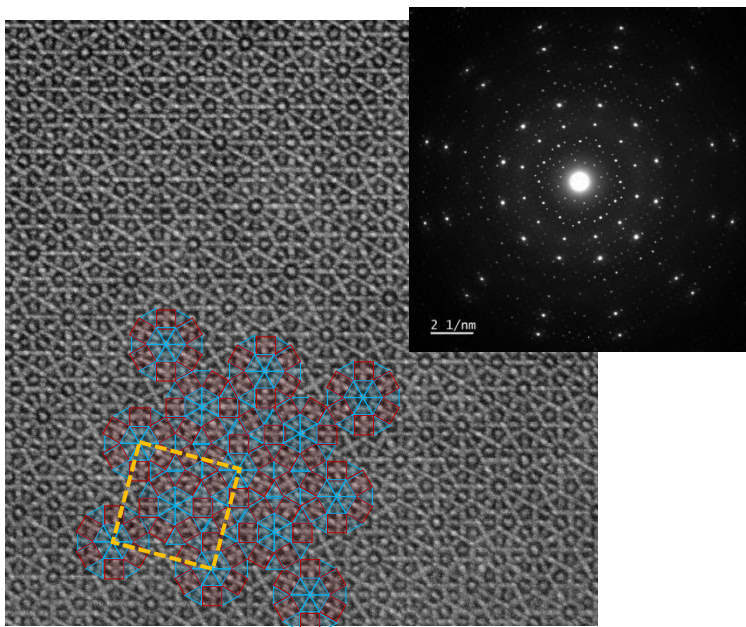
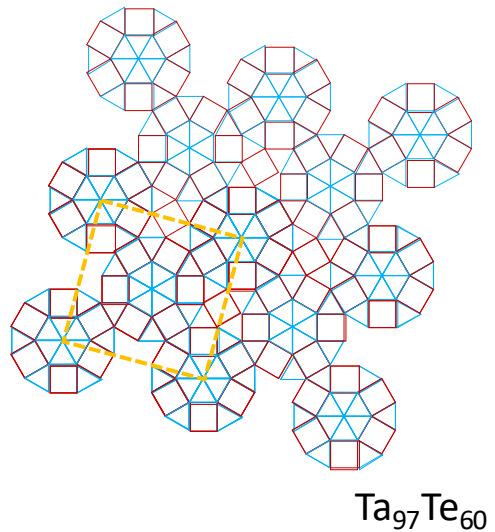
Dodecagonal building blocks



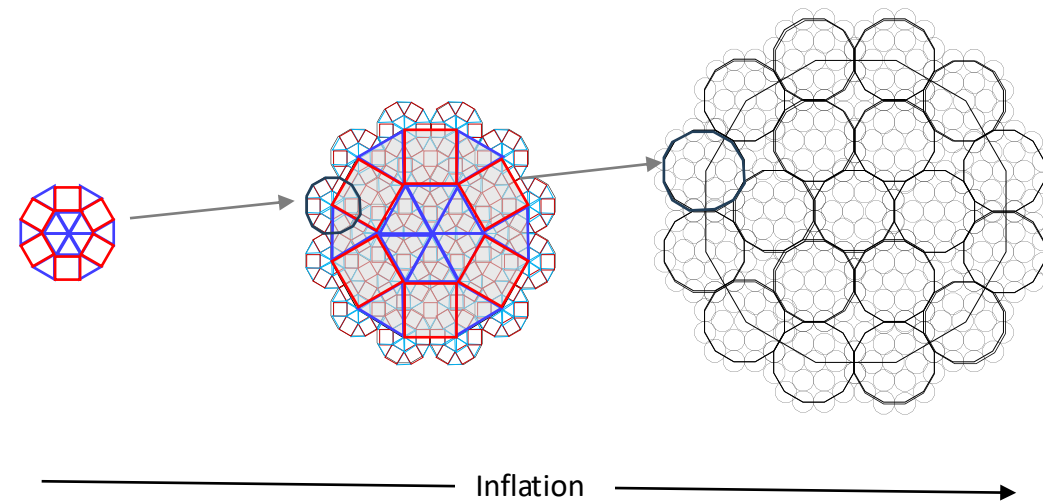
High-Angle Annular Dark-Field Scanning Transmission
Electron Microscopy (HAADF STEM)

