SPICE workshop on Quantum materials and quantum information science,
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Unconventional superconductivity and magnetism from quantum geometry

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Correlation
Design Science

科研費
KAKENHI

Collaborators

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Makoto Shimizu (Kyoto)

Quantum geometry

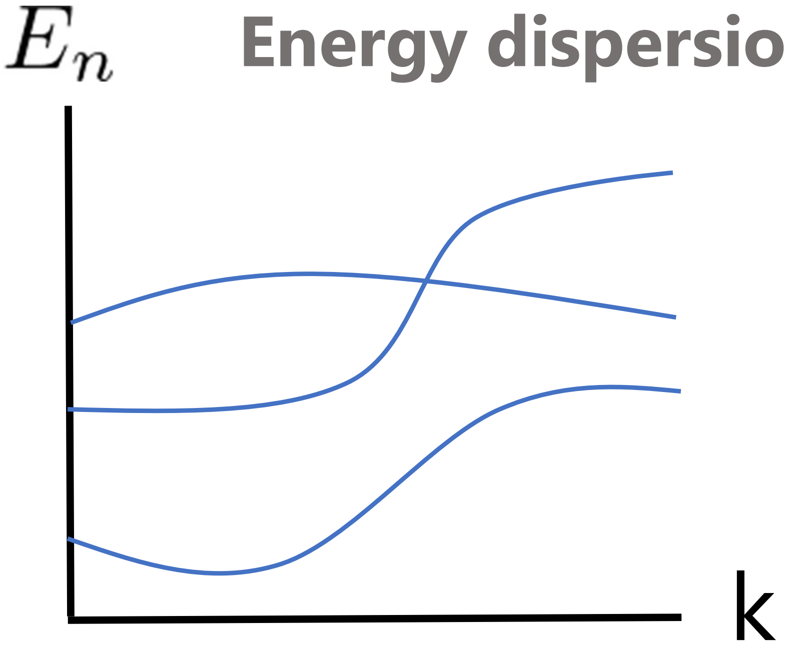
Adiabatic change of wave function $H(\lambda)\psi_{n\lambda}(x) = E_n(\lambda)\psi_{n\lambda}(x)$

Eigenstate Adiabatic parameter

[Quantum geometry of **Bloch electrons**]

$$\boxed{H_0(\boldsymbol{k})|u_n(\boldsymbol{k})\rangle = \epsilon_n(\boldsymbol{k})|u_n(\boldsymbol{k})\rangle}$$

Energy dispersion



Quantum geometry

Berry curvature :

$$\Omega_n^{\mu\nu} = \sum_{m(\neq n)}(A_{nm}^{\mu}A_{mn}^{\nu} - A_{nm}^{\nu}A_{mn}^{\mu})$$

Berry connection : $(A_{nm}^{\mu} = i\langle u_n|\partial_\mu u_m\rangle)$

- Quantum Hall effect
- Electric polarization *etc.*

Di Xiao, Ming-Che Chang, and Qian Niu, Rev. Mod. Phys. **82**, 1959 (2010).

Beyond Berry phase/curvature

Berry curvature does not represent the whole band geometry.

Quantum geometric tensor : $M_n^{\mu\nu} = \langle \partial_\mu u_n | 1 - P_n | \partial_\nu u_n \rangle$

$(P_n = |u_n\rangle\langle u_n|)$

$$= -\frac{i}{2}\Omega_n^{\mu\nu} + g_n^{\mu\nu}$$

Review: R. Resta, Eur. Phys. J. B 79, 121 (2011).

Quantum metric

Less studied in condensed matter physics, but recently …

Optical response, nonlinear transport, multipolar moment, Landau levels .....

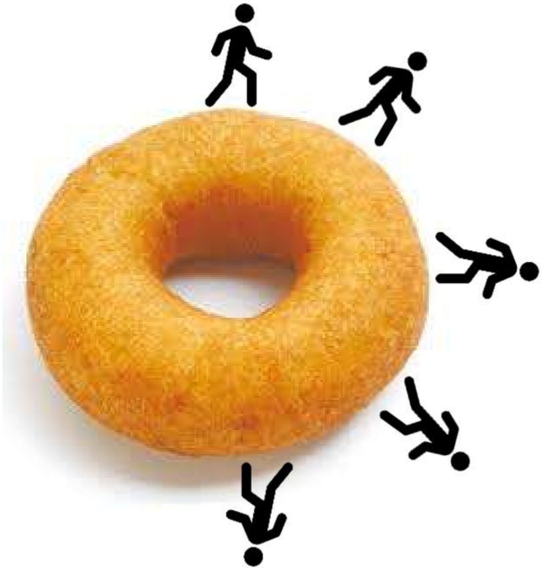
Physical meaning $\hat{x} = i\frac{\partial}{\partial k_x}$ in quantum mechanics

$$A_n^x = i\langle u_n | \frac{\partial}{\partial k_x} | u_n \rangle \simeq x \qquad g_n^{xx} \simeq x^2$$

Question: Magnetism and superconductivity ?

Contents

Quantum geometry



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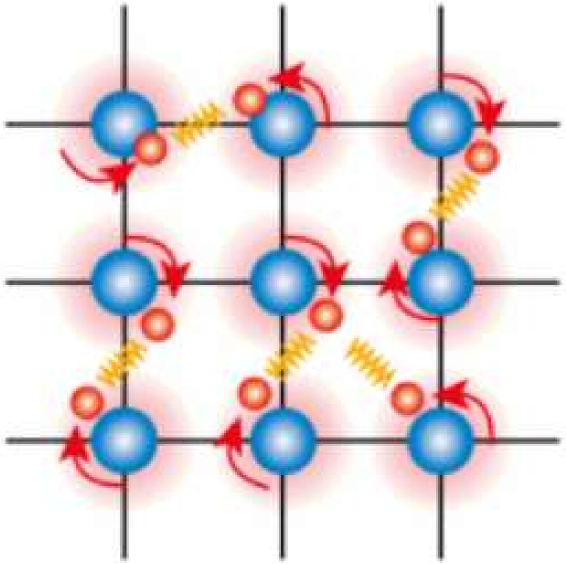
Correlation



Ferromagnetism, Odd-parity magnetism, Spin-triplet superconductivity

[Review] T. Kitamura, A. Daido, and YY, *Appl. Phys. Lett.* 128, 130501 (2026)

Taisei Kitamura, Akito Daido, YY, Phys. Rev. Lett. **132,** 036001 (2024).

Taisei Kitamura, Hiroki Nakai, Akito Daido, YY, arXiv:2505.01089

Kanta Kudo and YY, arXiv:2505.20907

Chang-geun Oh, Taisei Kitamura, Akito Daido, Jun-Won Rhim, YY, arXiv:2509.13618.

