

# Interactions and superconductivity in twisted an un-twisted graphene stacks



F. Guinea



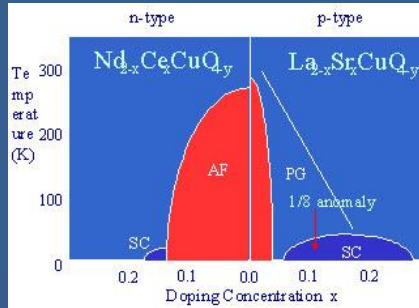
May 14th, 2026

## Outline

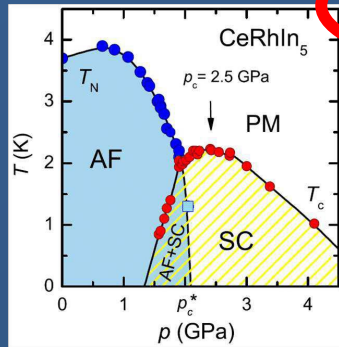
- Interactions and phase diagram in twisted graphene bilayers
- Superconductivity in graphene stacks.
- Novel features in rhombohedral graphene.

In coloboration with N. R. Walet (U. Manchester), T. Cea, P. Pantaleon, A. Jimeno-Pozo, H. Sainz-Cruz, G. Parra, J. Silva (Imdea). Also V. P. Phong (U. Penn), V. Crépel, L. Fu, L. Levitov (MIT), Y. Oreg, E. Peleg, A. Stern (Weizmann), D. Kaplan (Rutgers), E. Mele (UPenn), A. Bernevig (Princeton, DIPC).

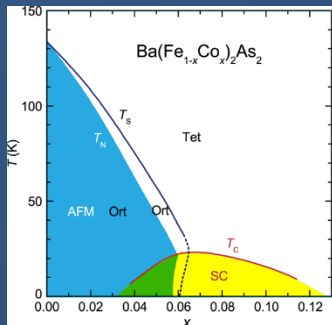
# Strongly correlated systems



Cuprate superconductors



Heavy fermion compounds

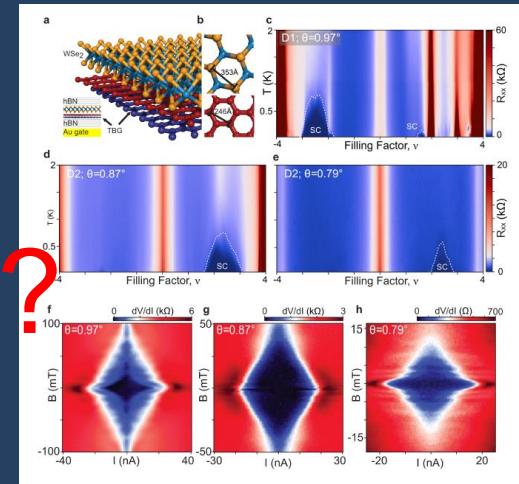


Pnictides

Pairing due to magnetic fluctuations  
Variations of the Hubbard model

**Article**  
**Superconductivity in metallic twisted bilayer graphene stabilized by WSe<sub>2</sub>**  
<https://doi.org/10.1038/s41586-020-2473-8> Harpreet Singh Arora<sup>1,2\*</sup>, Robert Polak<sup>1,2,3\*</sup>, Yiran Zhang<sup>1,2,3\*</sup>, Alex Thomson<sup>1,2,3\*</sup>, Youngjoon Choi<sup>1,2,3</sup>, Hyungin Kim<sup>1,2,3</sup>, Zhong Li<sup>4</sup>, Iham Zaky Wilson<sup>5</sup>, Xiaodong Xu<sup>6</sup>, Jun-Hwa Chu<sup>7</sup>, Kenji Watanabe<sup>8</sup>, Takashi Taniguchi<sup>8</sup>, Jason Allou<sup>9,10</sup> & Steven Nadj-Perge<sup>1,2,3,11</sup>  
Received: 31 January 2020  
Accepted: 24 May 2020

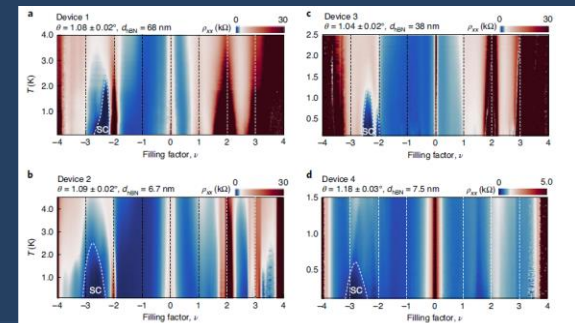
Nature **583**, 579 (2020)



**Independent superconductors and correlated insulators in twisted bilayer graphene**

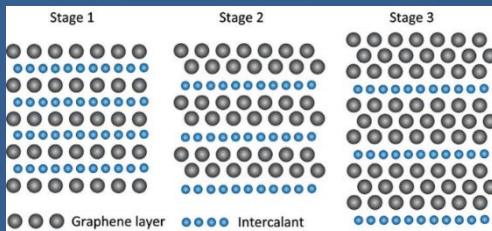
Yu Saito<sup>1</sup>, Jingyuan Ge<sup>2</sup>, Kenji Watanabe<sup>3</sup>, Takashi Taniguchi<sup>3</sup> and Andrea F. Young<sup>2,4</sup>

Nature Phys. **16**, 926 (2020)

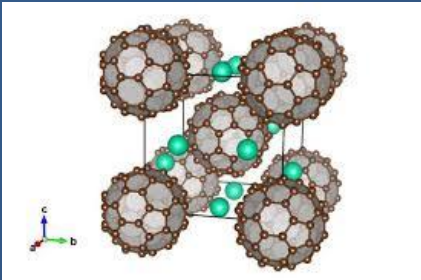


# Superconductivity and strongly correlated phases in carbon compounds

## Superconductivity



Graphite intercalation compounds, such as  $C_6Ca$ ,  
 $n \geq 10^{14} \text{ cm}^{-2}$ ,  $T_c \sim 1 - 10 \text{ K}$

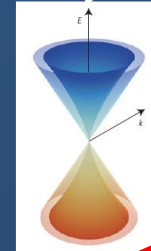
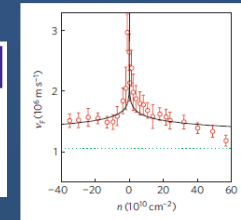


Alkali intercalated fullerenes, such as  $Rb_3C_{60}$ ,  
 $T_c \sim 10 - 40 \text{ K}$

## Renormalization of the Fermi velocity

**nature physics** LETTERS  
 PUBLISHED ONLINE 24 JULY 2011 | DOI:10.1038/NPHYS12345  
**Dirac cones reshaped by interaction effects in suspended graphene**

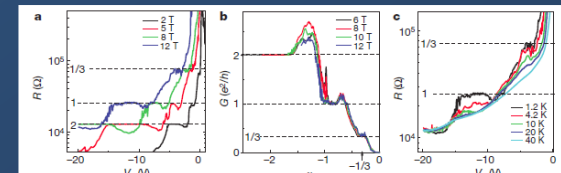
D. C. Elias<sup>1</sup>, R. V. Gorbachev<sup>1</sup>, A. S. Mayorov<sup>1</sup>, S. V. Morozov<sup>2</sup>, A. A. Zhukov<sup>3</sup>, P. Blake<sup>3</sup>,  
 L. A. Ponomarenko<sup>1</sup>, I. V. Grigorieva<sup>1</sup>, K. S. Novoselov<sup>1</sup>, F. Guinea<sup>4\*</sup> and A. K. Geim<sup>1,3</sup>



## Fractional quantum Hall effect and insulating phase of Dirac electrons in graphene

Xu Du<sup>1,†</sup>, Ivan Skachko<sup>1</sup>, Fabian Duerr<sup>1</sup>, Adina Luican<sup>1</sup> & Eva Y. Andrei<sup>1</sup>

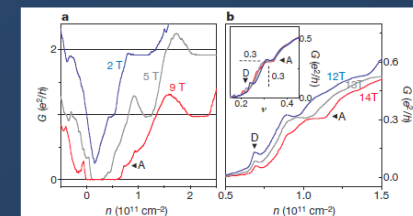
Nature **462**, 192 (2009)



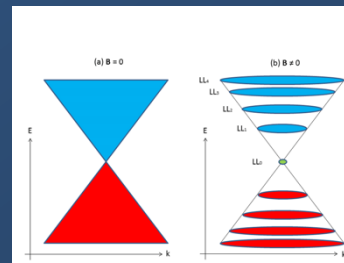
## Observation of the fractional quantum Hall effect in graphene

Kirill I. Bolotin<sup>1,\*†</sup>, Fereshte Ghahari<sup>1,4</sup>, Michael D. Shulman<sup>2</sup>, Horst L. Stormer<sup>1,2</sup> & Philip Kim<sup>1,2</sup>

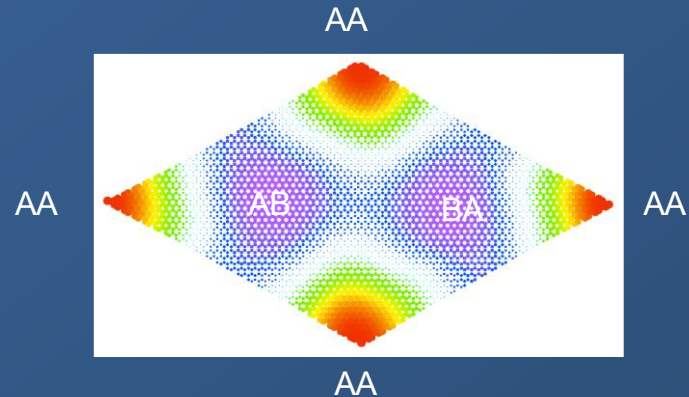
Nature **462**, 196 (2009)



## Landau levels



# Interactions in twisted bilayer graphene



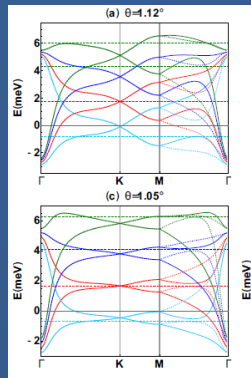
|                                    |                                                                                         |                                                      |
|------------------------------------|-----------------------------------------------------------------------------------------|------------------------------------------------------|
| Number of atoms:                   | $N \approx 10^4$                                                                        |                                                      |
| Moiré unit length:                 | $L_M \approx 15 \text{ nm}$                                                             | $\sim d\sqrt{N}$                                     |
| Radius of the charge distribution: | $L_c \approx 5 \text{ nm}$                                                              | $\sim d\sqrt{N}$                                     |
| Coulomb energy:                    | $E_C \approx \frac{e^2}{\epsilon L_c} \sim 10 - 50 \text{ meV}$                         | $\sim \frac{e^2}{\epsilon d} \frac{1}{\sqrt{N}}$     |
| Intraatomic (Hubbard) repulsion:   | $E_{\text{Hubbard}} = N \times \frac{U}{N^2} \approx 1 \text{ meV}$                     | $\sim \frac{e^2}{d} \times \frac{1}{N}$              |
| Electron-phonon coupling, LA:      | $g = \frac{D^2}{\lambda + 2\mu} \times \frac{1}{\sqrt{3}L_M^2/2} \approx 1 \text{ meV}$ | $\sim \frac{D^2}{\lambda + 2\mu} \times \frac{1}{N}$ |
| Electron-phonon coupling, TA:      | $g = \frac{(t\beta)^2}{\mu} \times \frac{1}{\sqrt{3}L_M^2/2} \approx 1 \text{ meV}$     | $\sim \frac{(t\beta)^2}{\mu} \times \frac{1}{N}$     |



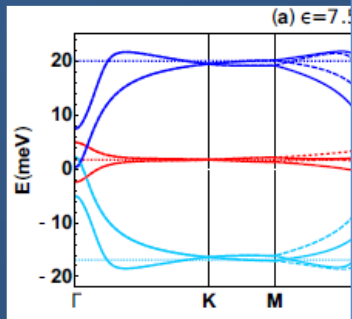
# Shapes and widths of the bands: theory

PHYSICAL REVIEW B 100, 205113 (2019)

Electronic band structure



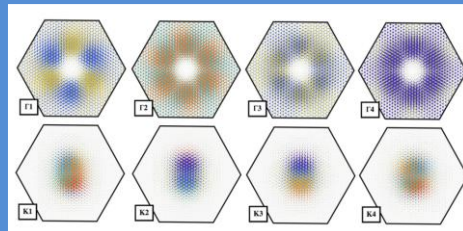
$$V_H(AA) - V_H(BA)$$



PHYSICAL REVIEW B 98, 235158 (2018)

Charge-transfer insulation in twisted bilayer graphene

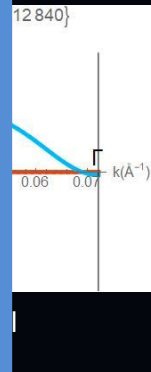
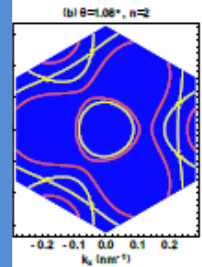
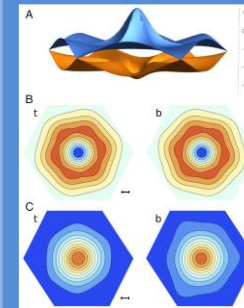
Louk Rademaker<sup>1,2</sup> and Paula Mellado<sup>2,3</sup>



Electrostatic effects, band distortions, and superconductivity in twisted graphene bilayers

Francisco Guinea<sup>a,b,1,2</sup> and Niels R. Walet<sup>b,1,2</sup>

13174-13179 | PNAS | December 26, 2018 | vol. 115 | no. 52

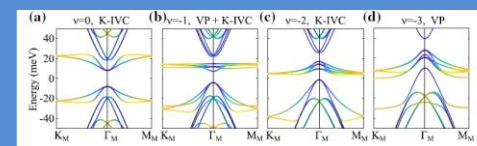
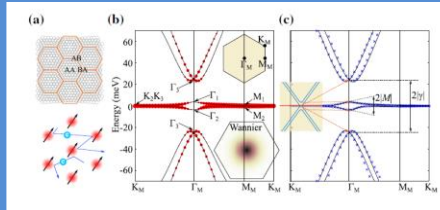