Approximants and Quasicrystals

Approximant tiling

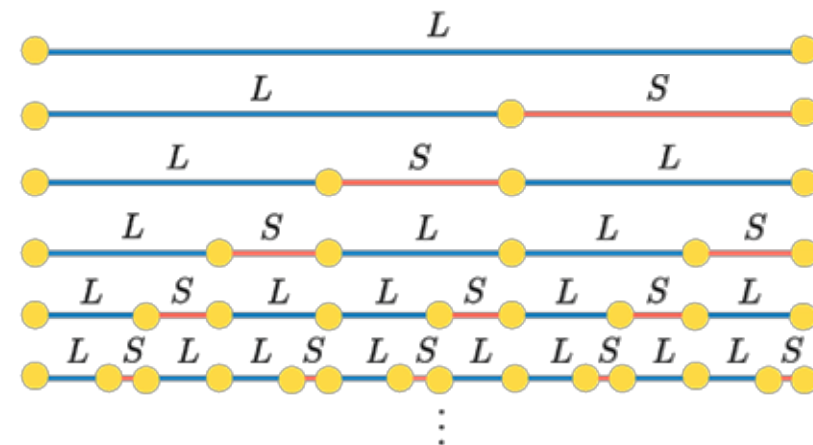


Quasicrystal Tiling



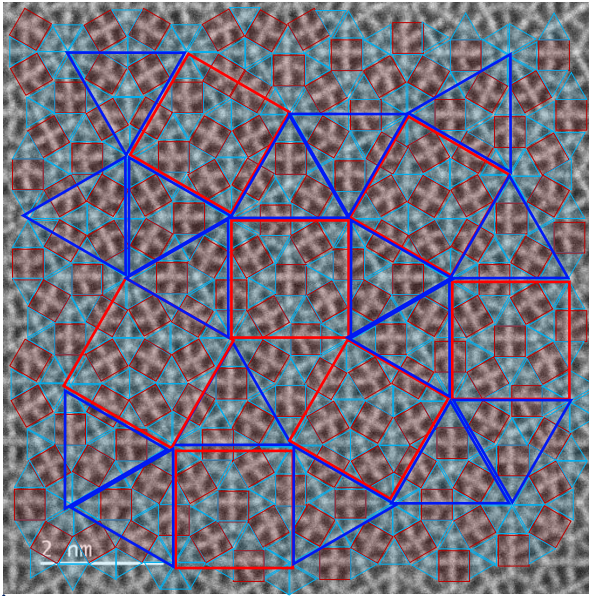
Fibonacci Chain

$$L \rightarrow L + S; \quad S \rightarrow L$$

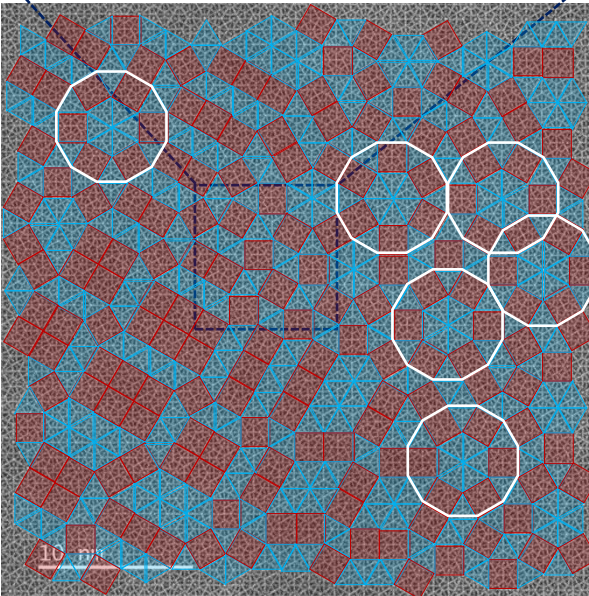


Inflation Rules: Second Order Tiling

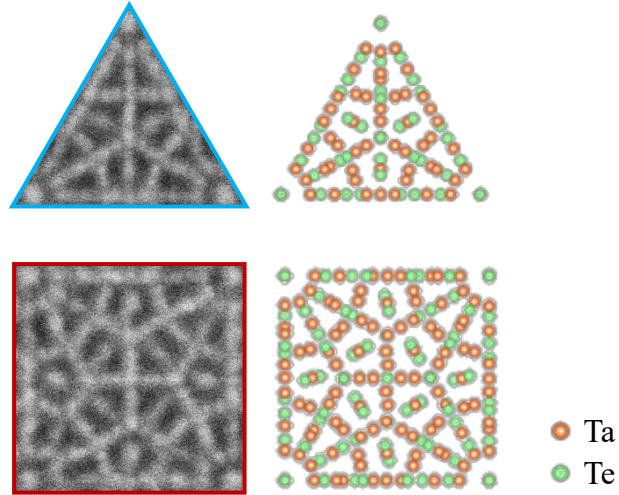
First tiling



Second tiling



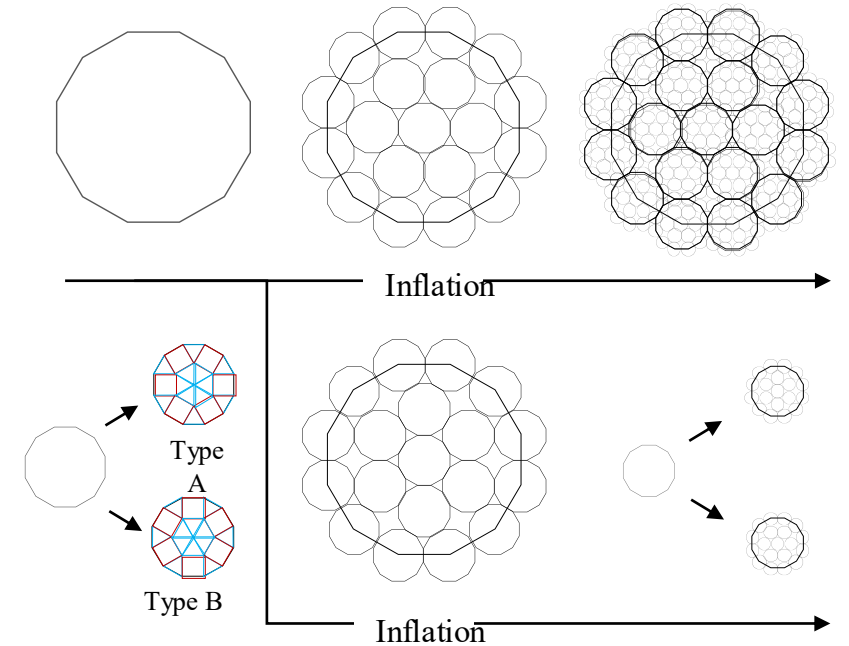
Second Order Tiles



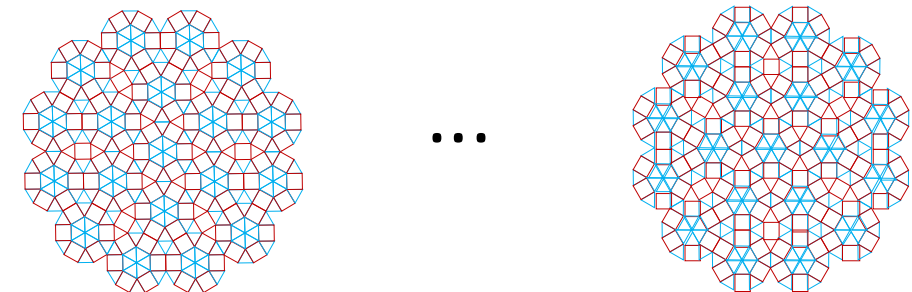
- Tiling can be done using larger squares and triangles with a scaling factor of $2 + \sqrt{3}$.
- Beyond the second-order tiling, it is not possible to proceed in the same way. The inflation rule does not apply.

**Failure of the inflation rule
after second order tiling!**

Ambiguity of Substitution Rule



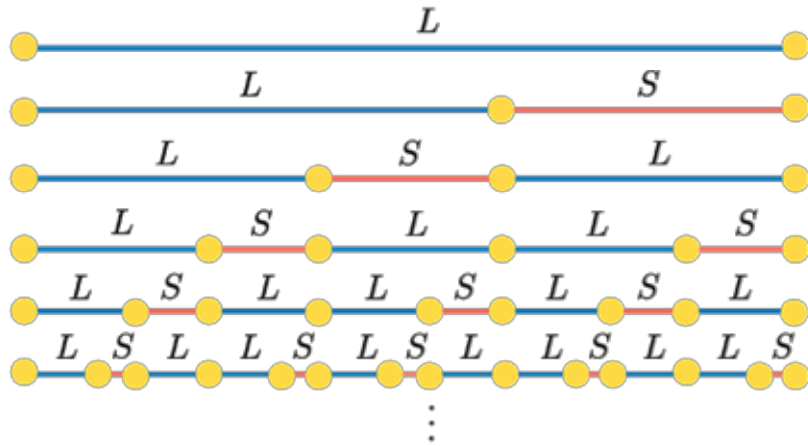
Many possible inflation possibility...



Cut and Projection Method: Fibonacci Chain

Fibonacci Chain

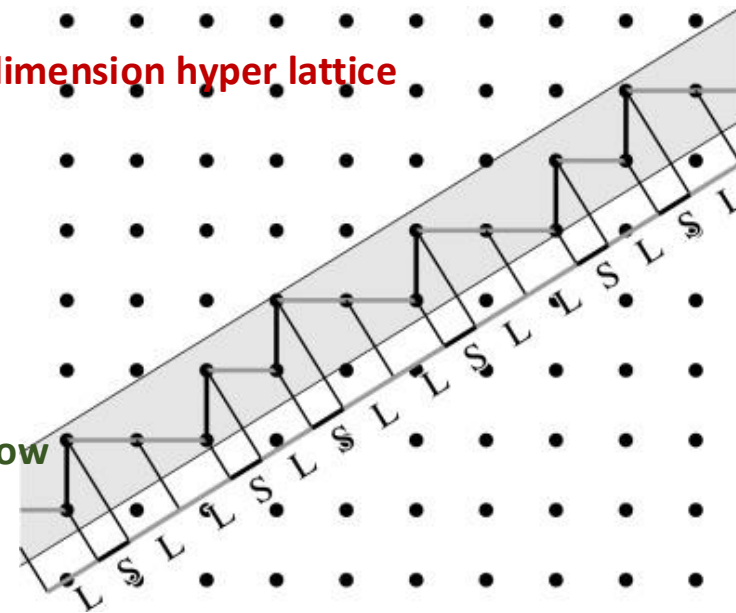
$L \rightarrow L+S; \quad S \rightarrow L$



A system without periodicity in real space can exhibit periodicity in higher-dimensional space.

Higher dimension hyper lattice

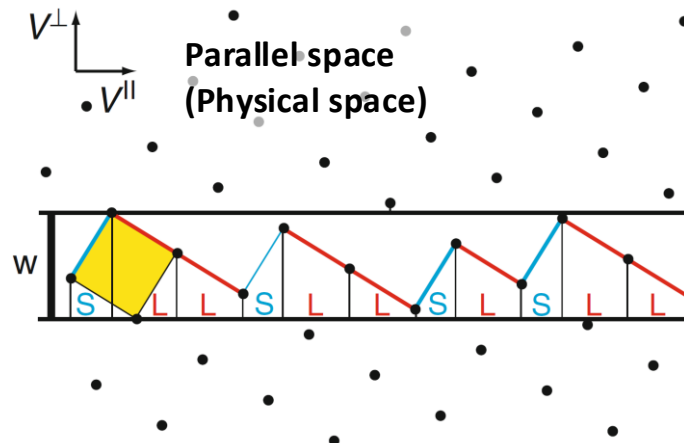
Acceptance window



Slope:

$$\tau = \frac{1 + \sqrt{5}}{2}$$

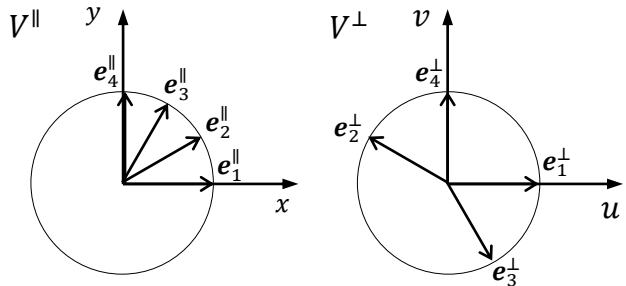
Perpendicular space



Acceptance window

Physical Space and Perpendicular Space: Toward Higher Dimension

The dodecagonal square-triangle tiling can be generated from 4D lattice.