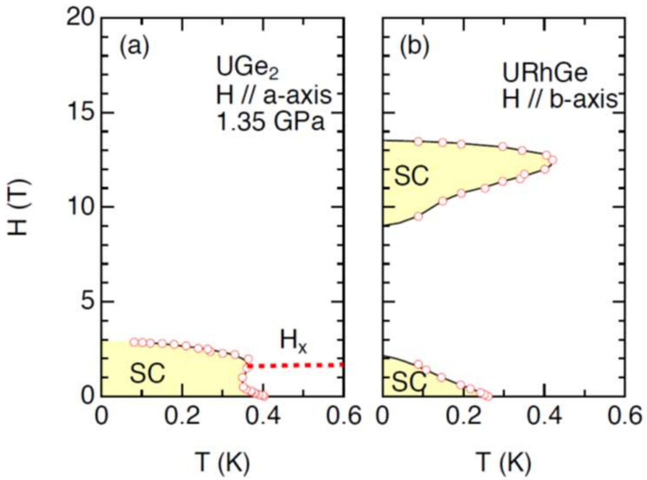
Makoto Shimizu, Chang-geun Oh, YY, arXiv:2602.14511

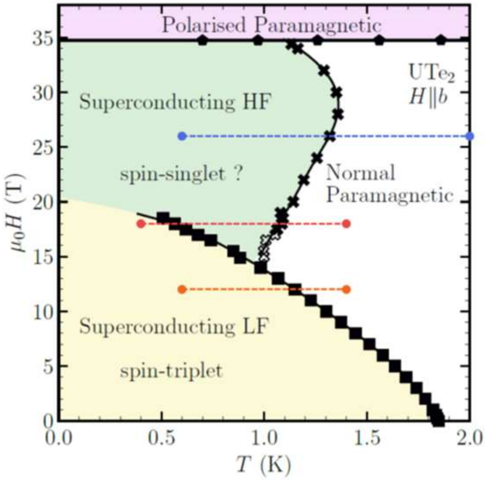
Spin-triplet superconductivity

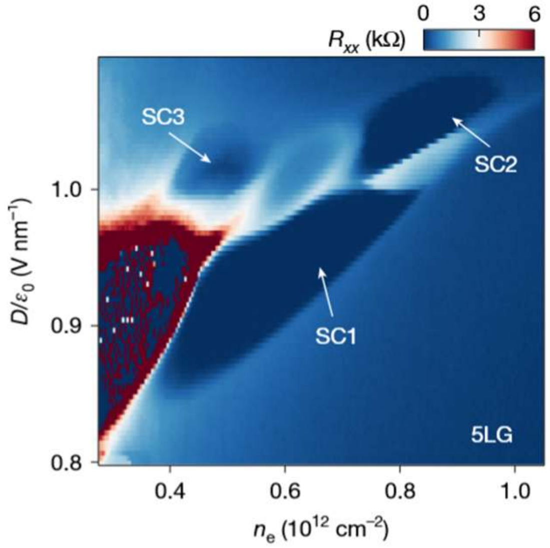
- ◆ Multiple superconducting phases, symmetry breaking
- ◆ Platform for the topological superconductivity

→ Can be applied to quantum computation
with Majora Fermions

Candidate materials are rare in nature







S. S. Saxena *et al.* Nature (2000), D. Aoki *et al.* Nature (2001), S. Ran *et al.* Science (2019), T. Han *et al.* Nature (2025)

Guiding principle for materials search

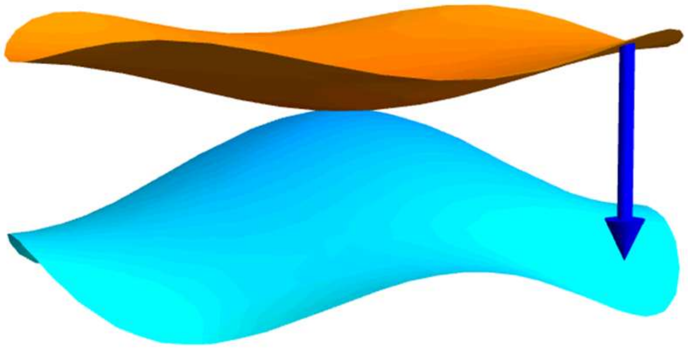
- ◆ **Ferromagnetic fluctuation is a canonical origin.**
- ◆ **Quantum geometric ferromagnetism can be a guiding principle.**

Our idea

Quantum distance and spin fluctuation

- Spin susceptibility of multiband free fermions

$$\chi^0(\boldsymbol{q}) = \sum_{nm} \int_{\mathrm{B.Z.}} d\boldsymbol{k} \underbrace{F_{nm}(\boldsymbol{k},\boldsymbol{q})}_{\text{Lindhard function}} \left(1 - \underbrace{D_{nm}(\boldsymbol{k},\boldsymbol{q})}_{\text{Quantum distance}}\right)$$



$\sim D_{nn}(\boldsymbol{k},\boldsymbol{q})$

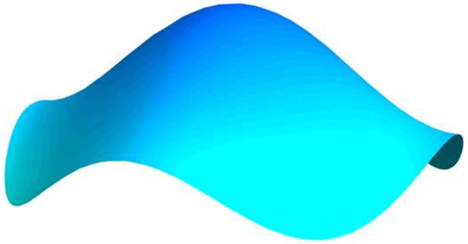
$$D_{nn}(\boldsymbol{k},\boldsymbol{q}) = 1 - |\langle u_n(\boldsymbol{k}+\boldsymbol{q})|u_n(\boldsymbol{k})\rangle|^2$$

$$\sim g_n^{\mu\nu}(\boldsymbol{k}) dq_\mu dq_\nu$$

Quantum distance always favors ferromagnetic fluctuation.