PHYSICAL REVIEW LETTERS 129, 047601 (2022)

Editors' Suggestion | Featured in Physics

Magic-Angle Twisted Bilayer Graphene as a Topological Heavy Fermion Problem

Zhi-Da Song<sup>1,2,\*</sup> and B. Andrei Bernevig<sup>2,3,4</sup>

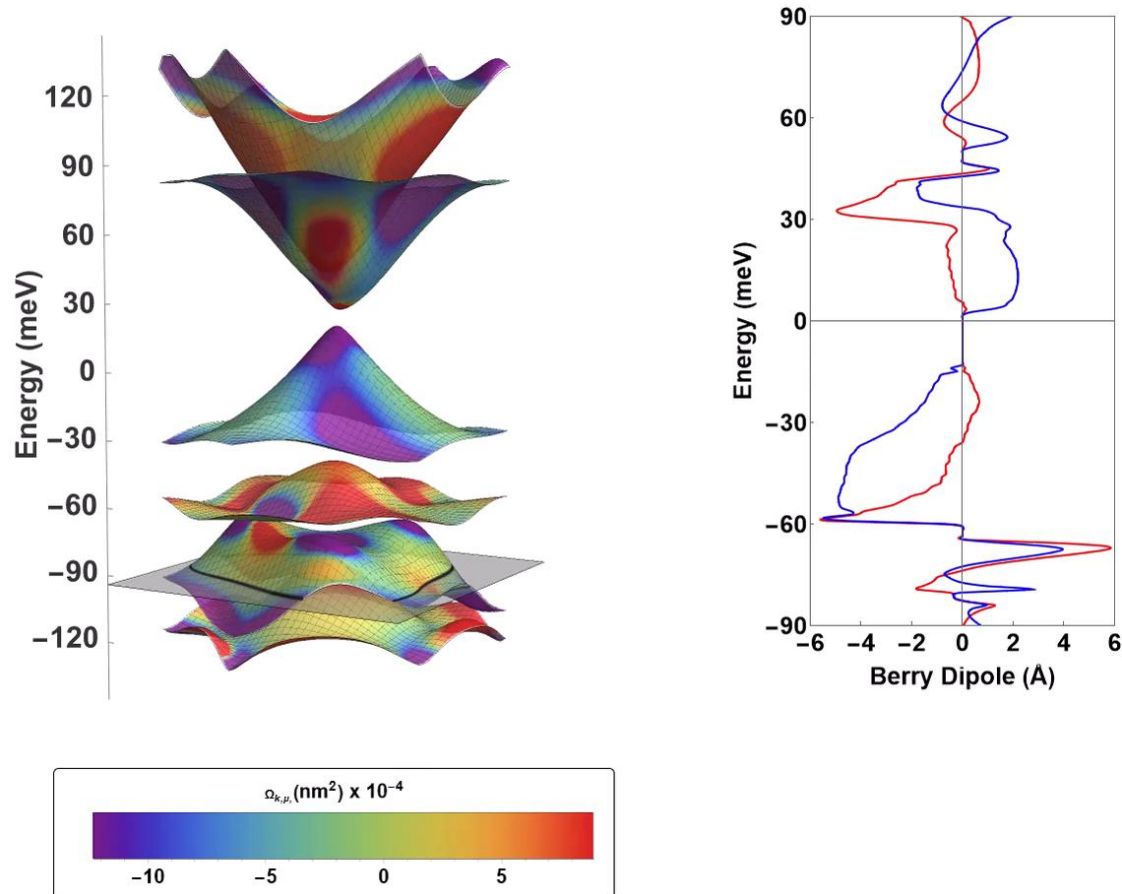


the density of states at the Fermi energy remains high (pinning of van Hove singularities).

## Interaction-enhanced topological Hall effects in strained twisted bilayer graphene

Pierre A. Pantaleón<sup>1,\*</sup> Võ Tiến Phong,<sup>2,†</sup> Gerardo G. Naumis,<sup>3</sup> and Francisco Guinea<sup>1,4,5</sup><sup>1</sup>IMDEA Nanoscience, E-28940 Madrid, Spain<sup>2</sup>Department of Physics, University of Science, Hanoi, Vietnam<sup>3</sup>CONICET, Instituto de Física de Mar del Plata, CC 37, 7600 Mar del Plata, Argentina<sup>4</sup>IMDEA Nanoscience, E-28940 Madrid, Spain<sup>5</sup>IMDEA Nanoscience, E-28940 Madrid, Spain

Fermi Level = -94.10 meV

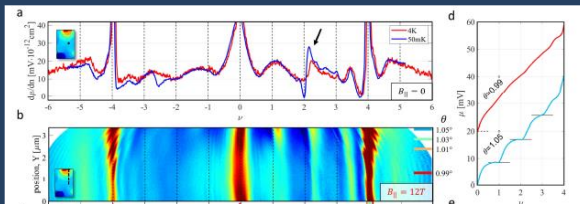


# Cascade of phase transitions in twisted bilayer graphene

## Article Cascade of phase transitions and Dirac revivals in magic-angle graphene

<https://doi.org/10.1038/s41586-020-2373-y> U. Zondiner<sup>1</sup>, A. Rozani<sup>1\*</sup>, D. Rodan-Legrain<sup>1\*</sup>, Y. Cao<sup>2</sup>, R. Quintero<sup>1</sup>, T. Taniguchi<sup>1</sup>, K. Watanabe<sup>1</sup>, Y. Oreg<sup>1</sup>, F. von Oppen<sup>1</sup>, A. Stern<sup>1</sup>, E. Berg<sup>1</sup>, P. Jarillo-Herrero<sup>1,2</sup> & S. Kivelson<sup>1,2</sup>  
Received: 25 November 2019

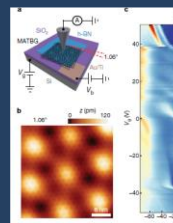
Nature **582**, 203 (2020).



## Article Cascade of electrostatic phase transitions in magic-angle twisted bilayer graphene

<https://doi.org/10.1038/s41586-020-2339-0> Dillon V. Sengupta<sup>1</sup>, U. Zondiner<sup>1</sup>, A. Rozani<sup>1</sup>, D. Rodan-Legrain<sup>1</sup>, Y. Cao<sup>2</sup>, R. Quintero<sup>1</sup>, T. Taniguchi<sup>1</sup>, K. Watanabe<sup>1</sup>, Y. Oreg<sup>1</sup>, F. von Oppen<sup>1</sup>, A. Stern<sup>1</sup>, E. Berg<sup>1</sup>, P. Jarillo-Herrero<sup>1,2</sup> & S. Kivelson<sup>1,2</sup>  
Received: 25 November 2019

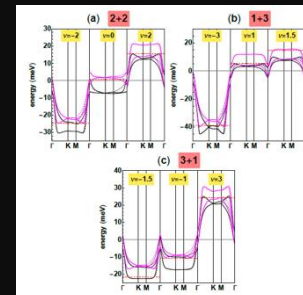
Nature **582**, 203 (2020).



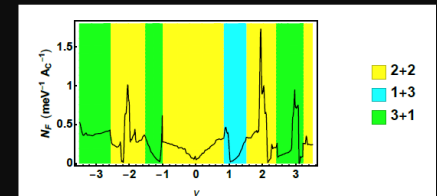
PHYSICAL REVIEW B **102**, 045107 (2020)

Band structure and insulating states driven by Coulomb interaction in twisted bilayer graphene

Tommaso Cea<sup>1</sup> and Francisco Guinea<sup>1,2</sup>



The mean field (HF) cascade



## Unconventional sequence of correlated Chern insulators in magic-angle twisted bilayer graphene

Andrew T. Pierce<sup>1\*</sup>, Yonglong Xie<sup>1,2\*</sup>, Jeong Min Park<sup>2\*</sup>, Eslam Khalaf<sup>1\*</sup>, Seung Hwan Lee<sup>1</sup>, Yuan Cao<sup>2</sup>, Daniel E. Parker<sup>1</sup>, Patrick R. Forrester<sup>1</sup>, Shaowen Chen<sup>1</sup>, Kenji Watanabe<sup>1</sup>, Takashi Taniguchi<sup>1</sup>, Pablo Jarillo-Herrero<sup>1,2</sup> & S. Kivelson<sup>1,2</sup>  
Received: 25 November 2019



Douglas R. Hartree



Vladimir A. Fock

- The long range Coulomb interaction gives a qualitative explanation of the band deformations, and of the broken symmetry phases of twisted bilayer graphene.
- It does not identify details of the broken symmetry phases.

# Superconductivity in twisted bilayer graphene. Some theoretical models

## Electron-phonon coupling:

F. Wu, E. Hwang, S. Das Sarma, Phys. Rev. B **99**, 165112 (2018)  
F. Wu, A. H. MacDonald, I. Martin, Phys. Rev. Lett. **121**, 257001 (2019)  
B. Lian, Z. Wang, B. A. Bernevig, Phys. Rev. Lett. **122**, 257002 (2019)

## Electron-electron coupling: effect of van Hove singularities:

Y. Lin, R. M. Nandkishore, Phys. Rev. B **21**, 214521 (2018)

T. Cea, F. G., Proc. Nat. Acad. Sci. **118**, e2107874118 (2021)  
V. T. Phong, P. A. Pantaleon, T. Cea, F. G., Phys. Rev. B **104**, L121116 (2022)  
T. Cea, P. A. Pantaleon, V. T. Phong, F. G., Phys. Rev. B **105**, 075432 (2022)  
A. Jimeno-Pozo, H. Sainz-Cruz, T. Cea, P. Pantaleon, F. G., Phys. Rev. B **107**, L161106 (2023)  
H. Sainz-Cruz, P. A. Pantaleon, F. G., Phys. Rev. Lett. **131**, 016003 (2023)  
V. T. Phong, H. Sainz-Cruz, E. J. Mele, F. G., arXiv:2307.03031 (2023), Phys. Rev. B, in press  
M. Long, A. Jimeno-Pozo, H. Sainz-Cruz, P. A. Pantaleon, F. G., Proc. Nat. Acad. Sci. **121**, e2405259121 (2024)

## Electron-electron interactions: diagrammatic methods (Kohn-Luttinger) based on the Coulomb interaction:

T. Cea, F. G., Proc. Nat. Acad. Sci. **118**, e2107874118 (2021)  
G. Savit, E. Berg, A. Stern, Y. Oreg, Phys. Rev. Lett. **127**, 247703 (2022)  
V. Kozii, M. P. Zaletel, N. Bulnick, Phys. Rev. B **106**, 235157 (2022)

# Long range interactions and superconductivity: the Kohn-Luttinger mechanism

## NEW MECHANISM FOR SUPERCONDUCTIVITY\*

W. Kohn

University of California, San Diego, La Jolla, California

and

J. M. Luttinger

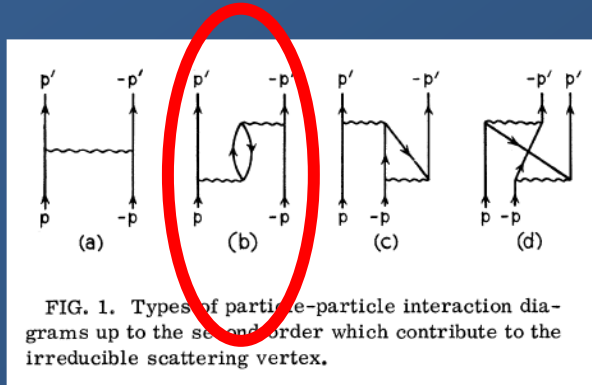
Phys. Rev. Lett. **15**, 524 (1965)



Walter Kohn



Joaquin M. Luttinger



## Superconductivity mediated by electron-hole pairs

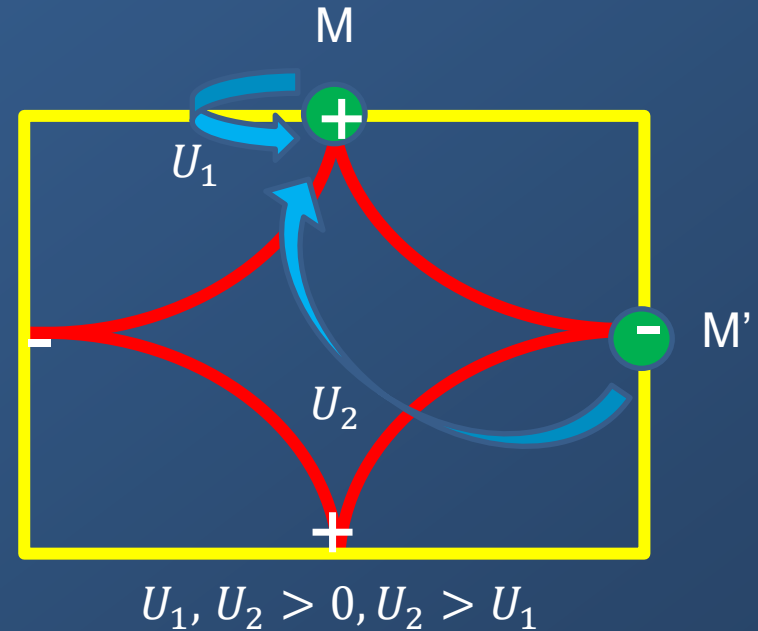
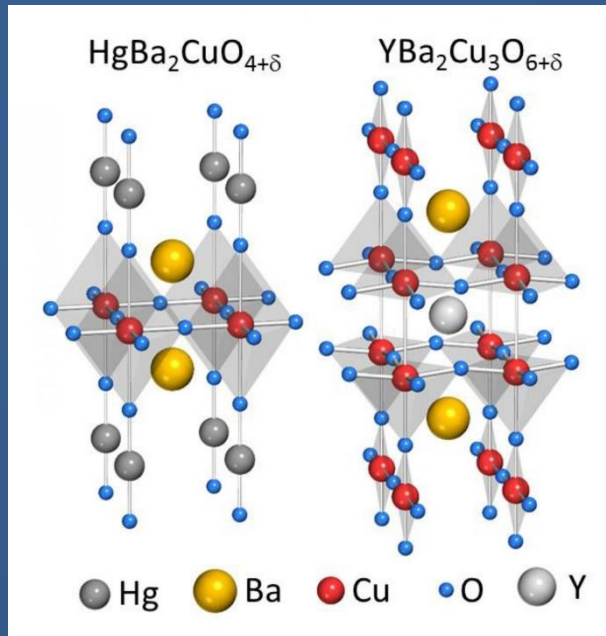
For electrons in metals we can make estimates using a screening Coulomb potential. It is easily seen that in the high-density limit  $u(0)$  dominates and becomes  $u(0) = \pi^2 / m k_F$ ; therefore,

$$(kT_c / \epsilon_0) \sim \exp[-(2l)^4].$$

We also mention that since the electron-phonon interaction is screened by the same kind of mechanism, it too should contribute to long-range effects.