2D physical Space

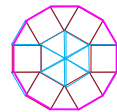
2D Perpendicular

4D basis vectors

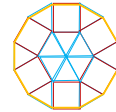
$$e_i = (e_i^{\parallel}, e_i^{\perp})$$

$$e_i^{\parallel} = R_{12}^{(i-1)} e_1^{\parallel}$$

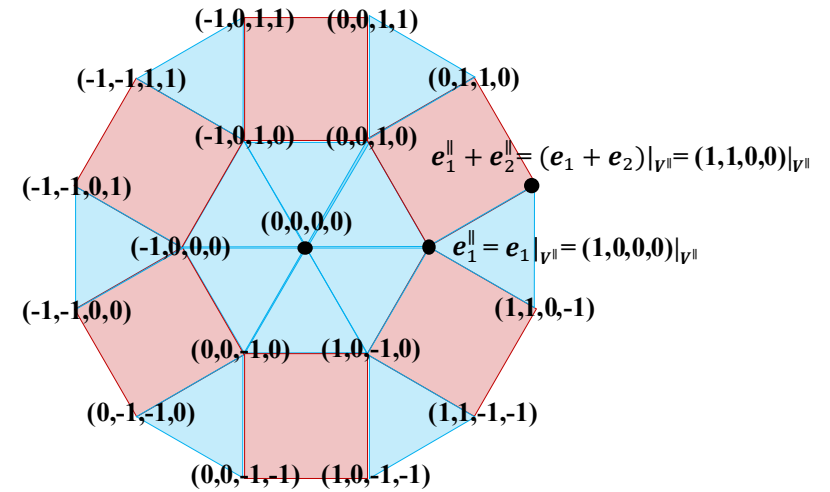
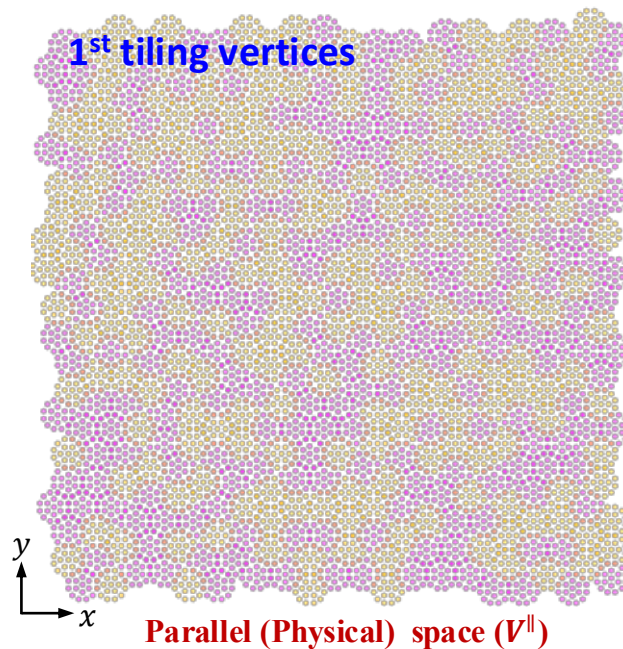
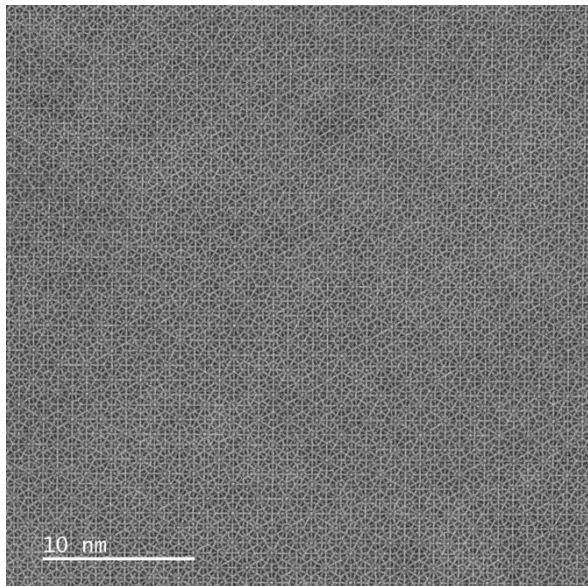
$$e_i^{\perp} = R_{12}^{5(i-1)} e_1^{\perp}$$