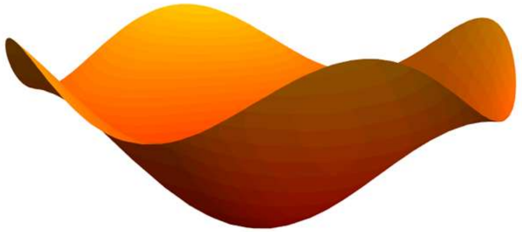
- Ferromagnetic or antiferromagnetic fluctuation

$\chi\ (\boldsymbol{q} = 0)$ is a (local) maximum



Ferromagnetic
Spin-triplet SC (HF SC …)

$\chi\ (\boldsymbol{q} \neq 0)$ is maximum



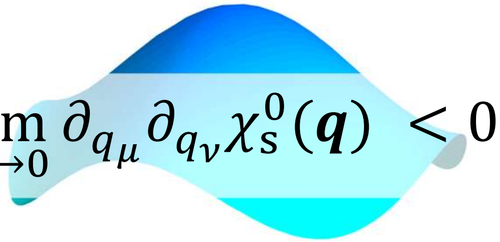
Antiferromagnetic
Spin-singlet SC (cuprates, Fe-based SC …)

Theoretical formula

Criterion for ferromagnetic fluctuation

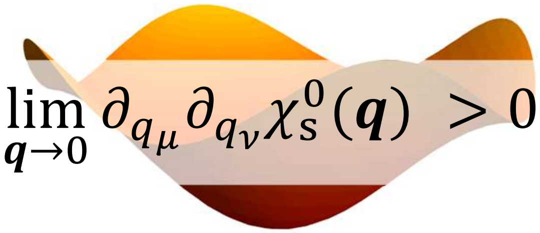
◆ Curvature of bare spin susceptibility

Ferromagnetic



$$\lim_{\boldsymbol{q}\to 0} \partial_{q_\mu}\partial_{q_\nu}\chi_{\mathrm{s}}^0(\boldsymbol{q}) < 0$$

Antiferromagnetic



$$\lim_{\boldsymbol{q}\to 0} \partial_{q_\mu}\partial_{q_\nu}\chi_{\mathrm{s}}^0(\boldsymbol{q}) > 0$$

$$\chi_{\mathrm{e}}^{0:\mu\nu} = \lim_{\boldsymbol{q}\to 0} \partial_{q_\mu}\partial_{q_\nu}\chi_{\mathrm{s}}^0(\boldsymbol{q}) = \chi_{\mathrm{geom}}^{\boldsymbol{0:\mu\nu}} + \underline{\chi_{\mathrm{band}}^{\boldsymbol{0:\mu\nu}}}$$

Conventional term

$$\chi_{\mathrm{geom}}^{0:\mu\nu} = \sum_n \int \frac{d\boldsymbol{k}}{(2\pi)^d}\left[f'\big(\epsilon(\boldsymbol{k})\big) g_n^{\mu\nu}(\boldsymbol{k}) + f\big(\epsilon(\boldsymbol{k})\big) X_n^{\mu\nu}(\boldsymbol{k})\right]$$

Quantum metric favors
ferromagnetic fluctuation

Berry connection polarizability
[Gao, Yan, and Niu (2014)] favors
antiferromagnetic fluctuation

(1) Flat band system, (2) Topological metal/semimetal

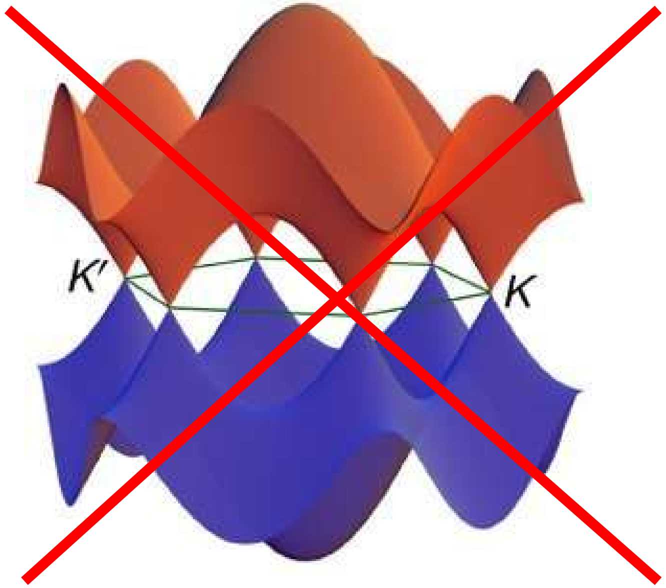
Guiding principle

Criterion for ferromagnetic fluctuation

$$\chi^{0:\mu\nu}_{\mathrm{geom}} = \sum_n \int \frac{d\boldsymbol{k}}{(2\pi)^d} \left[f'\left(\epsilon(\boldsymbol{k})\right) g^{\mu\nu}_n(\boldsymbol{k}) + f\left(\epsilon(\boldsymbol{k})\right) X^{\mu\nu}_n(\boldsymbol{k}) \right]$$

Quantum metric favors
ferromagnetic fluctuation

Berry connection polarizability favors
antiferromagnetic fluctuation



Topological materials may be good candidates.
But, Dirac electrons in Graphene … are not.

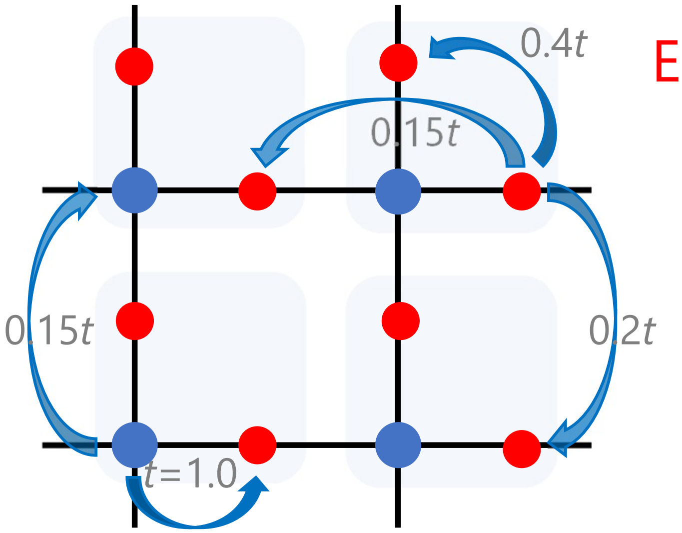
Quantum metric term is a Fermi-surface term.