# d-wave superconductivity in cuprates



For electrons in metals we can make estimates using a screening Coulomb potential. It is easily seen that in the high-density limit  $u(0)$  dominates and becomes  $u(0) = \pi^2 / m k_F$ ; therefore,

$$(kT_c / \epsilon_0) \sim \exp[-(2l)^4].$$

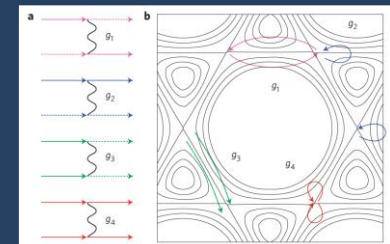
## ARTICLES

PUBLISHED ONLINE: 22 JANUARY 2012 | DOI:10.1038/NPHYS2208

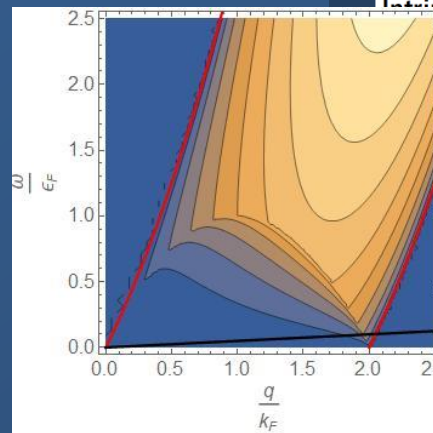
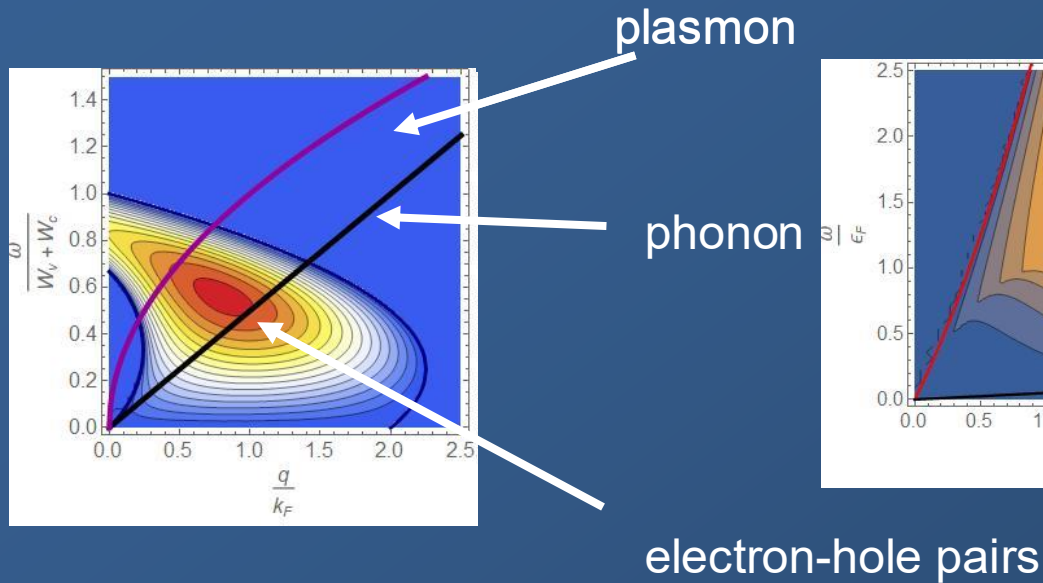
nature  
physics

## Chiral superconductivity from repulsive interactions in doped graphene

Rahul Nandkishore<sup>1</sup>, L. S. Levitov<sup>1</sup> and A. V. Chubukov<sup>2\*</sup>



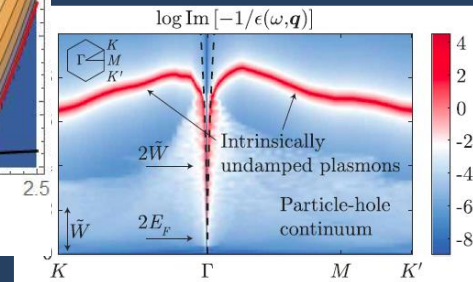
# Low energy excitations and charge excitations: Electron-hole pairs, plasmons, and phonons



## Intrinsically undamped plasmon modes in narrow bands

Bandowski<sup>a,1</sup> and Leonid Levitov<sup>a</sup>

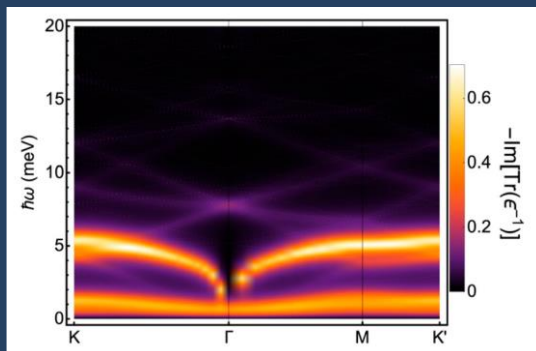
PNAS 116, 20829 (2019)



## Coulomb interaction, phonons, and superconductivity in twisted bilayer graphene

Tommaso Cea<sup>a,b</sup> and Francisco Guinea<sup>a,c,d,1</sup>

PNAS 118, e2107874118 (2021)



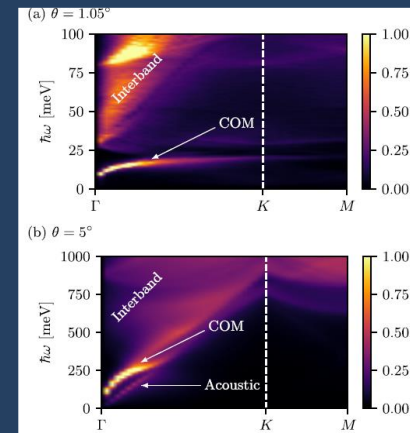
PHYSICAL REVIEW B 110, 045431 (2024)

Editors' Suggestion

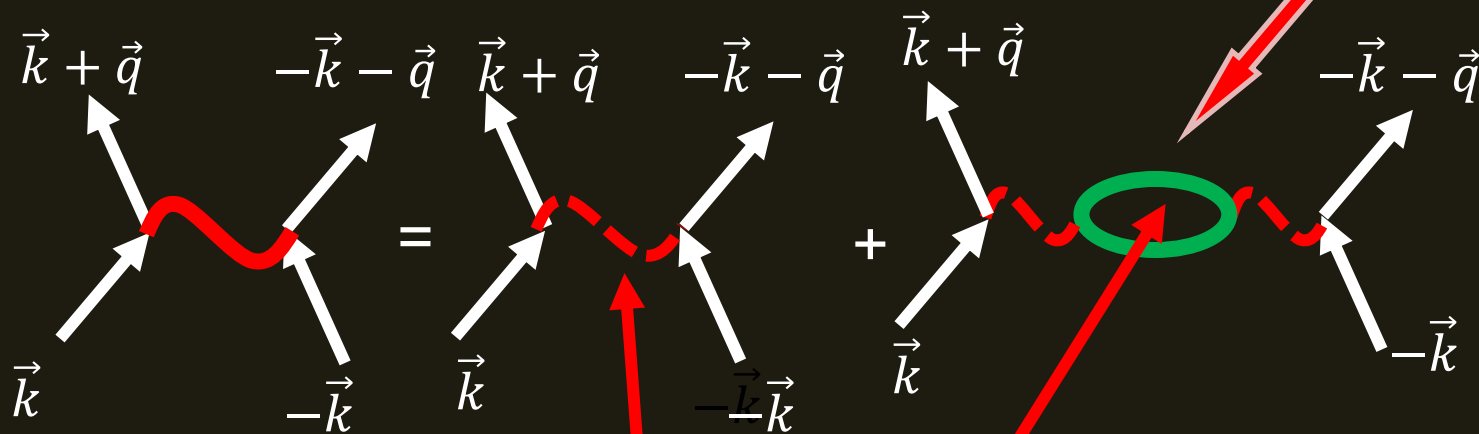
Featured in Physics

## Theory of intrinsic acoustic plasmons in twisted bilayer graphene

Lorenzo Cavicchi<sup>1</sup>, Iacopo Torre<sup>2,3</sup>, Pablo Jarillo-Herrero<sup>4</sup>, Frank H. L. Koppens<sup>3,5</sup> and Marco Polini<sup>3,6</sup>



# Repulsive interactions and pairing



- Longitudinal phonons modify the susceptibility.
- The bare interaction is the repulsive Coulomb coupling

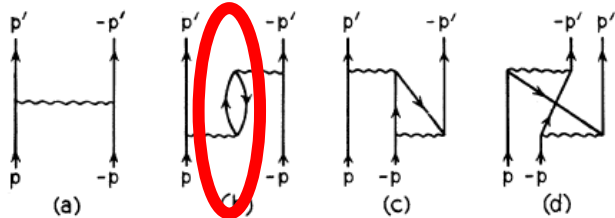


FIG. 1. Types of particle-particle interaction diagrams up to the second order which contribute to the irreducible scattering vertex.

## NEW MECHANISM FOR SUPERCONDUCTIVITY\*

W. Kohn

University of California, San Diego, La Jolla, California

and

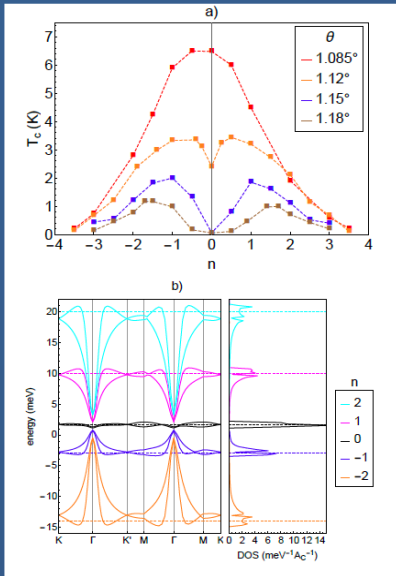
J. M. Luttinger

Phys. Rev. Lett. **15**, 524 (1965)

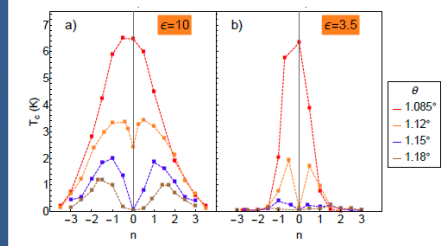
We also mention that since the electron-phonon interaction is screened by the same kind of mechanism, it too should contribute to long-range effects.

# Results: critical temperature

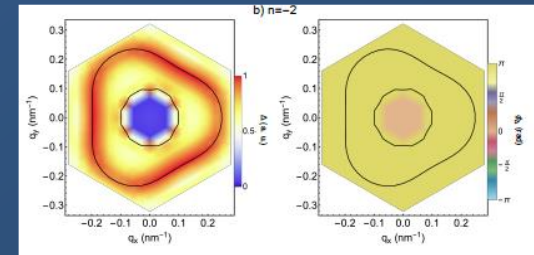
Critical temperatures



Bands, densities of states, magic angle



Dependence on dielectric constant



- Order parameter. Finite gap in each valley
- Consistent with spin singlet/valley triplet or with spin triplet/valley singlet superconductivity.

- The critical temperature is significantly enhanced by the electron-phonon interaction.
- Superconductivity correlates with the density of states at the Fermi level.
- The effect of external screening depends on the strength of the electron-phonon interaction.