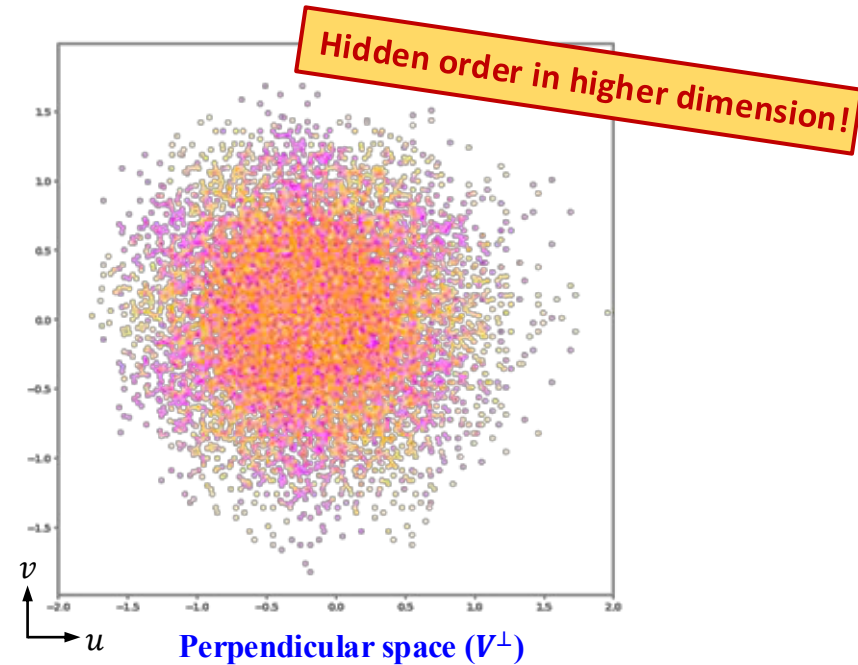
#(A) = 219



#(B) = 211



Coordination with basis $e_i (i = 1, 2, 3, 4)$

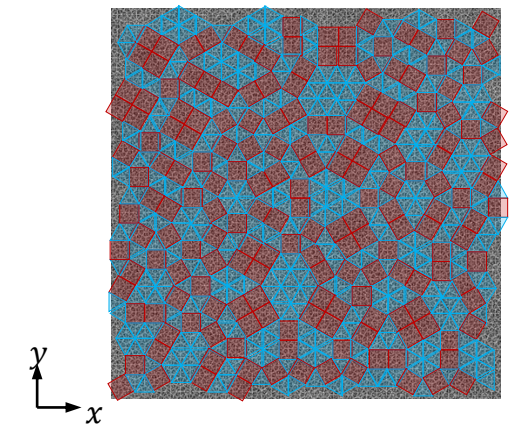
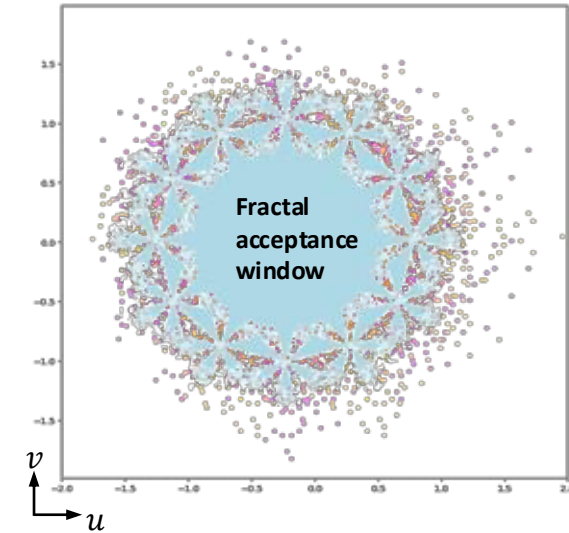
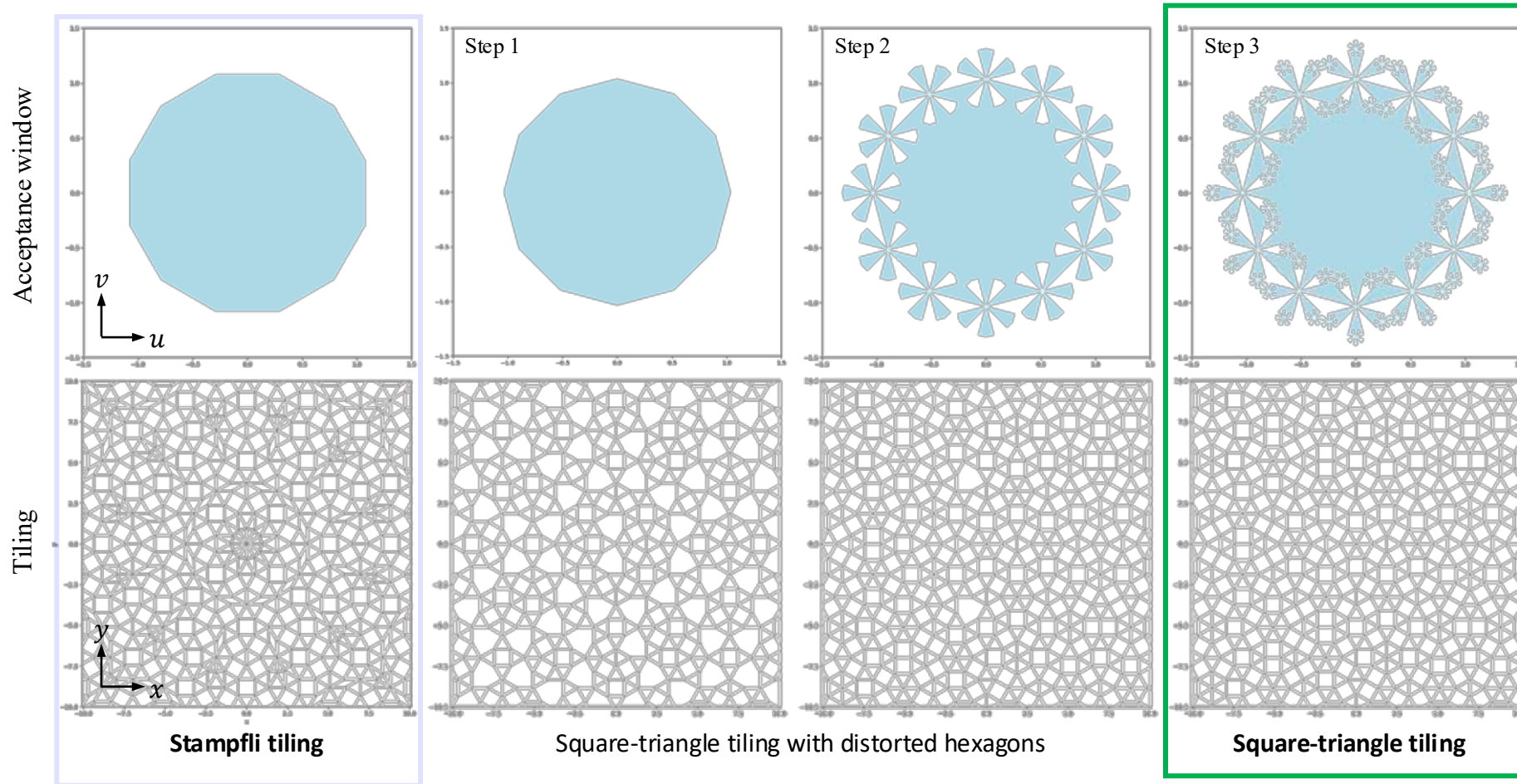


Cut and Projection Method: Fractal Acceptance Window

Dodecagonal Quasicrystal Tiling: 4D Hyper Lattice projection with acceptance windows

Inspired by Baake, et. al., Physica A 191 554 (1992)

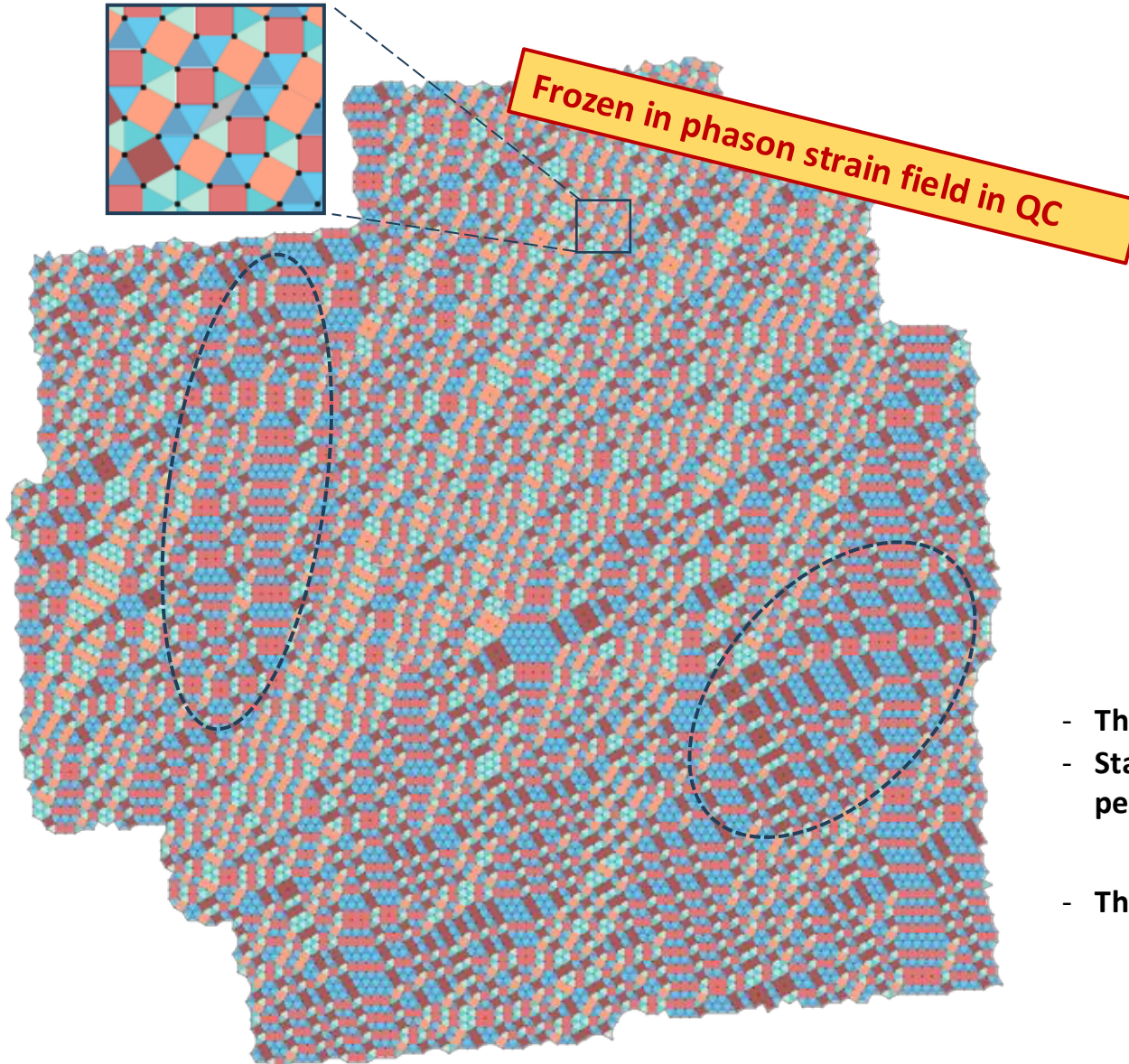
Fractal acceptance window works reasonable but not perfect.
Disorder in quasicrystal?



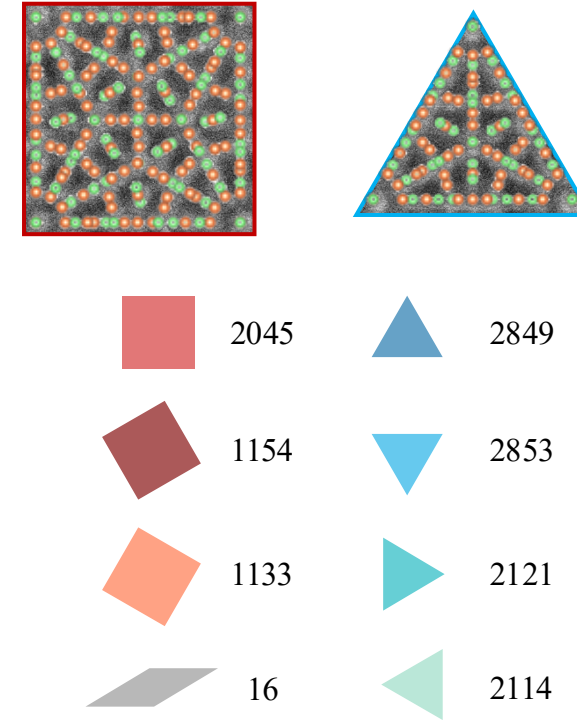
A **fractal-shaped** acceptance window is required to eliminate skinny rhombi in the Stampfli tiling.