(1) Flat band system, (2) Topological metal/semimetal

Topological metal + Large Fermi surface = Ferromagnetism

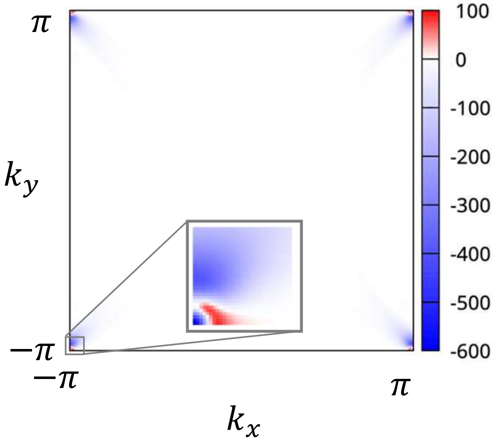
Model

Model of topological metal with band degeneracy

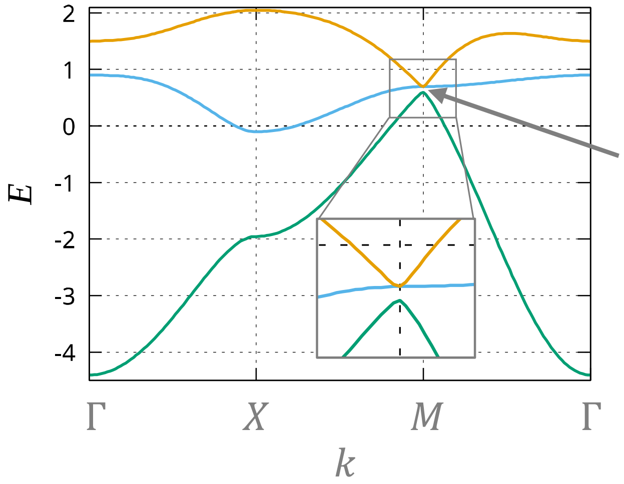


Extended Lieb lattice

Quantum geometric term

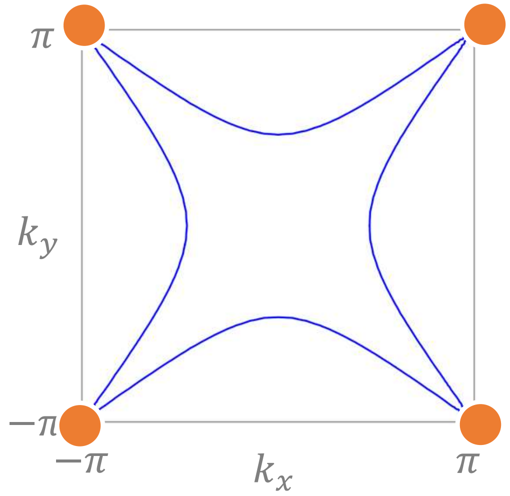


Energy dispersion



- ✓ No flat band
- ✓ Band degeneracy
- ✓ Singular saddle point

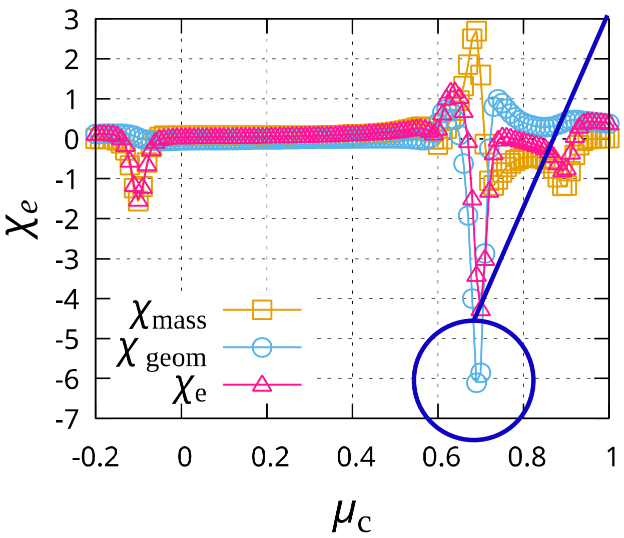
Fermi surface



Results: Quantum geometric ferromagnetism

Quantum-geometry-induced ferromagnetic fluctuation

- Generalized electric susceptibility = curvature of spin susceptibility



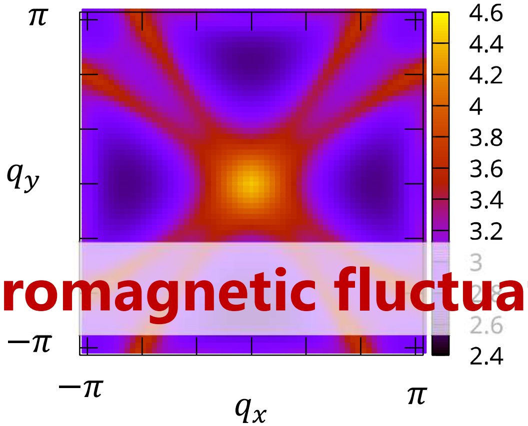
Enhancement of quantum geometric term.

Quantum metric gives large negative peak!

Quantum geometric ferromagnetism

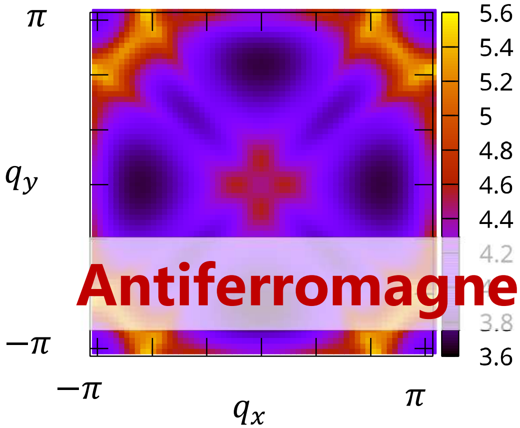
- Spin susceptibility

Total $\chi^0(\boldsymbol{q})$



Ferromagnetic fluctuation

Without quantum geometry



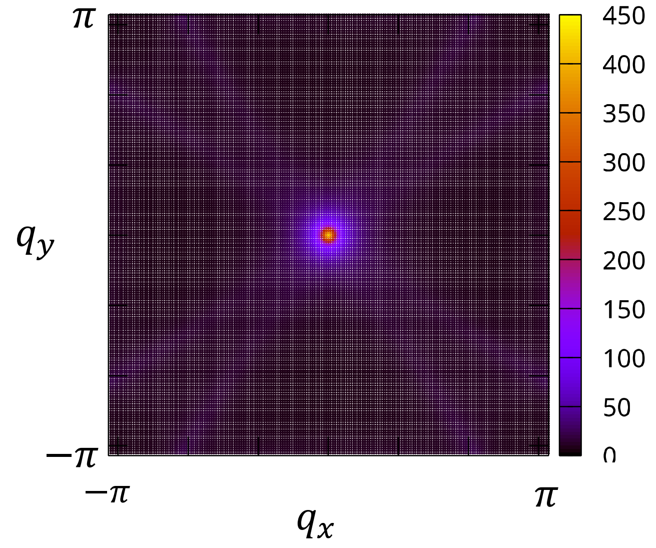
Antiferromagnetic fluctuation

calculated by $\sum_n \int_{\mathrm{B.Z.}} d\boldsymbol{k} F_{nm}(\boldsymbol{k},\boldsymbol{q})$