# Superconductivity in twisted trilayers

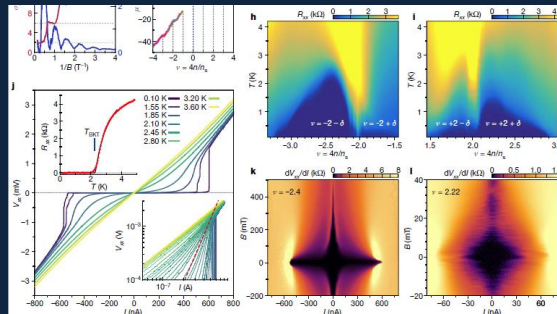
## Article

### Tunable strongly coupled superconductivity in magic-angle twisted trilayer graphene

<https://doi.org/10.1038/s41586-021-03192-0> Jeong Min Park<sup>1,4</sup>, Yuan Cao<sup>1,4,12</sup>, Kenji Watanabe<sup>1</sup>, Takashi Taniguchi<sup>1</sup> & Pablo Jarillo-Herrero<sup>1,2</sup>

Received: 26 October 2020

Nature | Vol 590 | 11 February 2021 | 249

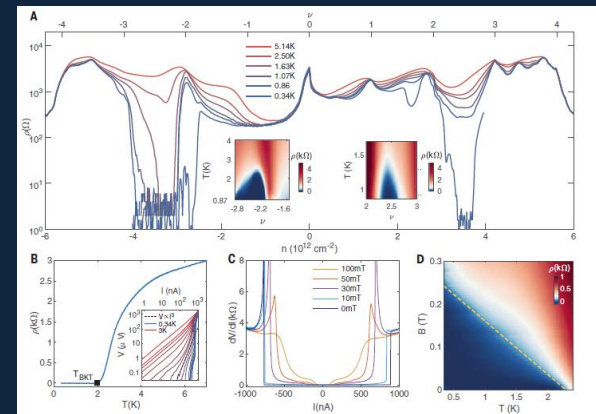


## SUPERCONDUCTIVITY

### Electric field-tunable superconductivity in alternating-twist magic-angle trilayer graphene

Zeyu Hao<sup>1,2</sup>, A. M. Zimmerman<sup>1,2</sup>, Patrick Ledwith<sup>1</sup>, Eslam Khalaf<sup>1</sup>, Danial Haie Najafabadi<sup>1</sup>, Kenji Watanabe<sup>1</sup>, Takashi Taniguchi<sup>1</sup>, Ashwin Vishwanath<sup>1</sup>, Philip Kim<sup>1,†</sup>

Hao et al., Science 371, 1133–1138 (2021) 12 March 2021



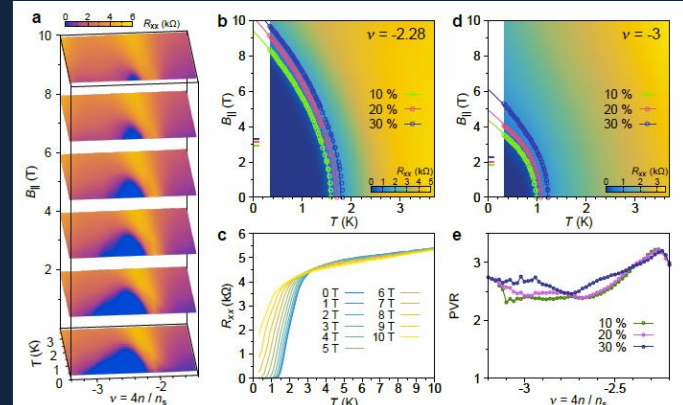
## Article

### Pauli-limit violation and re-entrant superconductivity in moiré graphene

<https://doi.org/10.1038/s41586-021-03685-y> Yuan Cao<sup>1,4,12</sup>, Jeong Min Park<sup>1,4,12</sup>, Kenji Watanabe<sup>1</sup>, Takashi Taniguchi<sup>1</sup> & Pablo Jarillo-Herrero<sup>1,2</sup>

Received: 4 March 2021

528 | Nature | Vol 595 | 22 July 2021





# Superconductivity in twisted multilayers

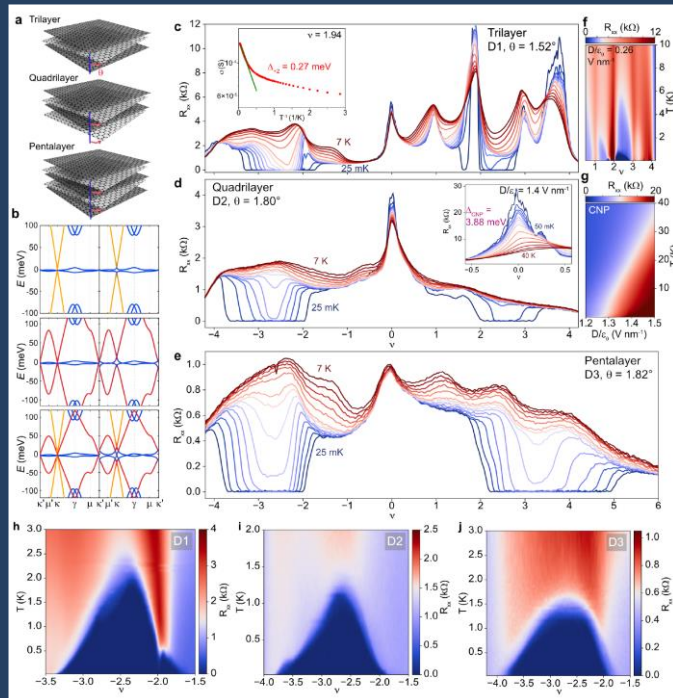
## RESEARCH

### SUPERCONDUCTIVITY

## Promotion of superconductivity in magic-angle graphene multilayers

Yiran Zhang<sup>1,2,3</sup>, Robert Polsh<sup>1,2</sup>, Cyprian Lewandowski<sup>2,3</sup>, Alex Thomson<sup>2,3,4</sup>, Yang Peng<sup>5</sup>, Youngjoon Choi<sup>1,2,3</sup>, Hyunjin Kim<sup>1,2,3</sup>, Kenji Watanabe<sup>6</sup>, Takashi Taniguchi<sup>6</sup>, Jason Alicea<sup>2,3</sup>, Felix von Oppen<sup>7</sup>, Gil Refael<sup>2,3</sup>, Stevan Nadj-Perge<sup>1,2,\*</sup>

Zhang et al., *Science* **377**, 1538–1543 (2022) 30 September 2022



nature  
materials

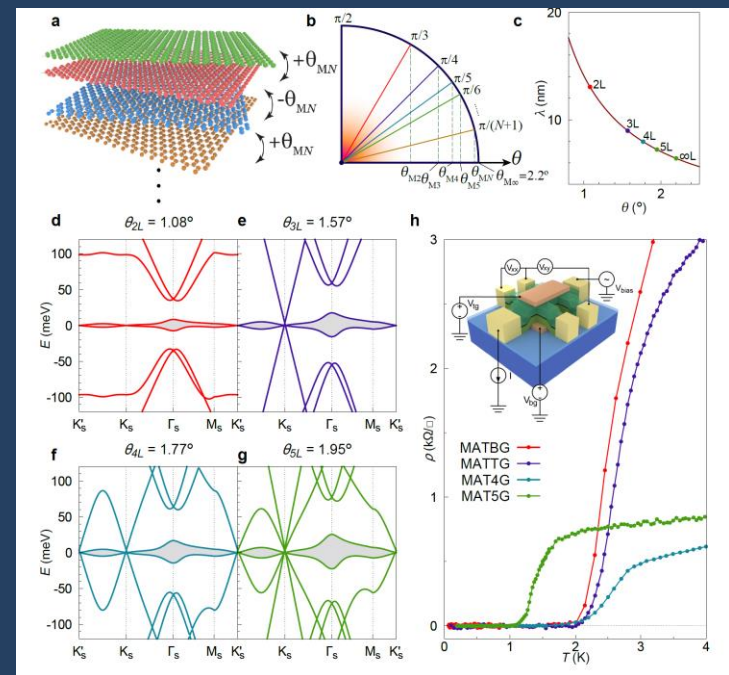
ARTICLES

<https://doi.org/10.1038/s41563-022-01287-1>

Check for updates

## Robust superconductivity in magic-angle multilayer graphene family

Jeong Min Park<sup>1,3</sup>, Yuan Cao<sup>1,3</sup>, Li-Qiao Xia<sup>1</sup>, Shuwen Sun<sup>1</sup>, Kenji Watanabe<sup>2</sup>, Takashi Taniguchi<sup>2</sup> and Pablo Jarillo-Herrero<sup>1</sup>



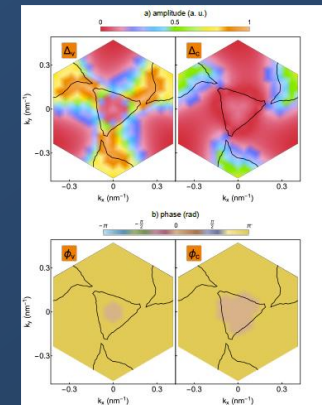
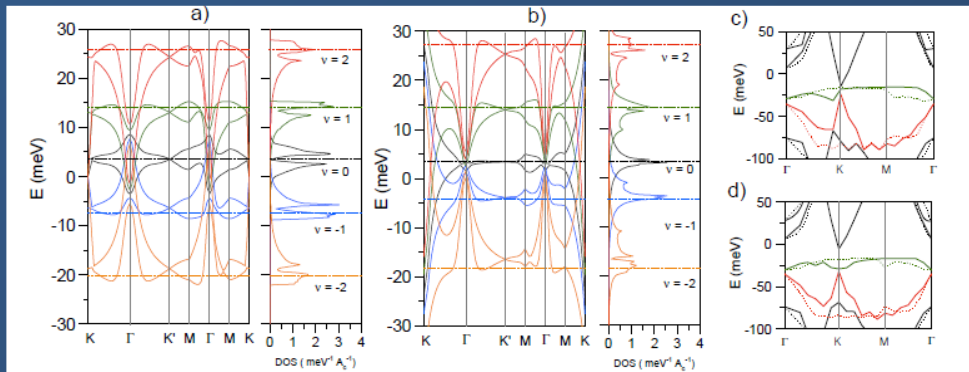
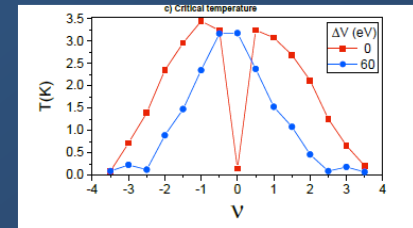
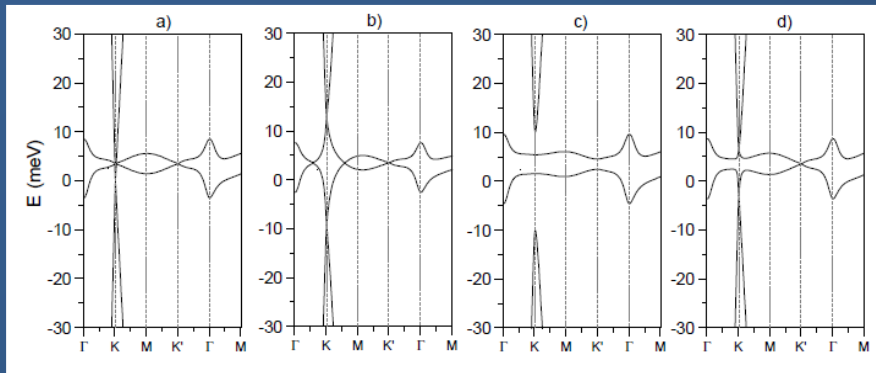
# Superconductivity in twisted trilayers: theory

PHYSICAL REVIEW B **104**, L121116 (2021)

Letter

## Band structure and superconductivity in twisted trilayer graphene

Võ Tiến Phong,<sup>1</sup> Pierre A. Pantaleón<sup>2</sup>, Tommaso Cea<sup>2</sup>, and Francisco Guinea<sup>2,3</sup>



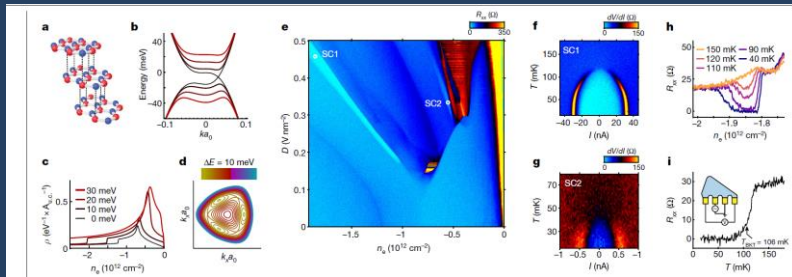
# Superconductivity in graphene without a moiré superlattice

## Article

### Superconductivity in rhombohedral trilayer graphene

<https://doi.org/10.1038/s41586-021-03926-0> Haoxin Zhou<sup>1</sup>, Tian Xie<sup>1</sup>, Takashi Taniguchi<sup>2</sup>, Kenji Watanabe<sup>3</sup> & Andrea F. Young<sup>1,2</sup>

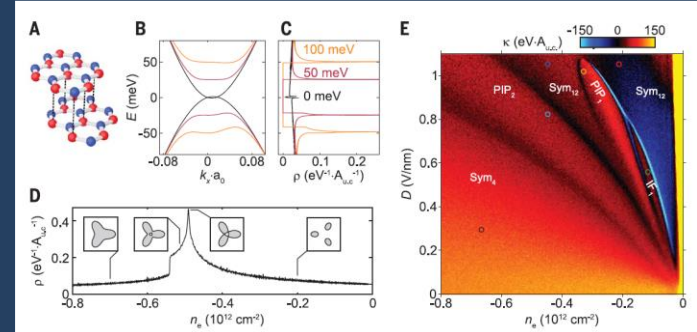
434 | Nature | Vol 598 | 21 October 2021



### Isospin magnetism and spin-polarized superconductivity in Bernal bilayer graphene

Haoxin Zhou<sup>1,2</sup>, Ludwig Holleis<sup>1</sup>, Yu Saito<sup>1</sup>, Liam Cohen<sup>1</sup>, William Huynh<sup>1</sup>, Caitlin L. Patterson<sup>1</sup>, Fangyuan Yang<sup>1</sup>, Takashi Taniguchi<sup>2</sup>, Kenji Watanabe<sup>3</sup>, Andrea F. Young<sup>1,2</sup>

Zhou *et al.*, *Science* **375**, 774–778 (2022) 18 February 2022



## Article

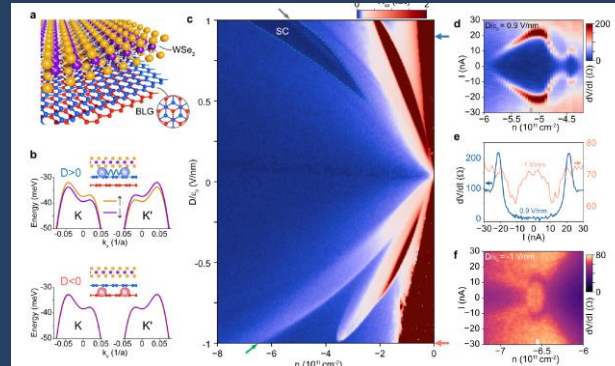
### Enhanced superconductivity in spin-orbit proximitized bilayer graphene