Second Order Tiling and Phason Flip Disorders Quasicrystals

- There exist defects that require rhombus tiles.



Second order tiles

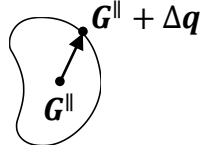


- The distribution of second-order tiles is not isotropic.
- Statistically, it is not maximally entropic and does not exhibit perfect 12-fold symmetry.
- There exist domains composed of only specific types of tiles.

Temperature Dependent Diffraction: Phonon versus Phason Elastic Constants

$$I(\mathbf{G}^{\parallel} + \Delta\mathbf{q}) \propto \exp\left[-\frac{1}{2}\Delta\mathbf{q}^T C^{-1}(\mathbf{G})\Delta\mathbf{q}\right]$$

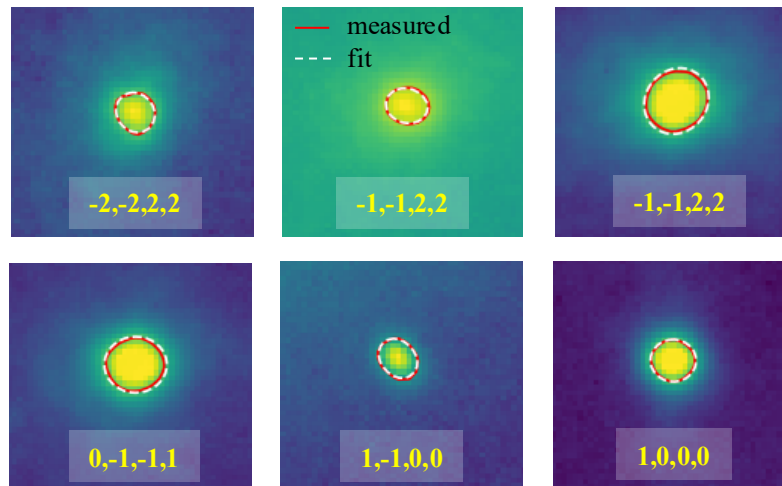
$\mathbf{G} = \mathbf{G}^{\parallel} + \mathbf{G}^{\perp}$



$$C = C_{\text{phonon}} + C_{\text{phason}} + C_{\text{coupling}}$$

$$= C_{\text{phonon}}(\mathbf{G}^{\parallel}, T) + C_{\text{phason}}(\mathbf{G}^{\perp}) + C_{\text{coupling}}(\mathbf{G}^{\parallel}, \mathbf{G}^{\perp}, T)$$

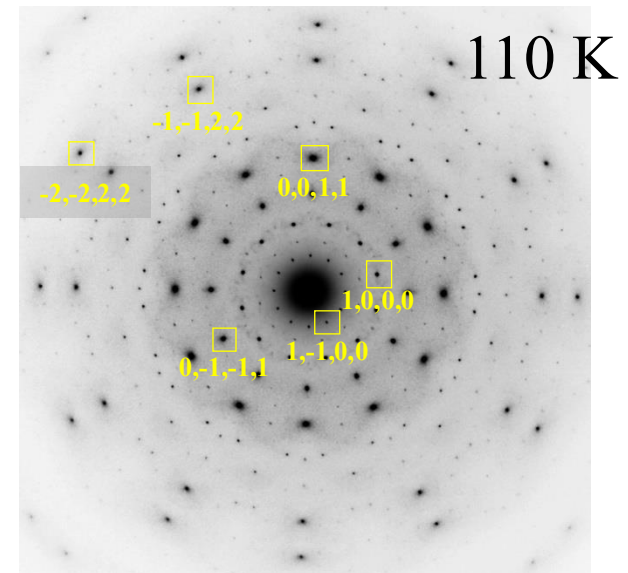
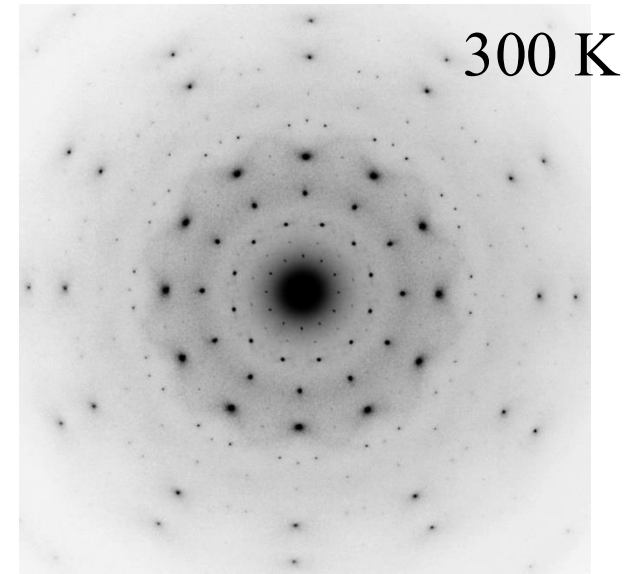
- If the phason contribution is temperature-independent, temperature-dependent diffraction patterns can be used to isolate the peak broadening caused by phonon fluctuations and phonon-phason coupling.



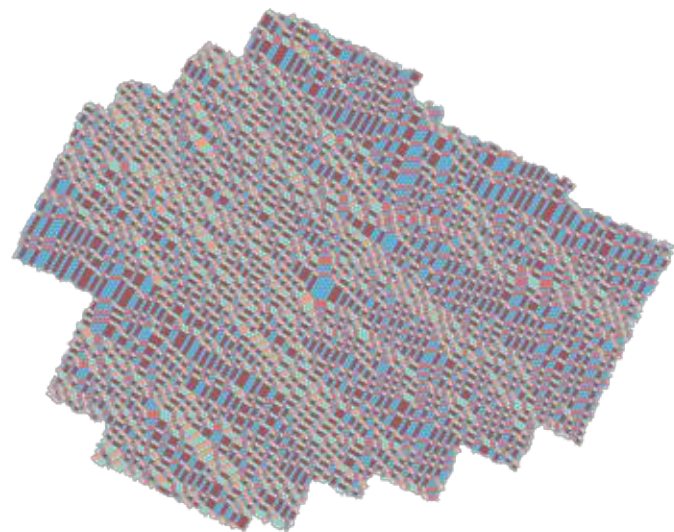
$$\begin{pmatrix} K_{tt} & K_{tr} & K_{ts} \\ K_{tr} & K_{rr} & K_{rs} \\ K_{ts} & K_{rs} & K_{ss} \end{pmatrix} = \begin{pmatrix} 1.042 & 0.002 & -0.167 \\ 0.002 & 1.012 & -0.013 \\ -0.167 & -0.013 & 0.948 \end{pmatrix} \xrightarrow{12\text{-fold}} \begin{pmatrix} K_{tt} & 0 & 0 \\ 0 & K_{tt} & 0 \\ 0 & 0 & K_{ss} \end{pmatrix}$$

$K_{tt} \approx K_{rr} > K_{ss} \gg K_{tr}, K_{ts}, K_{rs}$

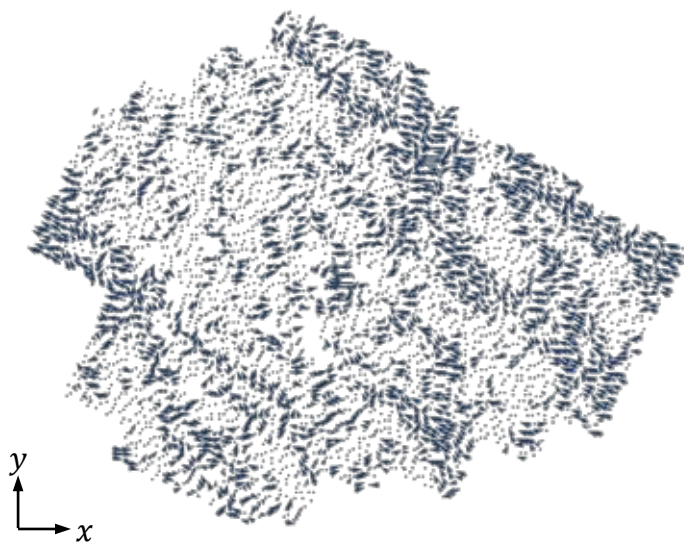
- The phason elastic constant becomes comparable to that of the 12-fold quasicrystalline phase.