Superconductivity in Hubbard model on extended Lieb lattice

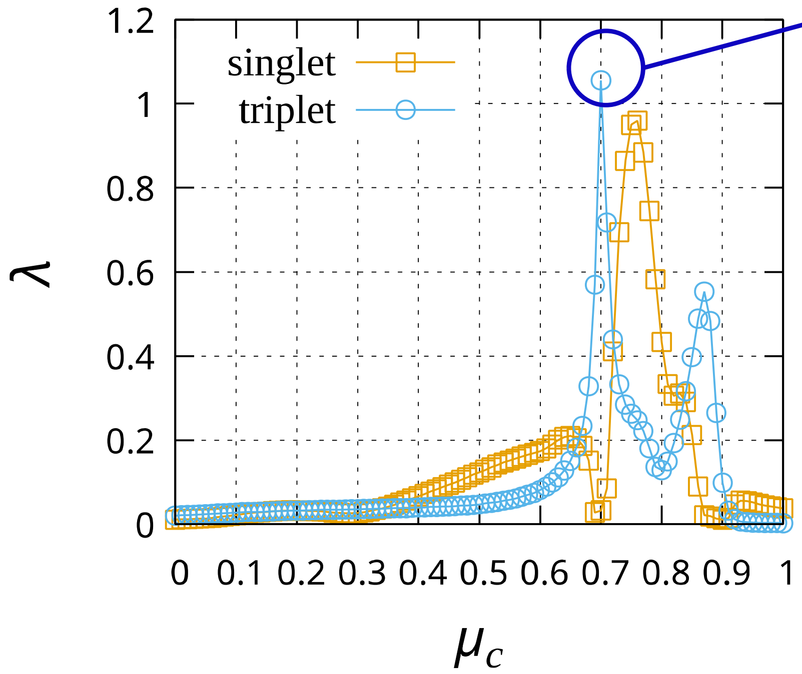
Spin-triplet superconductivity

◆ Random phase approximation



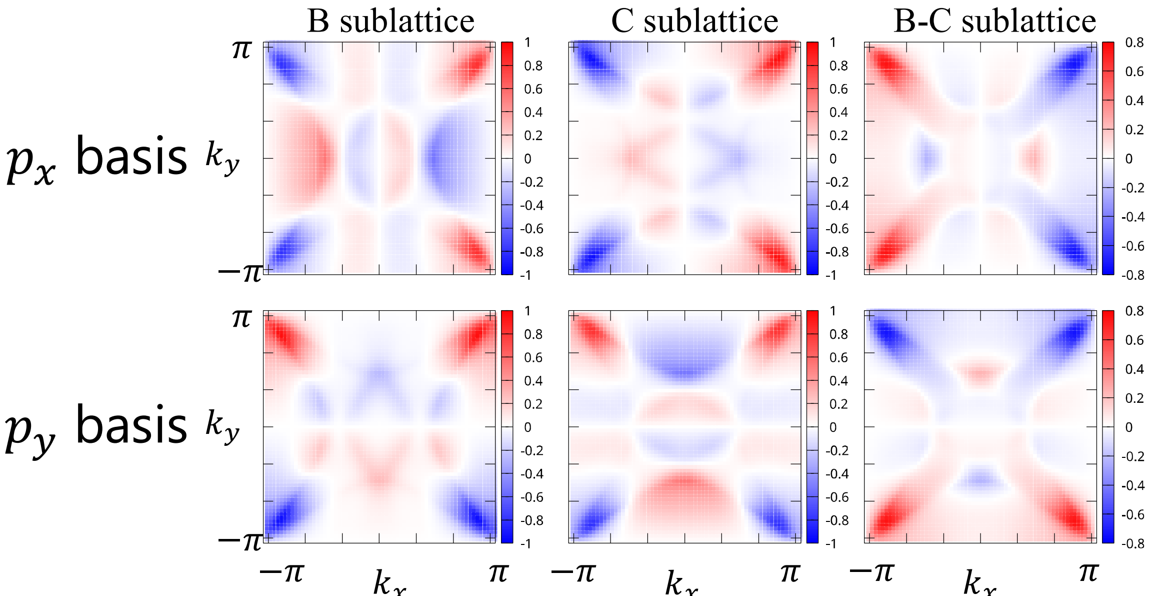
Ferromagnetic fluctuation is strongly enhanced by Coulomb interaction.

◆ Eliashberg equation



P-wave superconductivity

Two basis functions : p_x and p_y



Question:

What is a signature of quantum geometric magnetism?

Is there a difference from conventional magnetism?

Enforcing symmetry

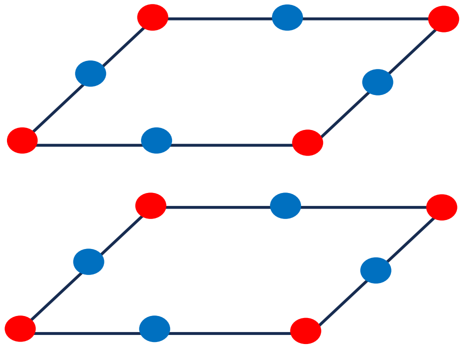
Kanta Kudo and YY, arXiv:2505.20907

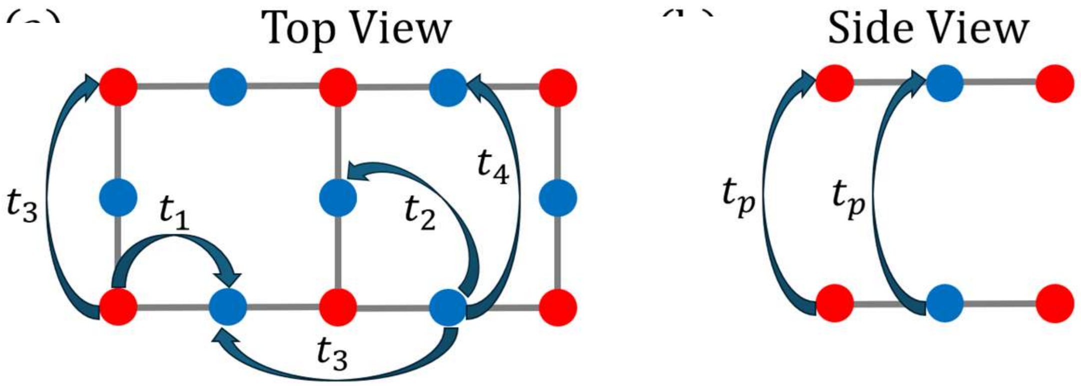
Quantum geometric odd-parity multipole order

Question: What is a signature of quantum geometric magnetism?
Is there a difference from conventional magnetism?

Answer: Additional degrees of freedom.