<https://doi.org/10.1038/s41586-022-05446-x> Yiran Zhang<sup>1,2,3</sup>, Robert Polski<sup>1,2</sup>, Alex Thomson<sup>1,2,4</sup>, Étienne Lantagne-Hurtubise<sup>1,2,3</sup>, Cyprian Lewandowski<sup>1,2</sup>, Haoxin Zhou<sup>1,2</sup>, Kenji Watanabe<sup>3</sup>, Takashi Taniguchi<sup>3</sup>, Jason Alicea<sup>1,2</sup> & Stevan Nadj-Perge<sup>1,2,3</sup>

Received: 5 May 2022

Accepted: 14 October 2022

268 | Nature | Vol 613 | 12 January 2023



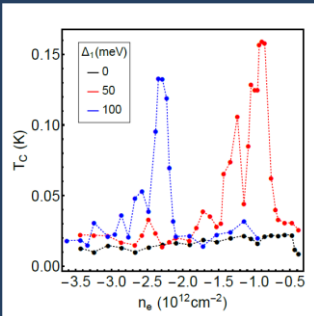
# Superconductivity in graphene without a moiré superlattice: theory

PHYSICAL REVIEW B **105**, 075432 (2022)

Superconductivity from repulsive interactions in rhombohedral trilayer graphene:  
A Kohn-Luttinger-like mechanism

Tommaso Cea<sup>1</sup> and Pierre A. Pantaleón<sup>2</sup>  
Andalus Nanoscience, February 9, 2023 Madrid, Spain

Vũ Tiến Phong  
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104, USA  
Francisco Guinea



PHYSICAL REVIEW B **107**, L161106 (2023)

Letter

Editors' Suggestion

Superconductivity from electronic interactions and spin-orbit enhancement  
in bilayer and trilayer graphene

Alejandro Jimeno-Pozo<sup>1,2</sup>, Héctor Sainz-Cruz<sup>1</sup>, Tommaso Cea<sup>1,2</sup>, Pierre A. Pantaleón<sup>1</sup>, and Francisco Guinea<sup>1,3</sup>

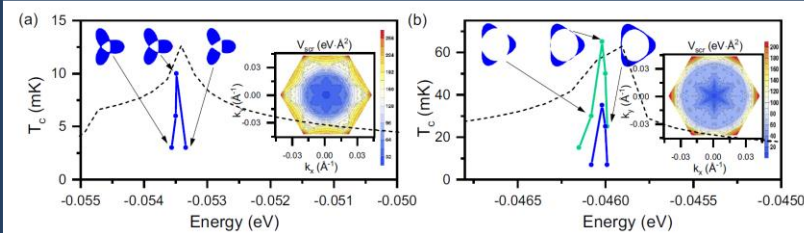


FIG. 2. Superconducting critical temperature due to only the screened long-range Coulomb interaction (blue), and including also the short-range interaction (green), and DOS (a.u., dashed black) versus Fermi energy in (a) BBG with hole doping, near the vHs ( $n_e \approx -0.87 \times 10^{12} \text{ cm}^{-2}$ ), we observe a  $T_c \approx 10 \text{ mK}$ , and (b) RTG with hole doping, near the vHs ( $n_e \approx -0.6 \times 10^{12} \text{ cm}^{-2}$ ),  $T_c \approx 33 \text{ mK}$  and it raises to  $T_c \approx 65 \text{ mK}$  after including the short-range interaction. The top insets show the shape of the Fermi surfaces, which experience a change in topology at the vHs due to Lifshitz transitions. Right insets: screened Coulomb potential at the energy for which  $T_c$  is maximum.

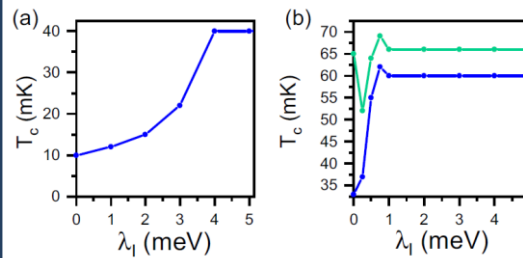
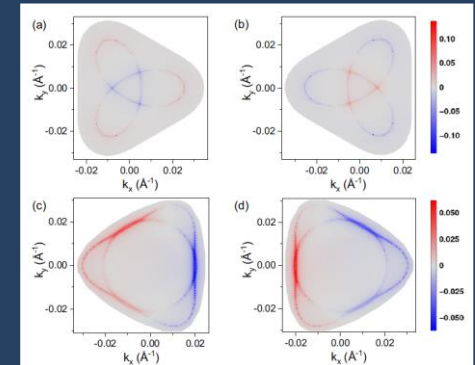
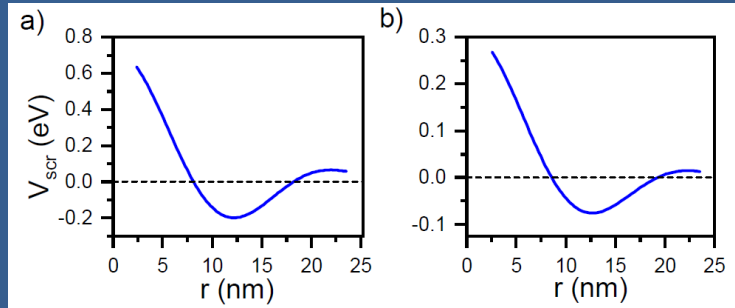


FIG. 3. Using spin-orbit coupling enhancement of critical temperature in hole-doped (a) BBG, for which  $T_c$  augments by a factor of 4 and saturates at  $\lambda_I = 4 \text{ meV}$ . (b) RTG, for which  $T_c$  due to long-range interactions (blue) augments by a factor of 2 and saturates at  $\lambda_I = 1 \text{ meV}$ . In contrast, Ising SOC is detrimental to short-range interactions, so  $T_c$  does not increase much when they are included (green).





# Superconductivity graphene without a moiré superlattice: effective attraction



RESEARCH ARTICLE | AUGUST 13 2013

## Superconductivity from repulsive interaction

Saurabh Maiti; Andrey V. Chubukov

AIP Conf. Proc. 1550, 3–73 (2013)

<https://doi.org/10.1063/1.4818400>

PHYSICAL REVIEW B **88**, 054515 (2013)

## Interactions and superconductivity in heavily doped $\text{MoS}_2$

R. Roldán,<sup>1</sup> E. Cappelluti,<sup>1,2</sup> and F. Guinea<sup>1</sup>

PHYSICAL REVIEW B **107**, L041111 (2023)

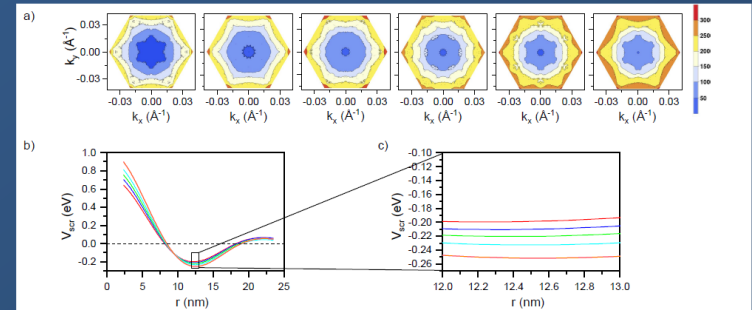
Letter

## Superconductivity induced by the intervalley Coulomb scattering in a few layers of graphene

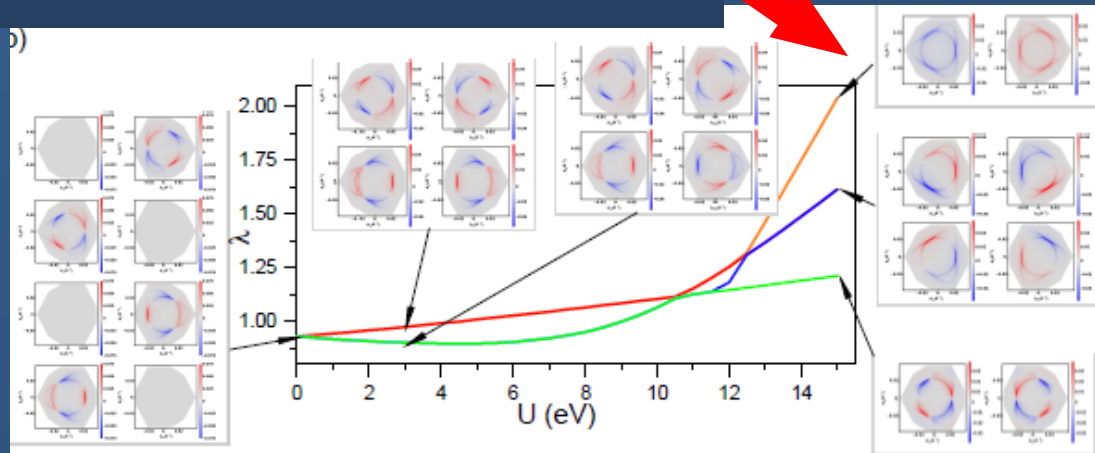
Tommaso Cea

hexagonal trilayer

in real space



Effect of spin-orbit coupling



$$V_{scr}(\vec{q}) = \frac{V_C(\vec{q})}{1 - V_C(\vec{q})\chi(\vec{q})} \approx \frac{1}{|\chi(\vec{q})|}$$

SOC, due to flavor polarization, lowers screening, and increases both repulsion and attraction

Order parameter and intervalley interactions



# Charge fluctuations, phonons, and superconductivity in multilayer graphene

Ziyan Li<sup>1,2</sup>, Xueheng Kuang<sup>1,2</sup>, Alejandro Jimeno-Pozo<sup>2</sup>, Héctor Sainz-Cruz<sup>2</sup>,  
Zhen Zhan<sup>2,1,\*</sup>, Shengjun Yuan<sup>1,3</sup>, and Francisco Guinea<sup>2,4</sup>

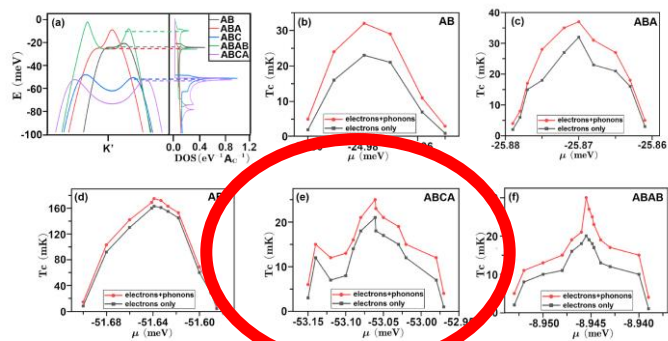


FIG. 4. (a) Band structures and DOS near the CNP in five different multilayer graphene stacks, due to electron interactions only or taking into account also the effects of electron-acoustic phonon coupling.

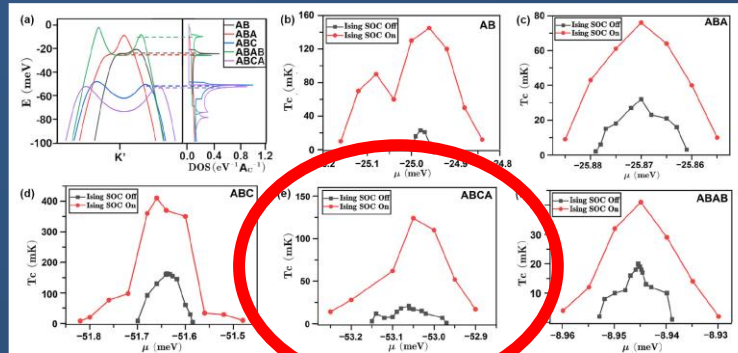


FIG. 5. (a) Bands and DOS near VHS in multilayer graphene, with displacement field  $\Delta_1 = 50$  meV. (b)–(f) Critical temperature of multilayer graphene with and without Ising SOC, with Fermi energy near VHS.

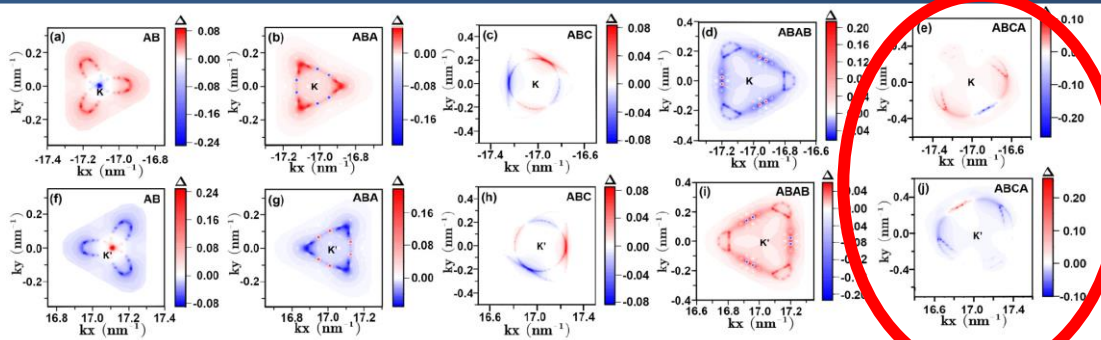


FIG. 6. (a)–(e) Superconducting order parameters near  $K$  valley in the first BZ of multilayer graphene with interlayer potential  $\Delta_1 = 50$  meV and maximum critical temperatures  $T_c$ . (f)–(j) Order parameters near  $K'$  valley in the first BZ.

## Quarter metal phase

- Lack of Cooper divergence, 1<sup>st</sup> order transition?
- Does the symmetries change in the superconducting phase?
- Role of trigonal distortion.