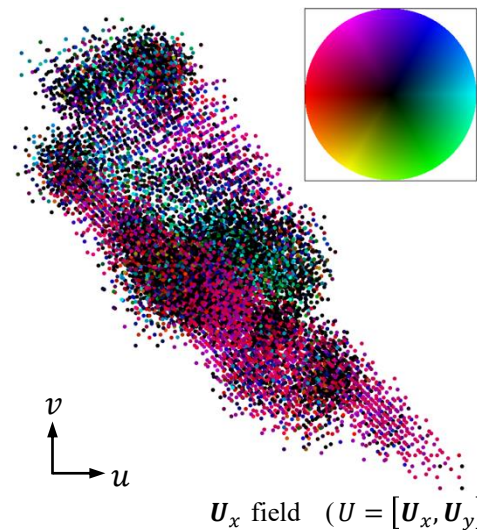
Phason Strain Field in Second Order Tiling



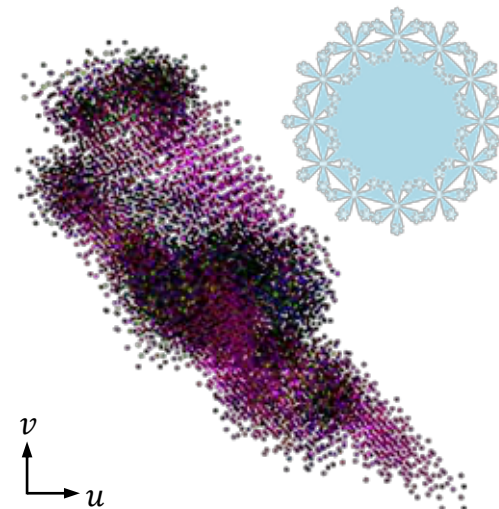
phason strain field in real space



phason strain field in perpendicular space



U_x field ($U = [U_x, U_y]$)



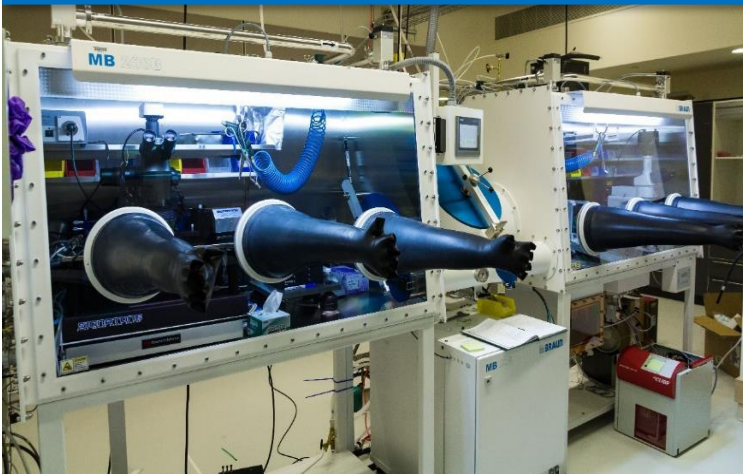
U_y field

- The direction of the local phason strain field follows the elongation observed in perpendicular space.

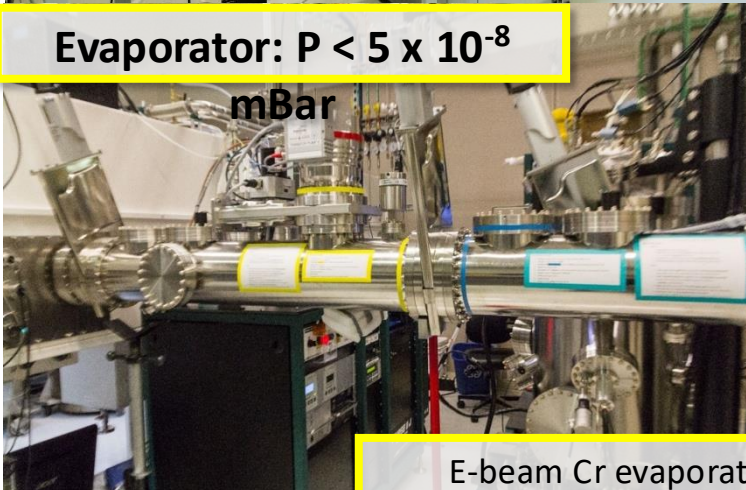
- Larger local phason strain is observed in approximant-like domains, while QC-like domains retain relatively small phason strain.

Device Fabrication

Glovebox: H₂O Concentration 5 - 40 ppb



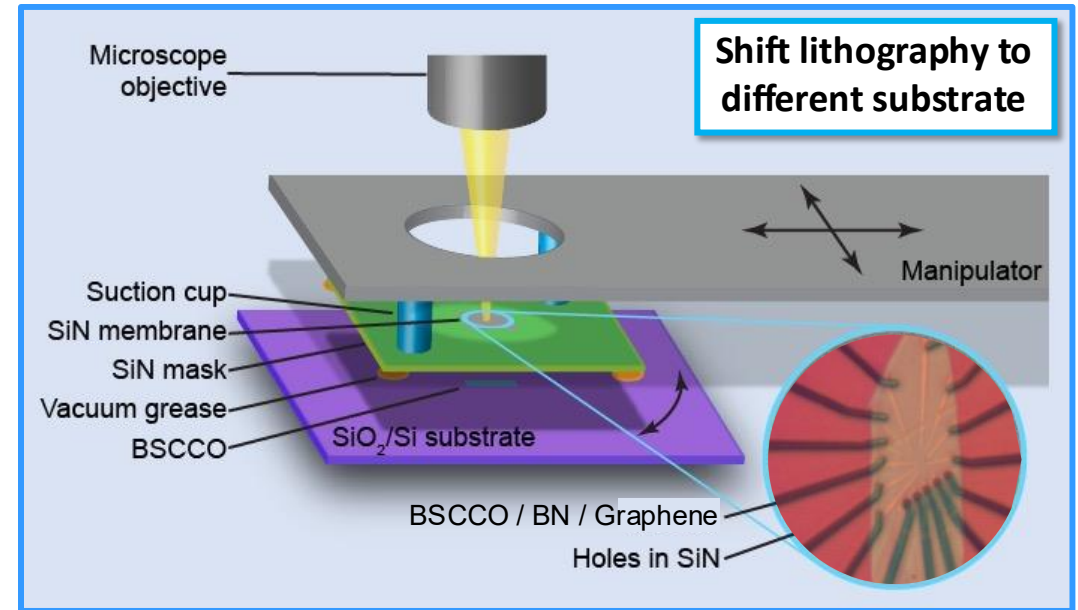
Evaporator: $P < 5 \times 10^{-8}$ mBar



Direct connection to argon glovebox

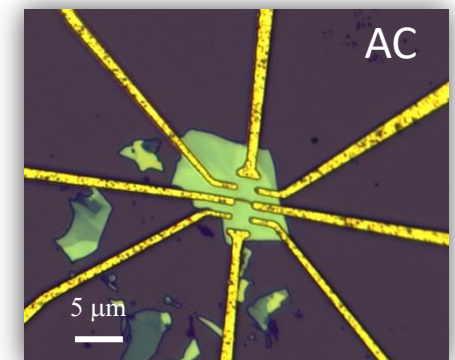
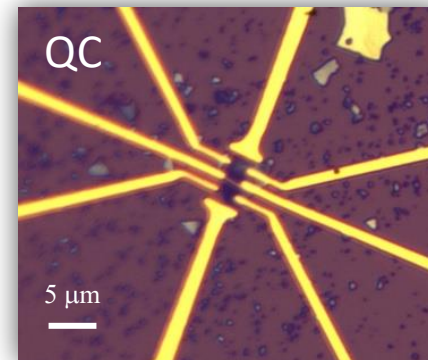
E-beam Cr evaporation source
Small Thermal Au evaporation source
Argon ion milling
Thermoelectric cooled sample stage