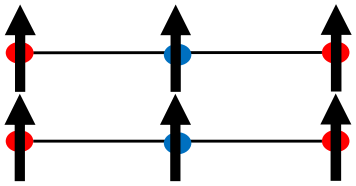
- Bilayer Lieb lattice



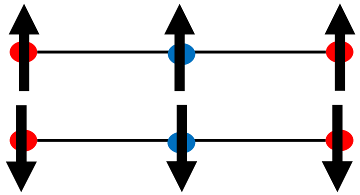


- Possible ferroinc order with q=0

Ferromagnet



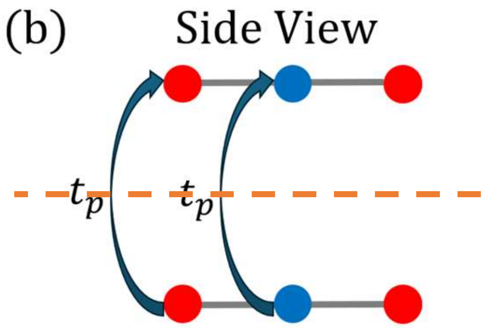
PT-symmetric antiferromagnet



Odd-parity magnetic
multipole order

Quantum metric contribution

◆ Symmetry



$$[\hat{H},\hat{\sigma}] = 0 \Longrightarrow \text{mirror symmetry}$$

$$\begin{cases} \hat{\sigma}|u_n^p(\boldsymbol{k})\rangle = p|u_n^p(\boldsymbol{k})\rangle \quad (p=\pm 1) \\ \hat{H}(\boldsymbol{k})|u_n^p(\boldsymbol{k})\rangle = \epsilon_n^p(\boldsymbol{k})|u_n^p(\boldsymbol{k})\rangle \end{cases}$$

n: band index
p: mirror eigenvalue

◆ Curvature of susceptibility

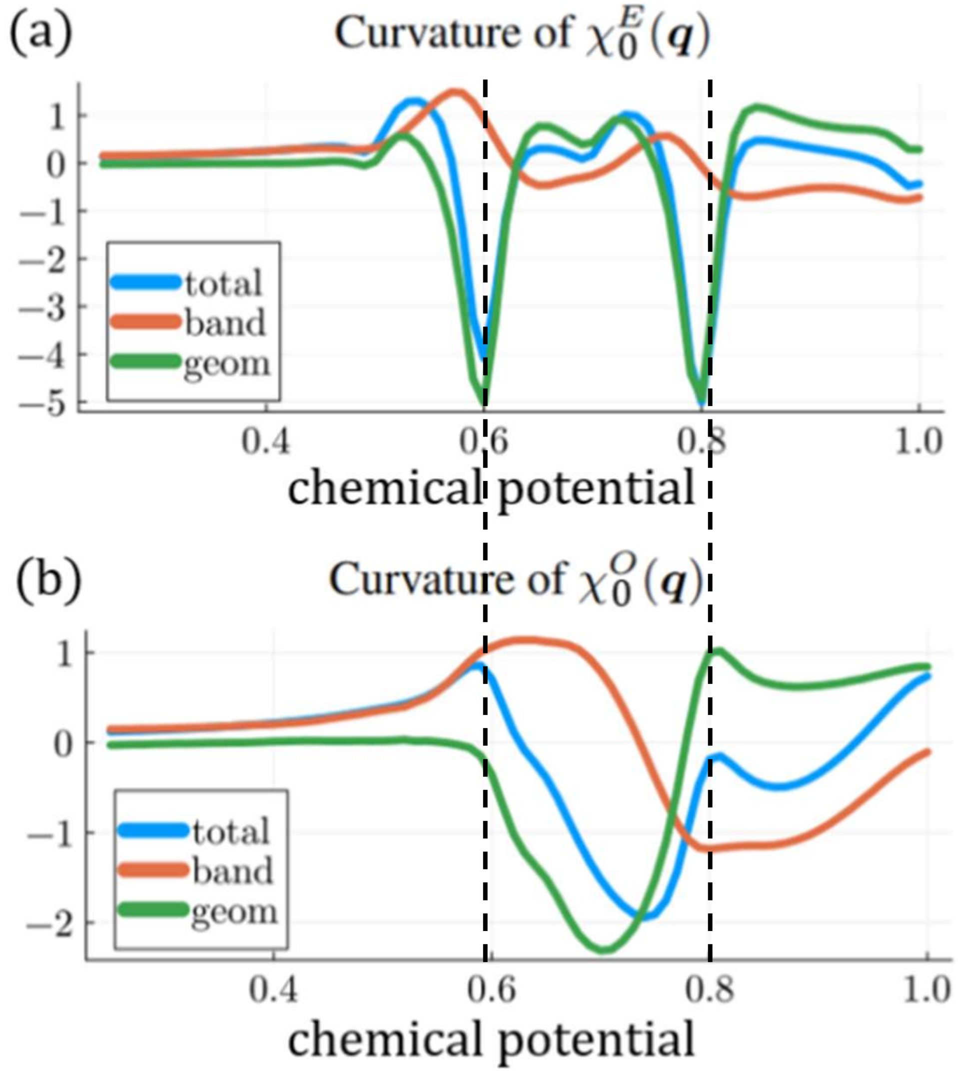
$$\partial_i\partial_j\chi^{E/O}\Big|_{\boldsymbol{q}=\boldsymbol{0}} = \partial_i\partial_j\chi^{E/O}_{band}\Big|_{\boldsymbol{q}=\boldsymbol{0}} + \partial_i\partial_j\chi^{E/O}_{geom}\Big|_{\boldsymbol{q}=\boldsymbol{0}}$$

$$\partial_i\partial_j\chi^{E/O}_{geom}\Big|_{\boldsymbol{q}=\boldsymbol{0}} = \sum_{n,m,p}\int\frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{f\left(\epsilon_m^{\pm p}(\boldsymbol{k})\right) - f\left(\epsilon_n^{p}(\boldsymbol{k})\right)}{\epsilon_n^p(\boldsymbol{k}) - \epsilon_m^{\pm p}(\boldsymbol{k})}\left(-\delta_{nm}\underline{g_n^{ij}(\boldsymbol{k})} + (1-\delta_{nm})g_{nm}^{ij}(\boldsymbol{k})\right)$$

Quantum metric in mirror subspace

Quantum metric term is Fermi surface term for spin fluctuation
interband term for multipole fluctuation

Quantum metric contribution



Band crossing energy

Geometric ferromagnetism:
enhanced on band crossing points

Geometric odd-parity monopole:
enhanced between band crossing points

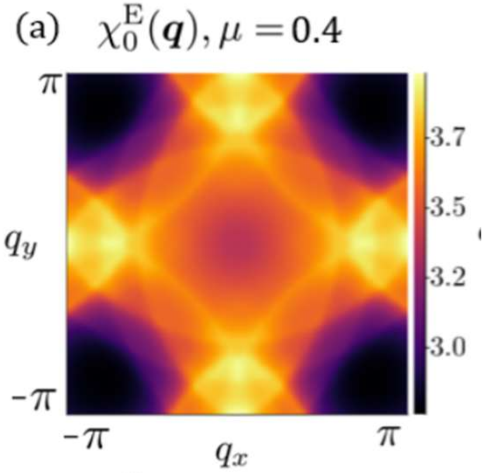
Not Ferromagnetism but odd-parity multipole magnetism is favored by quantum metric effects in a wide parameter range.