# Tuning repulsive interactions and superconductivity

nature physics

Article

<https://doi.org/10.1038/s41567-026-03243-1>

## Double-edged role of interactions in superconducting twisted bilayer graphene

Received: 22 December 2024

Accepted: 5 March 2026

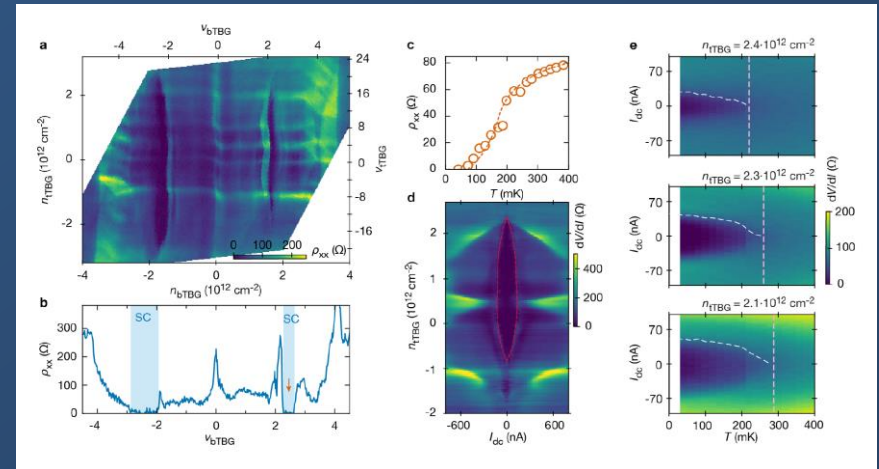
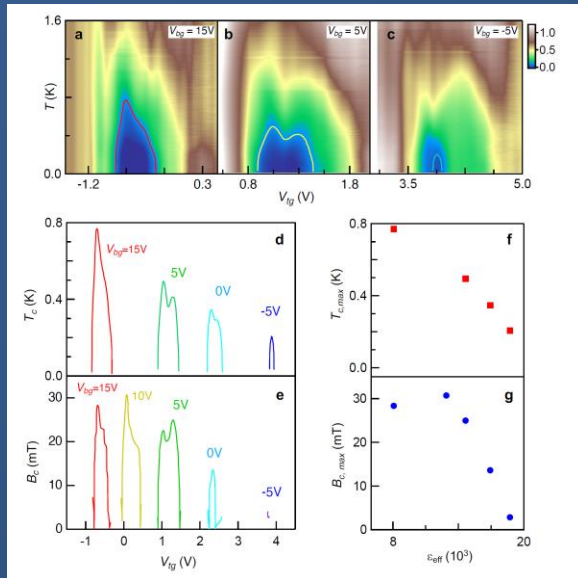
Published online: 07 April 2026

Xueshi Gao<sup>1</sup>, Alejandro Jimeno-Pozo<sup>2</sup>, Pierre A. Pantaleon<sup>2</sup>, Aatmaj Rajesh<sup>1</sup>, Emilio Codecido<sup>1,3</sup>, Daria L. Sharifi<sup>1</sup>, Zheneng Zhang<sup>1</sup>, Youwei Liu<sup>1</sup>, Kenji Watanabe<sup>4</sup>, Takashi Taniguchi<sup>5</sup>, Marc W. Bockrath<sup>1</sup>, Francisco Guinea<sup>2,6</sup> & Chun Ning Lau<sup>1</sup>

## Coulomb screening of superconductivity in magic-angle twisted bilayer graphene

Julien Barrier<sup>1,2</sup>, Liangtao Peng<sup>3</sup>, Shuigang Xu<sup>2</sup>, V.I. Fal'ko<sup>1,2,4</sup>, K. Watanabe<sup>5</sup>, T. Taniguchi<sup>5</sup>, A.K. Geim<sup>1,2†</sup>, S. Adam<sup>3,6,7†</sup> and Alexey I. Berdyugin<sup>6,3,1†</sup>

arXiv:2412.01577

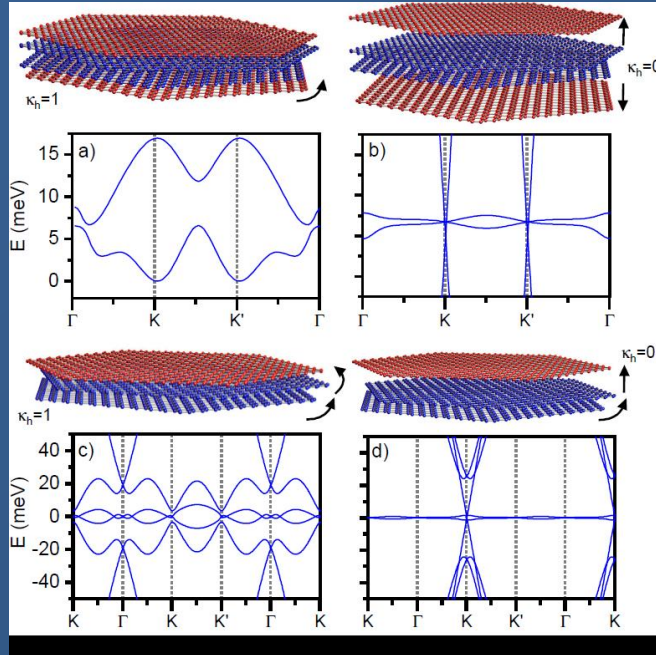


# Twisted double bilayer and helical trilayer graphene

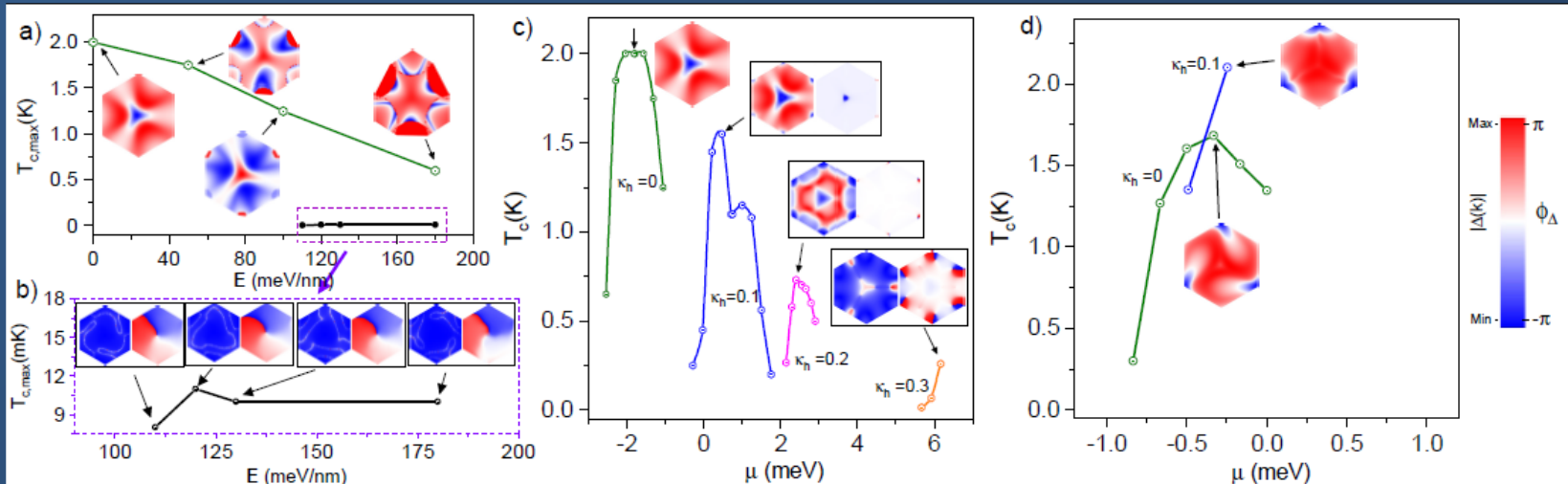
## Interlayer Coupling Driven Superconductivity in Twisted Double Bilayer Graphene

Min Long,<sup>1,2,\*</sup> Alejandro Jimeno-Pozo,<sup>3,\*</sup> Héctor Sainz-Cruz,<sup>3</sup> Pierre A. Pantaleón,<sup>1,†</sup> and Francisco Guinea<sup>1,4,5</sup>

arXiv:2401.09667, PNAS **121** (32) e2405259121 (2024)



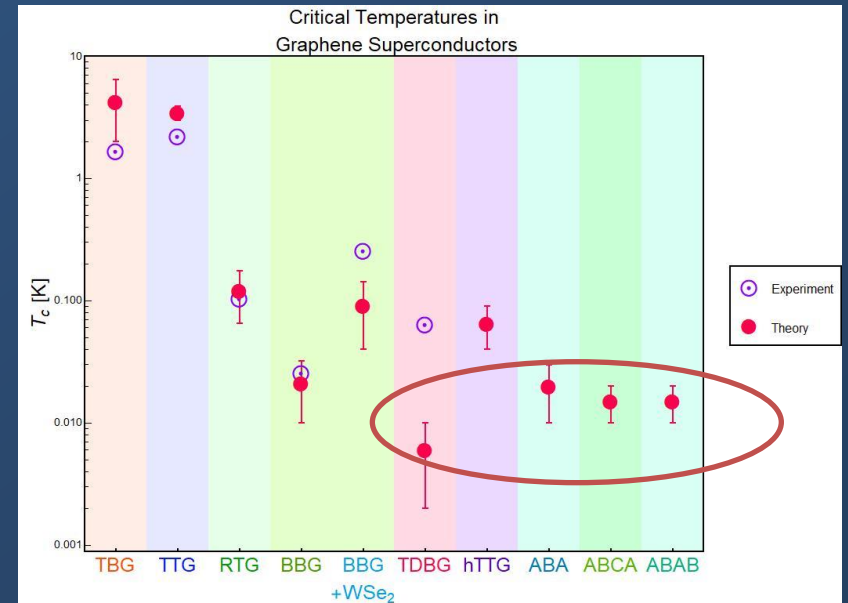
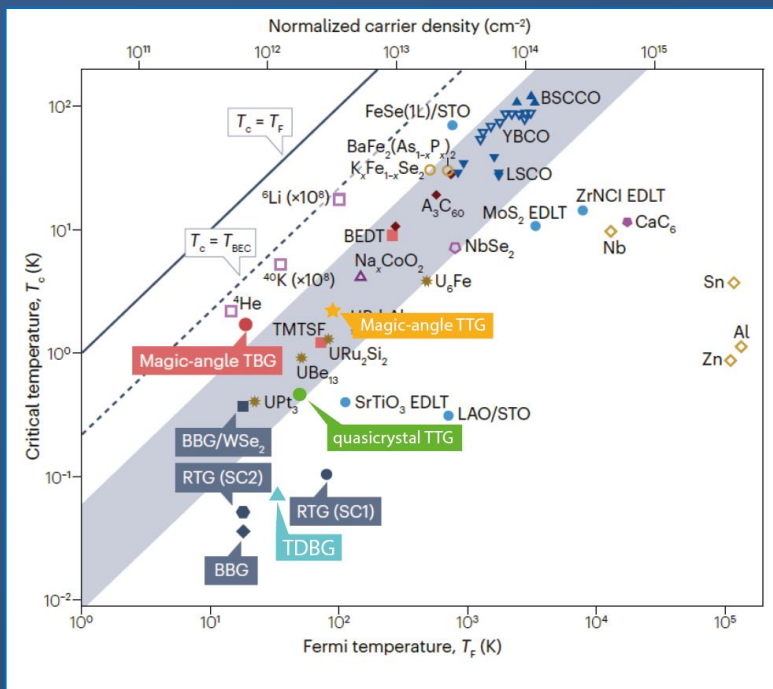
|      | ee+ph+G            | ee+G               | ee+ph              | ee               |
|------|--------------------|--------------------|--------------------|------------------|
| TBG  | $\sim 2\text{K}$   | $\sim 10\text{mK}$ | $\sim 10\text{mK}$ | $< 1\text{mK}$   |
| TDBG | $\sim 10\text{mK}$ | $< 1\text{mK}$     | $< 1\text{mK}$     | $\ll 1\text{mK}$ |
| hTTG | $\ll 1\text{mK}$   | $\ll 1\text{mK}$   | ??                 | ??               |



# Long range electron-electron interactions and superconductivity in graphene stacks

- Strongest interaction in twisted bilayer graphene
- Few adjustable parameters,  $\epsilon \approx 6 - 10$
- Describes well general trends
- Broken symmetry phases not included

| Electronic bands |                     |                |                |                |               |              |              | Coulomb potential |            | Deformation potential and elastic constants |                              |                          |
|------------------|---------------------|----------------|----------------|----------------|---------------|--------------|--------------|-------------------|------------|---------------------------------------------|------------------------------|--------------------------|
| $a(\text{\AA})$  | $v_F(eV\text{\AA})$ | $\gamma_1(eV)$ | $\gamma_3(eV)$ | $\gamma_4(eV)$ | $\Delta'(eV)$ | $g_{AA}(eV)$ | $g_{AB}(eV)$ | $d_G(nm)$         | $\epsilon$ | $D(eV)$                                     | $\lambda(eV\text{\AA}^{-2})$ | $\mu(eV\text{\AA}^{-2})$ |
| 2.46             | 5.253               | 0.4            | 0.32           | 0.044          | 0.05          | 0.797        | 0.0975       | 40                | 10         | 20                                          | 3.25                         | 9.44                     |

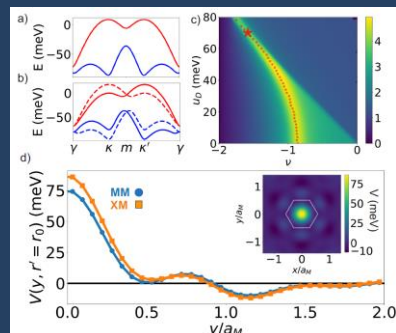




# Other recent RPA+KL calculations of superconductivity

## Topological superconductivity from repulsive interactions in twisted WSe<sub>2</sub>

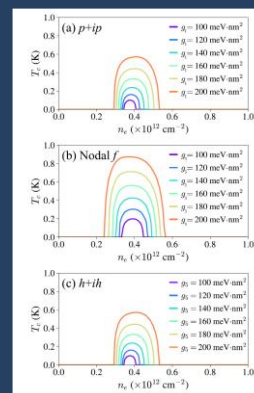
Daniele Guerzi,<sup>1,\*</sup> Daniel Kaplan,<sup>2,†</sup> Julian Ingham,<sup>3,1</sup> J. H. Pixley,<sup>2,1</sup> and Andrew Millis<sup>3,1</sup>



arXiv:2408.16075

## Intravalley spin-polarized superconductivity in rhombohedral tetralayer graphene

Yang-Zhi Chou<sup>1,\*</sup>, Jihang Zhu,<sup>1,†</sup> and Sankar Das Sarma<sup>1,2</sup>