- *Stacking*
- *Selection*
- ✓ *Deposition*
- ✓ *Etch*
- ✓ *Packaging*



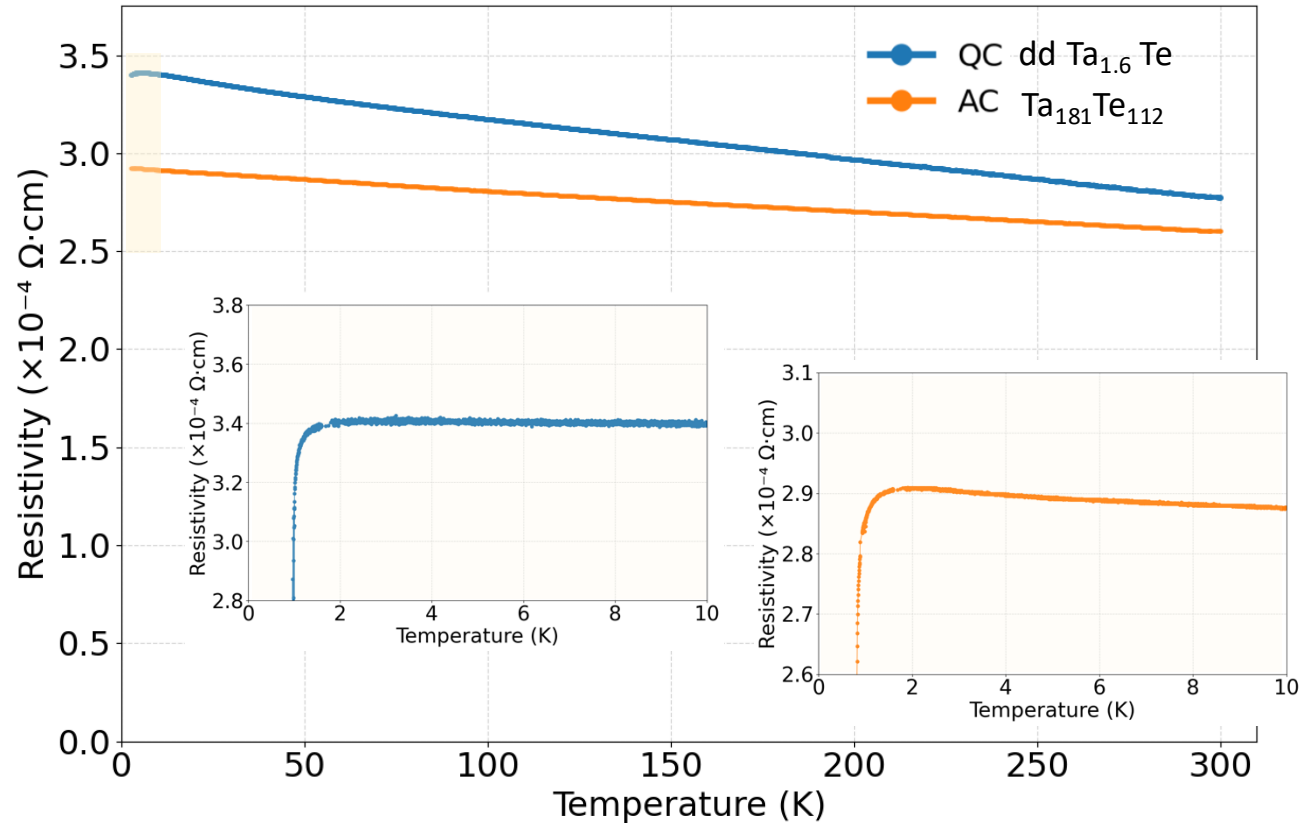
Technique advantages:

No air exposure
No heating
No solvent, no residue.



Electrical Transport Characteristics

Temperature Dependent Resistivity



QC: dd $\text{Ta}_{1.6}\text{Te}$

$t \sim 10 \text{ nm}, L \sim 6 \mu\text{m}, W \sim 4 \mu\text{m}$

$n = 3.506 \times 10^{22} \text{ cm}^{-3}$

$\mu = 0.525 \text{ cm}^2/\text{Vs}$

$k_F l = 3.544$

AC: $\text{Ta}_{181}\text{Te}_{112}$

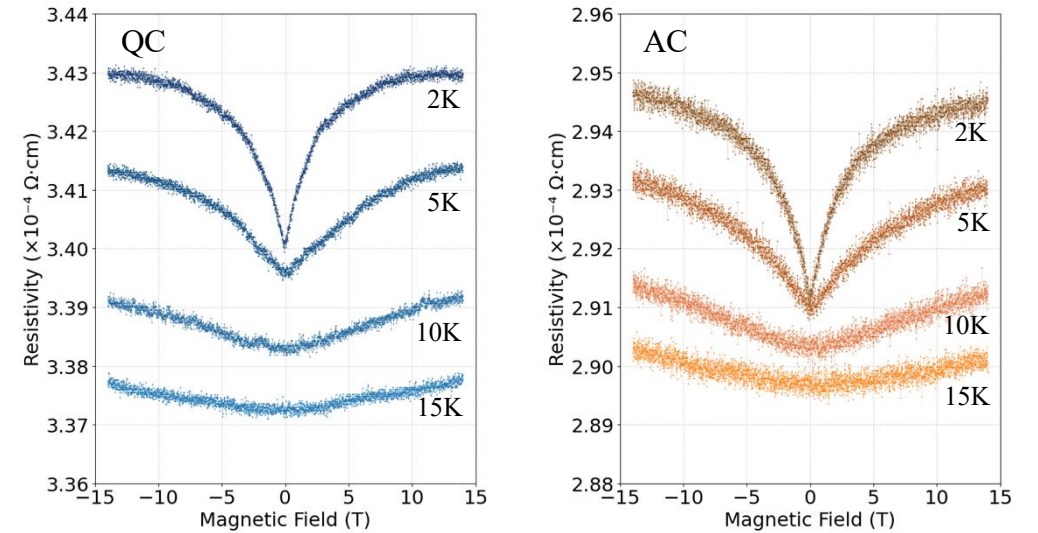
$t \sim 150 \text{ nm}, L \sim 8 \mu\text{m}, W \sim 5 \mu\text{m}$