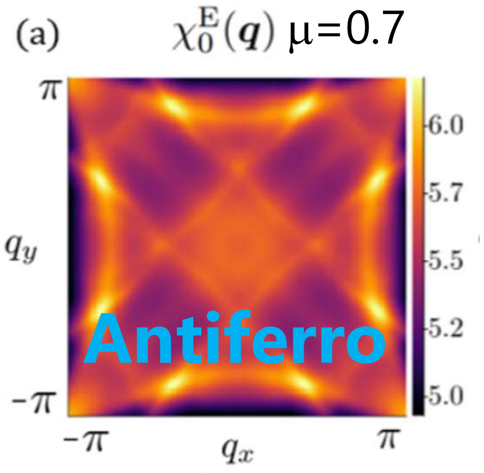
Signature of Quantum geometric magnetism

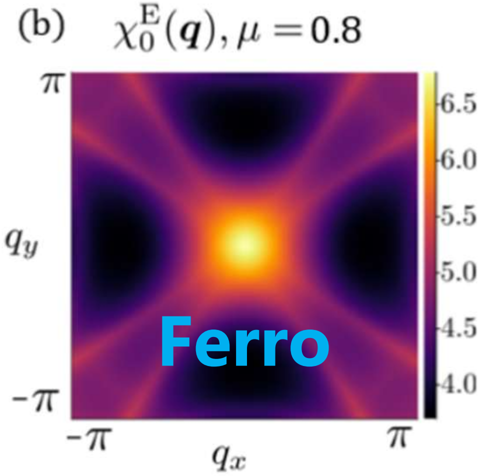
- Spin and multipole susceptibility for *weak inter-layer coupling*

Quantum geometry: negligible enhanced enhanced

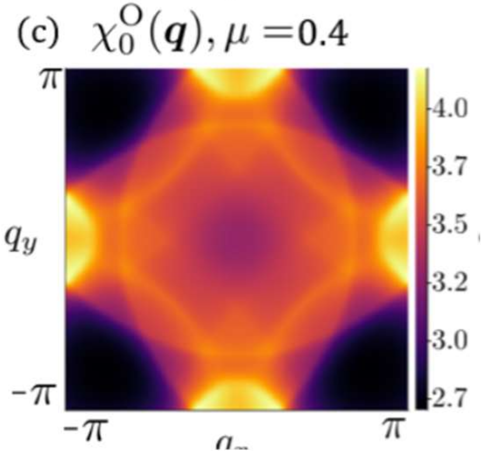
Spin susceptibility

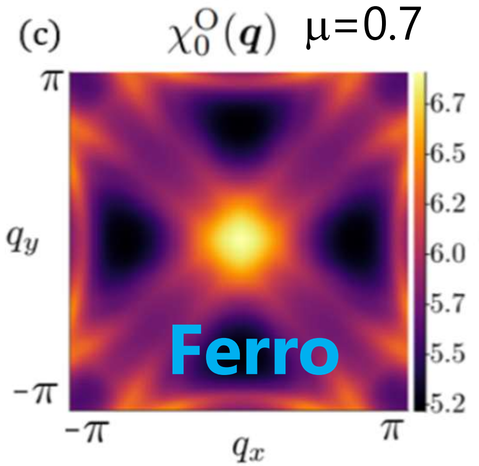


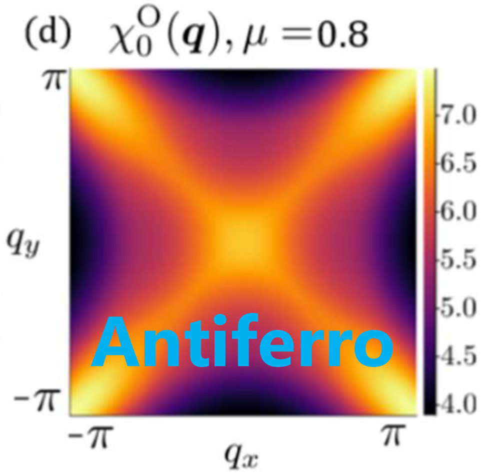




Monopole susceptibility







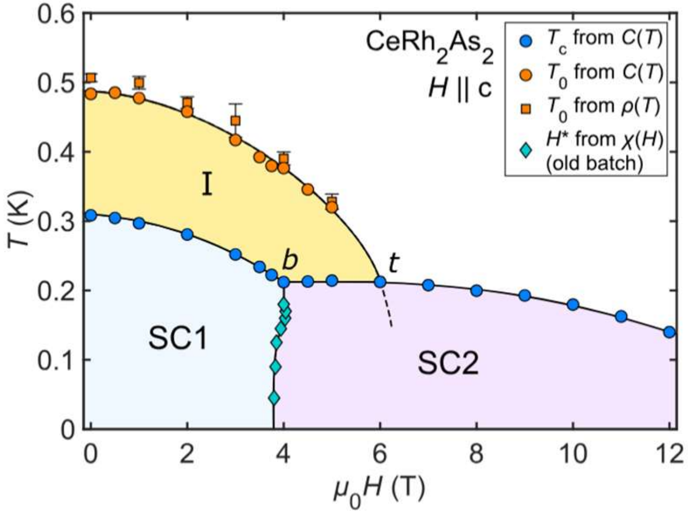
A signature of quantum geometry:
"Qualitatively different spin and multipole correlations"

can be verified by neutron scattering

Potential platforms

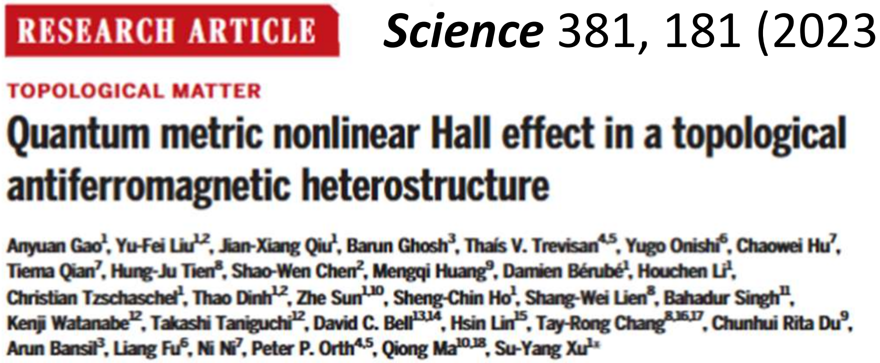
$CeRh_2As_2$ K. Nogaki and YY, arXiv:2510.24289