Phys. Rev. B **111**,174523 (2025)

nature communications

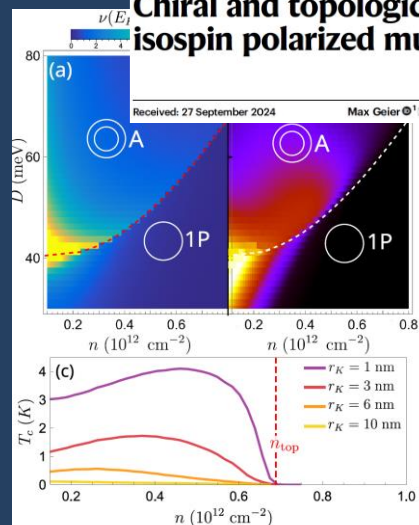
Article

<https://doi.org/10.1038/s41467-025-66902-4>

## Chiral and topological superconductivity in isospin polarized multilayer graphene

Received: 27 September 2024

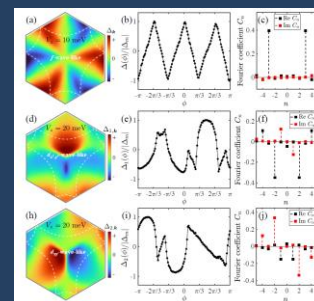
Max Geier<sup>1,\*</sup>, Margarita Davydova<sup>1,2</sup> & Liang Fu<sup>1,†</sup>



PHYSICAL REVIEW LETTERS **135**, 246002 (2025)

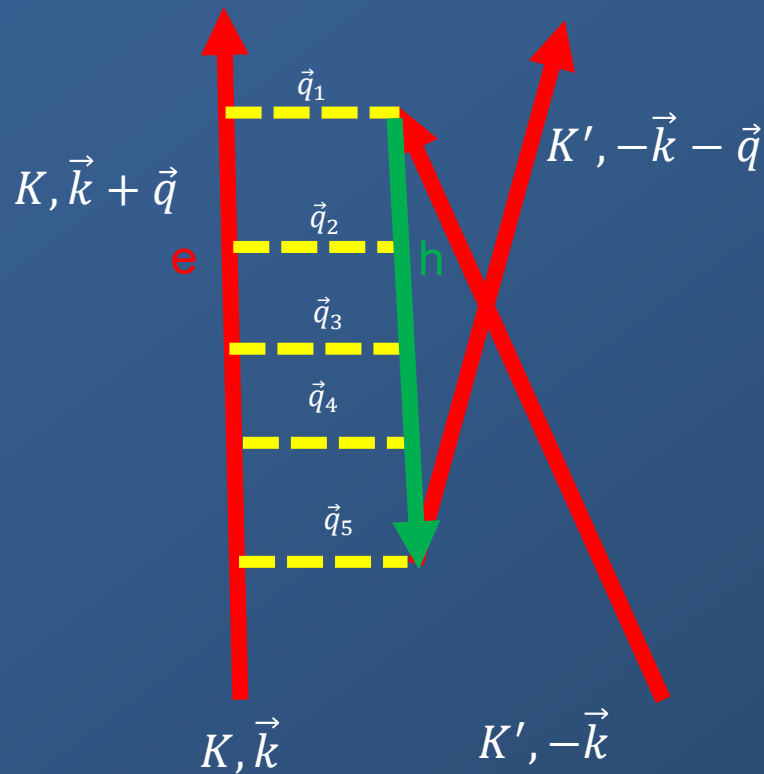
## Topological Chiral Superconductivity Mediated by Intervalley Antiferromagnetic Fluctuations in Twisted Bilayer WSe<sub>2</sub>

Wei Qin<sup>1,\*</sup>, Wen-Xuan Qiu,<sup>2,\*</sup> and Fengcheng Wu<sup>2,‡</sup>



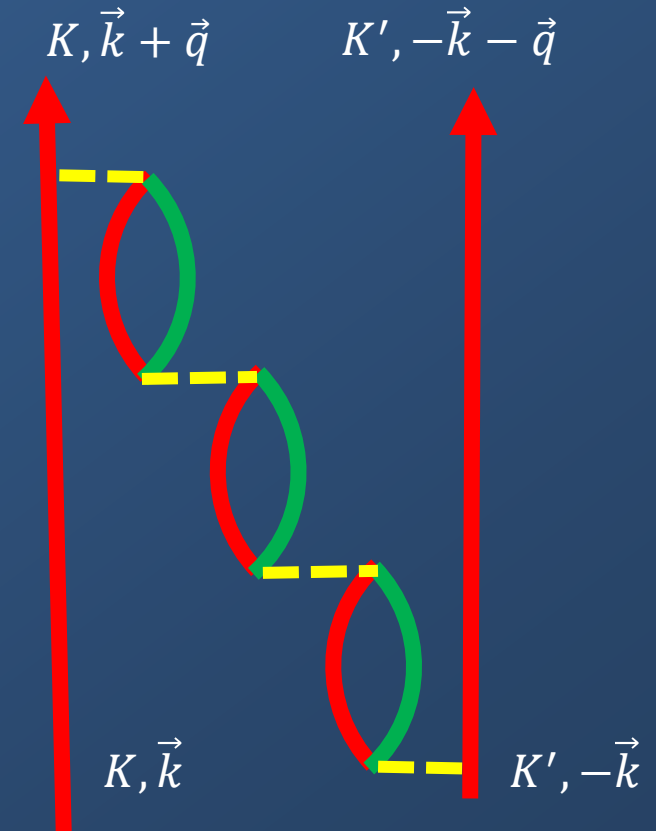


# Soft spin and valley modes, broken symmetries



$$V_{\vec{G}, \vec{G}'}(\vec{q}, \omega) = V_{\vec{G}, \vec{G}'}^0(\vec{q}, \omega) \left( 1 - \Pi_{\vec{G}, \vec{G}'}(\vec{q}, \omega) \right)^{-1}$$

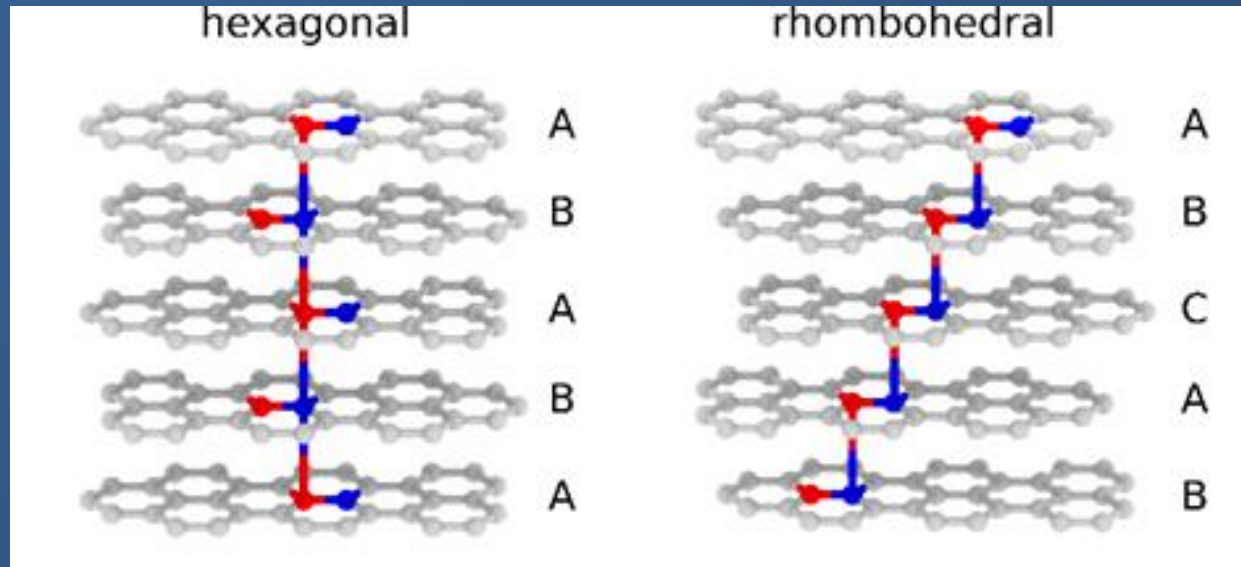
Spin, valley, ..., waves



$$V_{\vec{G}, \vec{G}'}(\vec{q}, \omega) = V_{\vec{G}, \vec{G}'}^0(\vec{q}, \omega) \left( 1 + \Pi_{\vec{G}, \vec{G}'}(\vec{q}, \omega) \right)^{-1}$$

Charge fluctuations

## Three dimensional rhombohedral graphite

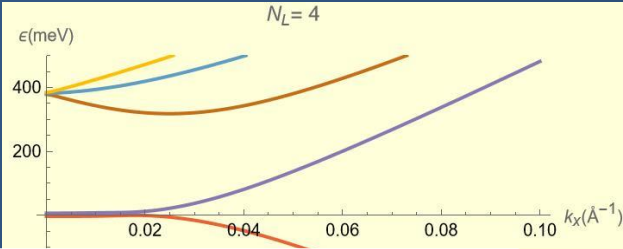
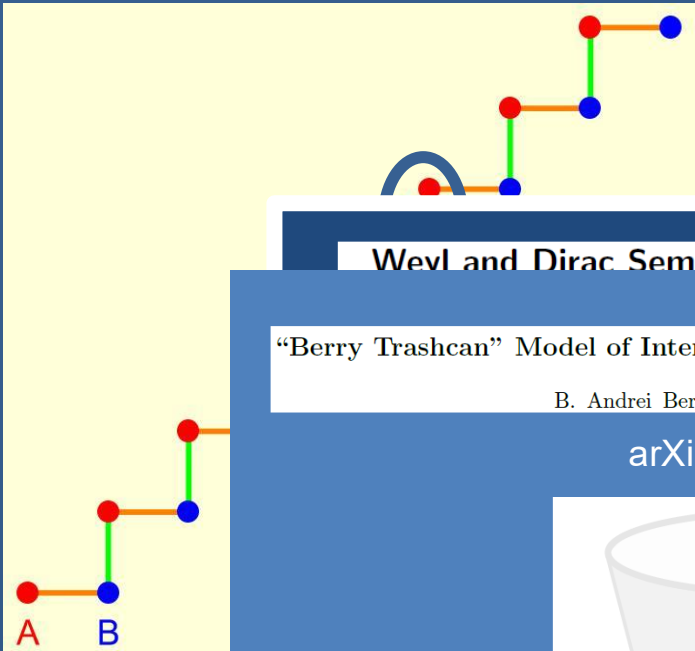


Bernal graphite ABABAB

Rhombohedral graphite ABABAB

About 10% of natural graphite shows rhombohedral stacking (M. Dresselhaus, private communication)

# Flat bands in thick rhombohedral multilayers



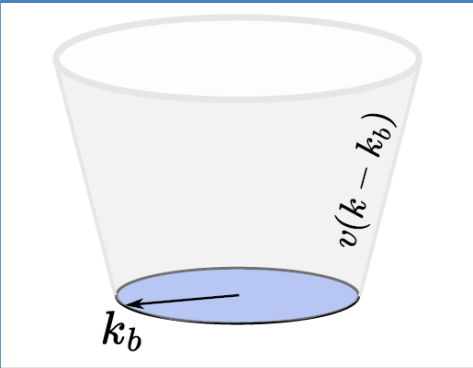
## Weyl and Dirac Semimetals in Three Dimensional Solids\*

“Berry Trashcan” Model of Interacting Electrons in Rhombohedral Graphene

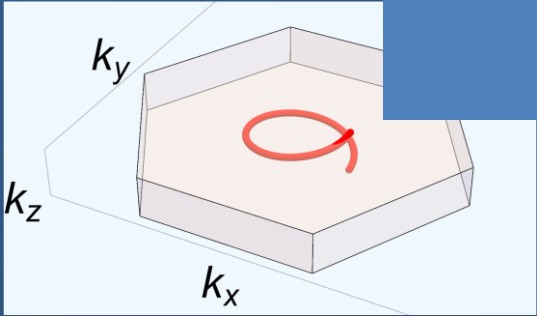
B. Andrei Bernevig<sup>1,2,3</sup> and Yves H. Kwan<sup>4</sup>

arXiv:2503.09692

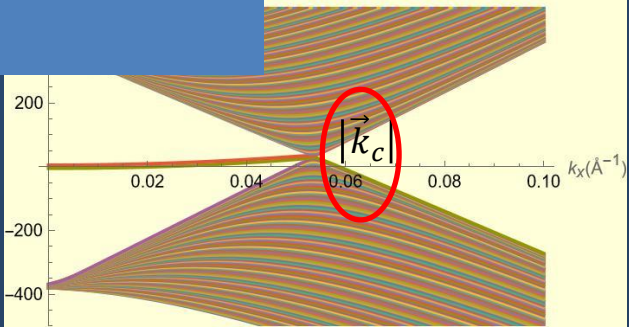
University, Baltimore,



Su-Schrieffer



$$\rho_c = \frac{|\vec{k}_c|^2}{4\pi} \approx 10^{13} \text{cm}^{-2}$$



## Flat Bands in Topological Media<sup>†</sup>

T. T. Heikkilä<sup>a</sup>, N. B. Kopnin<sup>a, b</sup>, and G. E. Volovik<sup>a, b</sup>

<sup>a</sup> Low Temperature Laboratory, Aalto University, School of Science and Technology,  
P.O. Box 15100, FI-00076 AALTO, Finland

<sup>b</sup> Landau Institute for Theoretical Physics, Russian Academy of Sciences, Moscow, 119334 Russia  
e-mail: tero.heikkila@tkk.fi, kopnin@boojum.hut.fi, volovik@boojum.hut.fi

Received June 22, 2011

Topological media are systems whose properties are protected by topology and thus are robust to deformations of the system. In topological insulators and superconductors the bulk-surface and bulk-vortex correspondence gives rise to the gapless Weyl, Dirac or Majorana fermions on the surface of the system and inside vortex cores. Here we show that in gapless topological media, the bulk-surface and bulk-vortex correspondence is more effective: it produces topologically protected gapless fermions without dispersion—the flat band. Fermion zero modes forming the flat band are localized on the surface of topological media with protected nodal lines [A. P. Schnyder and S. Ryu, Phys. Rev. B **84**, 060504(R) (2011); T. T. Heikkilä and G. E. Volovik, JETP Lett. **93**, 59 (2011)] and in the vortex core in systems with topologically protected Fermi points (Weyl points) [G. E. Volovik, JETP Lett. **93**, 66 (2011)]. Flat band has an extremely singular density of states, and we show that this property may give rise in particular to surface superconductivity which could exist even at room temperature.