$n = 5.944 \times 10^{21} \text{ cm}^{-3}$

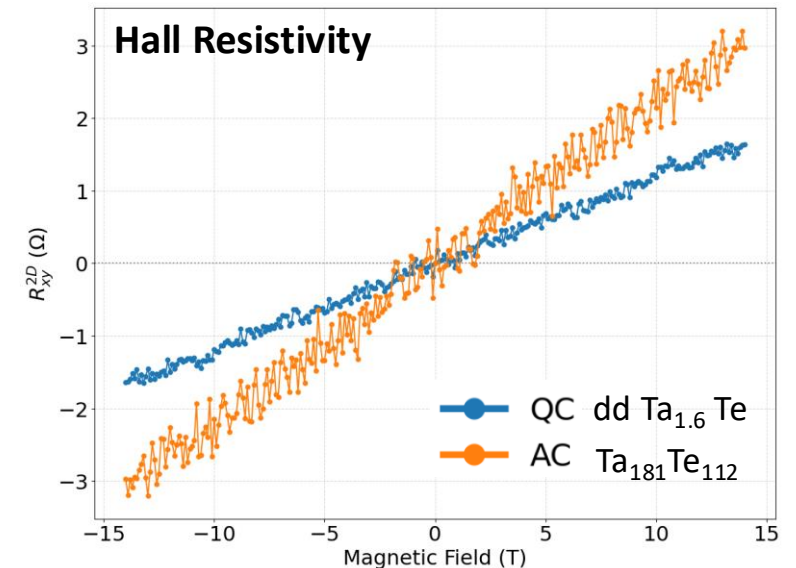
$\mu = 3.619 \text{ cm}^2/\text{Vs}$

$k_F l = 7.481$

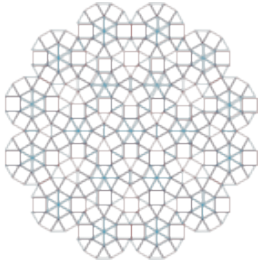
Magneto Resistivity



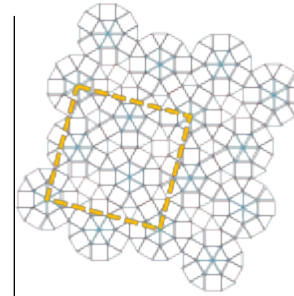
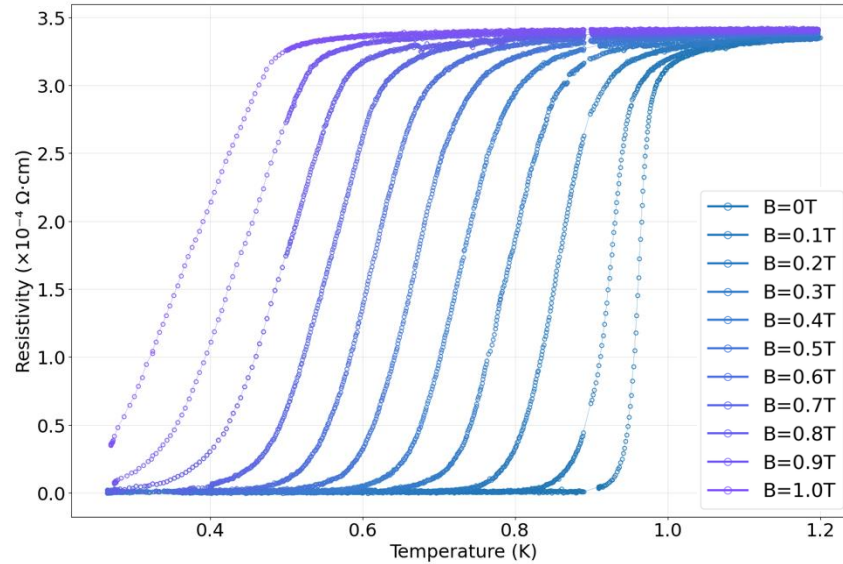
Hall Resistivity



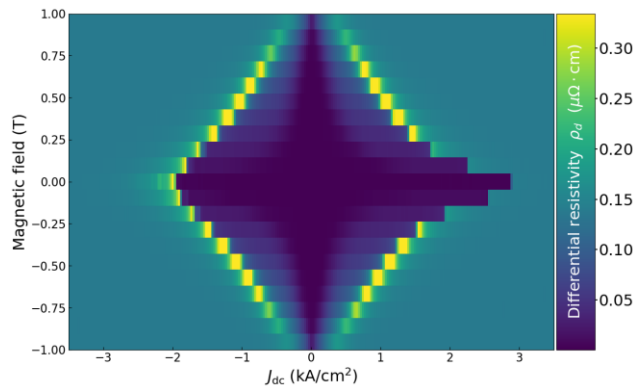
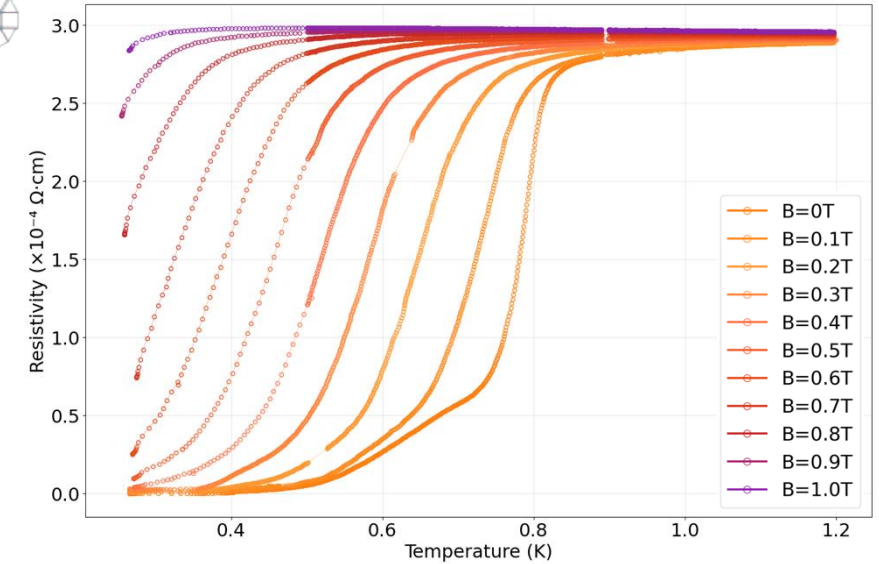
Superconductivity in QC and AC



QC ($\text{Ta}_{(3+2\sqrt{3})/4}\text{Te}$)

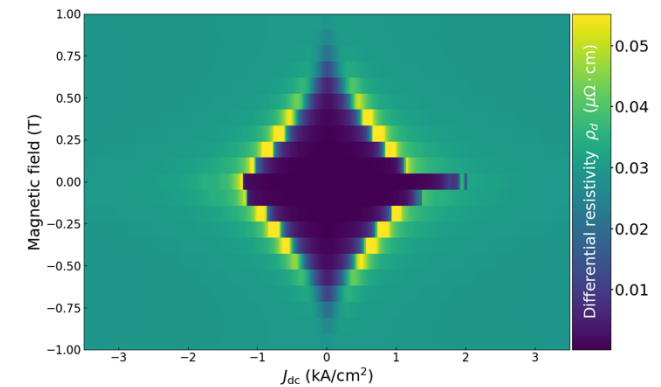


AC ($\text{Ta}_{181}\text{Te}_{112}$)



$$T_c(\text{QC}) > T_c(\text{AC})$$

$$\xi(\text{QC}) < \xi(\text{AC})$$

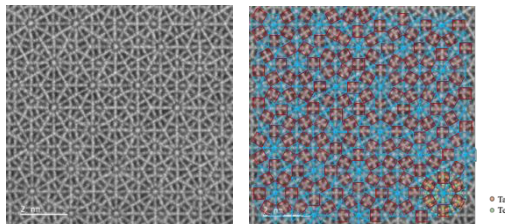


Coherence length $\xi = \sqrt{\frac{\Phi_0}{2\pi H_{c2}(T)}} \approx 18.2 \text{ nm}$ ($T = 0.3\text{K}$)

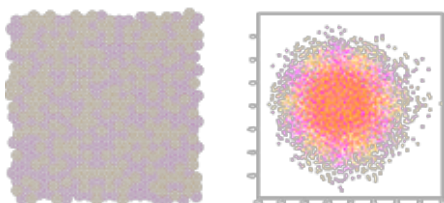
- Superconducting T_c of QC TaTe: $\sim 0.9 \text{ K}$
- Superconductivity has also been observed in the approximant crystal.

Summary and Outlook

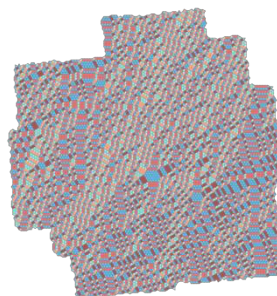
- Atomically thin 2D QC and AC TaTe samples were analyzed using TEM.



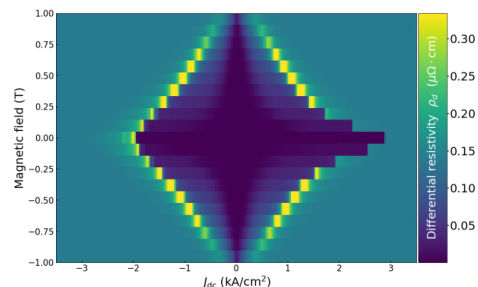
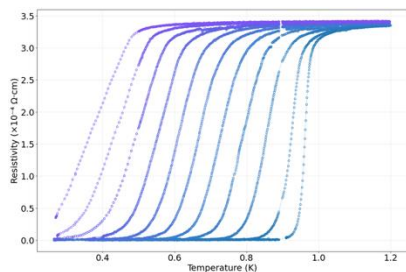
- Information from higher-dimensional space was utilized in the analysis.



- Quasicrystals contain phason strain disorders



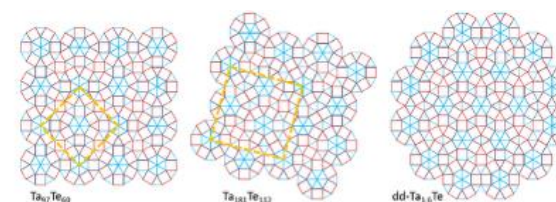
- Superconductivity in QC and AC has been observed



Outlook

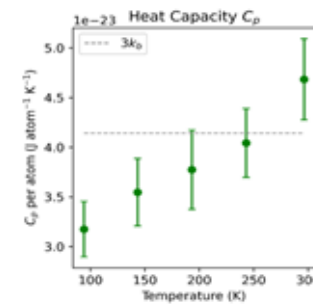
Can we control phason disorder in 2D dd-quasicrystals?

Systematic studies of ACs and QC in the 2D limits



Thermodynamic measurement: sensing higher dimensional effect

Breakdown of Dulong-Petit law in QC specific heat



Quasicrystal moire