Magnetic monopole superconductor



S. Khim *et al.*, Science **373**, 1012 (2021)

$MnBi_2Te_4$



RESEARCH ARTICLE

TOPOLOGICAL MATTER

Quantum metric nonlinear Hall effect in a topological antiferromagnetic heterostructure

Anyuan Gao[1], Yu-Fei Liu[1,2], Jian-Xiang Qiu[1], Barun Ghosh[3], Thaís V. Trevisan[4,5], Yugo Onishi[6], Chaowei Hu[7], Tiema Qian[7], Hung-Ju Tien[8], Shao-Wen Chen[2], Mengqi Huang[9], Damien Bérubé[1], Houchen Li[1], Christian Tzschaschel[1], Thao Dinh[1,2], Zhe Sun[1,10], Sheng-Chin Ho[1], Shang-Wei Lien[8], Bahadur Singh[11], Kenji Watanabe[12], Takashi Taniguchi[12], David C. Bell[13,14], Hsin Lin[15], Tay-Rong Chang[8,16,17], Chunhui Rita Du[9], Arun Bansil[3], Liang Fu[6], Ni Ni[7], Peter P. Orth[4,5], Qiong Ma[10,18], Su-Yang Xu[1*]

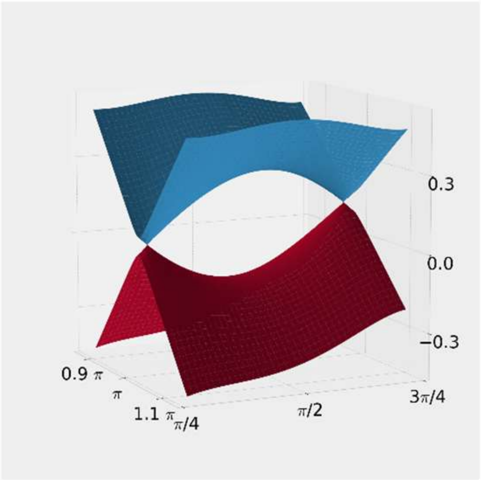
Science 381, 181 (2023)

Theory: Y. Gao, S. A. Yang, and Q. Niu, PRL 112, 166601 (2014)

>100 odd-parity magnetic materials
H. Watanabe and YY, PRB (2018)
[Review article]
H. Watanabe and YY, JPCM (2023)

CuMnAs

PT-symmetric Dirac electrons



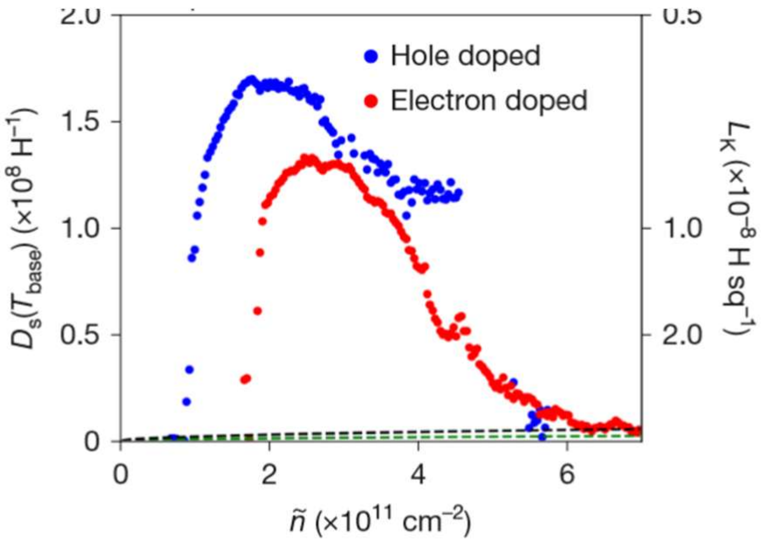
Šmejkal et al., PRL 118, 106402 (2017)

Quantum geometry can lead to odd-parity multipole order, and accordingly, emergent responses.

Unconventional SC from quantum geometry

- **Superfluid weight**

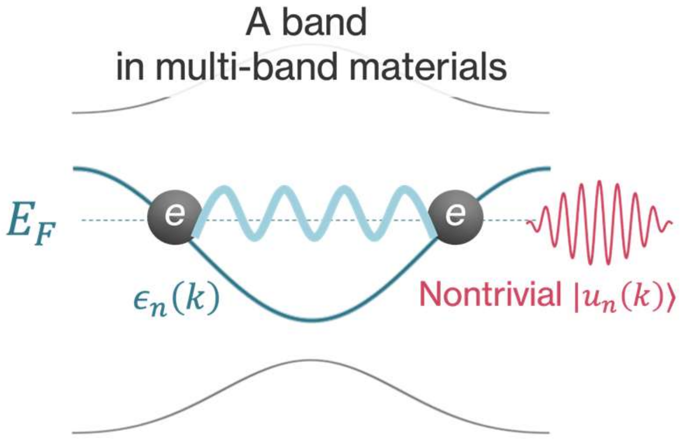
S. Peotta and P. Törmä, Nat. Commun. (2015)



TBG: Tanaka and Wang *et al*., *Nature* **638**, 99 (2025)

Scaling law: Hirobe, Kitamura and YY, arXiv:2505.13065

- **Topological superconductivity**



A. Daido *et al*., PRB (2024)

- **FFLO superconductivity**
- **Anapole superconductivity**

$$F(q) = F(0) + \underline{T^{\rm ana}_{\mu}} q_\mu + \frac{1}{2} D^{\rm s}_{\mu\nu} q_\mu q_\nu$$

Lifshitz invariant: pure quantum geometry

- **Surface acoustic wave response** Matsumoto et al., arXiv:2505.21436
- **Superconducting piezoelectric effect** Chazono et al., PRB (2023)
- **Nonlinear optical responses** Watanabe and YY, PRX (2021), PRB (2022)

Summary

Goal: Designing magnetism and superconductivity by utilizing quantum geometry

This talk: Quantum geometry × Electron correlation

Ferromagnetism, odd-parity magnetism, Spin-triplet SC

Design principles: Topological metal + electron correlation

Thank you very much!