DOI: 10.1134/S0021364011150045

- Metallic bulk:  
Finite density of states  
the gate is screened with  
 $l_{TF} \sim (e^2 N(E_F))^{-1}$ .
- Semiconducting bulk:  
The potential induced by the gate  
(valence) band down in the  
region and screen the

- Rhombohedral graphene:  
Vanishing density of states density of states at the Fermi level,  
but zero gap.  
There is a flat band at the surface, even in the absence of a  
gate. The band is at the neutrality point (nodal system,  
drumhead band).



# Superconductivity in untwisted tetralayers

## Signatures of chiral superconductivity in rhombohedral graphene

<https://doi.org/10.1038/s41586-025-09169-7>

Received: 1 September 2024

Accepted: 16 May 2025

Published online: 22 May 2025

Tonghang Han<sup>1,6</sup>, Zhengguang Lu<sup>1,2,6</sup>, Zach Hadjiri<sup>1,6</sup>, Lihan Shi<sup>1</sup>, Zhenghan Wu<sup>1</sup>, Wei Xu<sup>1</sup>, Yuxuan Yao<sup>1</sup>, Armel A. Cotten<sup>3</sup>, Omid Sharifi Sedeih<sup>3</sup>, Henok Weldeyesus<sup>3</sup>, Jixiang Yang<sup>1</sup>, Junseok Seo<sup>1</sup>, Shenyong Ye<sup>1</sup>, Muyang Zhou<sup>1</sup>, Haoyang Liu<sup>2</sup>, Gang Shi<sup>2</sup>, Zhenqi Hua<sup>2</sup>, Kenji Watanabe<sup>4</sup>, Takashi Taniguchi<sup>5</sup>, Peng Xiong<sup>2</sup>, Dominik M. Zumbühl<sup>2</sup>, Liang Fu<sup>1</sup> & Long Ju<sup>1,6</sup>

Nature **643**, 654 (2025)

4 layers

## Superconductivity and quantized anomalous Hall effect in rhombohedral graphene

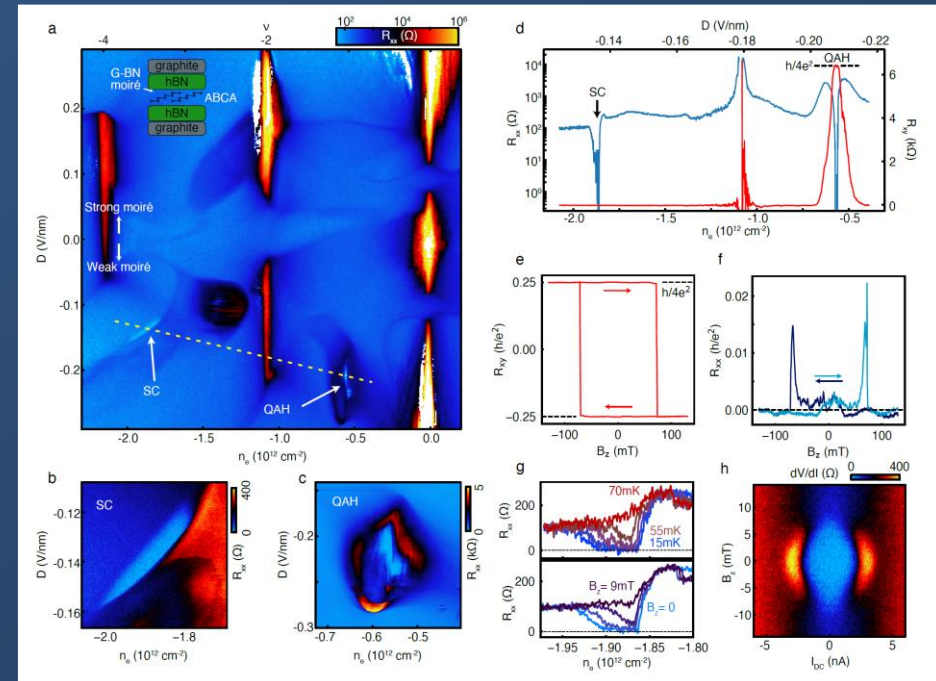
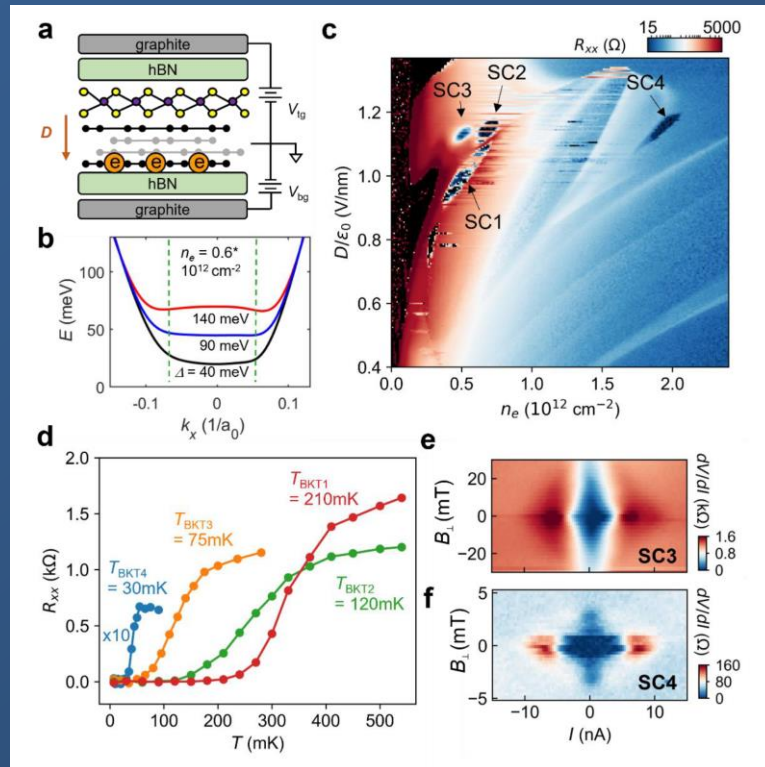
<https://doi.org/10.1038/s41586-025-08621-y>

Received: 22 August 2024

Accepted: 9 January 2025

Youngjoon Choi<sup>1,4</sup>, Ysun Choi<sup>1,4</sup>, Marco Valentini<sup>1,4</sup>, Caitlin L. Patterson<sup>1</sup>, Ludwig F. W. Holleis<sup>1</sup>, Owen I. Sheekey<sup>1</sup>, Hari Stoyanov<sup>1</sup>, Xiang Cheng<sup>1</sup>, Takashi Taniguchi<sup>5</sup>, Kenji Watanabe<sup>5</sup> & Andrea F. Young<sup>1,6</sup>

Nature **639**, 342 (2025)

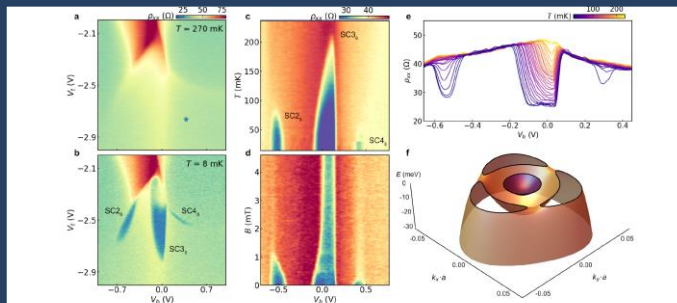


# More experiments

## Superconductivity from dual-surface carriers in rhombohedral graphene

Manish Kumar<sup>1\*</sup>, Derek Waleffe<sup>1\*</sup>, Anna Okounkova<sup>1\*</sup>, Raveel Tejani<sup>2,3</sup>, Võ Tiến Phong<sup>4,5</sup>, Kenji Watanabe<sup>6</sup>, Takashi Taniguchi<sup>7</sup>, Cyprian Lewandowski<sup>4,5</sup>, Joshua Folk<sup>2,3†</sup>, and Matthew Yankowitz<sup>1,8†</sup>

arXiv:2507.18598

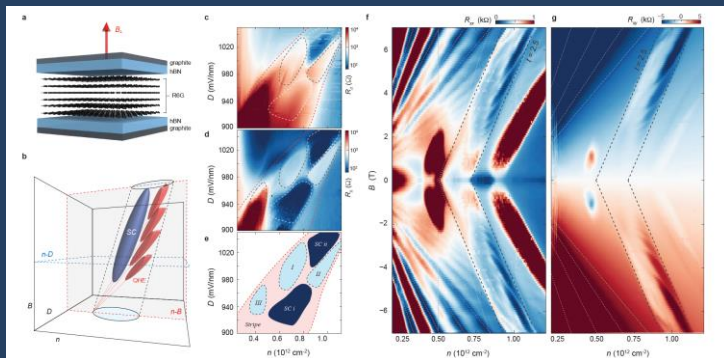


6 layers

## A Hierarchy of Topological and Superconducting States in Rhombohedral Hexalayer Graphene

Ron Q. Nguyen<sup>1,\*</sup>, Hai-Tian Wu<sup>1,\*</sup>, Erin Morissette<sup>1,\*</sup>, Naiyuan J. Zhang<sup>1,\*</sup>, Peiyu Qin<sup>1</sup>, K. Watanabe<sup>2</sup>, T. Taniguchi<sup>3</sup>, Aaron W. Hui<sup>1</sup>, D. E. Feldman<sup>1,5</sup>, and J.I.A. Li<sup>1,4†</sup>

arXiv:2507.22026



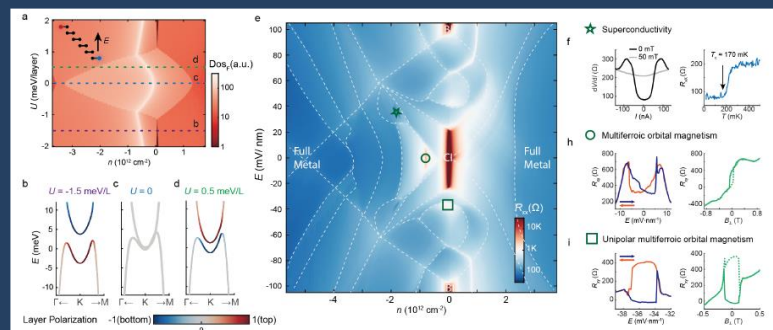
6 layers

## Superconductivity and Ferroelectric Orbital Magnetism in Semimetallic Rhombohedral Hexalayer Graphene

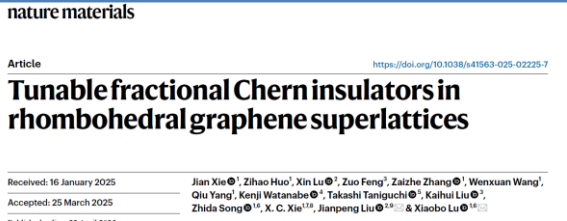
Jinghao Deng<sup>1#</sup>, Jiabin Xie<sup>1#</sup>, Hongyuan Li<sup>1#</sup>, Takashi Taniguchi<sup>2</sup>, Kenji Watanabe<sup>3</sup>, Jie Shan<sup>4,5</sup>, Kin Fai Mak<sup>1,5</sup>, Xiaomeng Liu<sup>1\*</sup>

arXiv:2508.15909

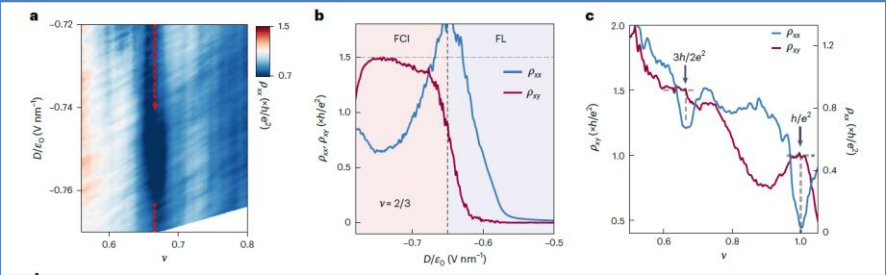
6 layers



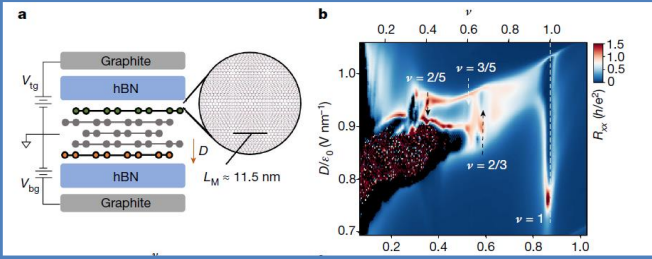
# More experiments



6 layers



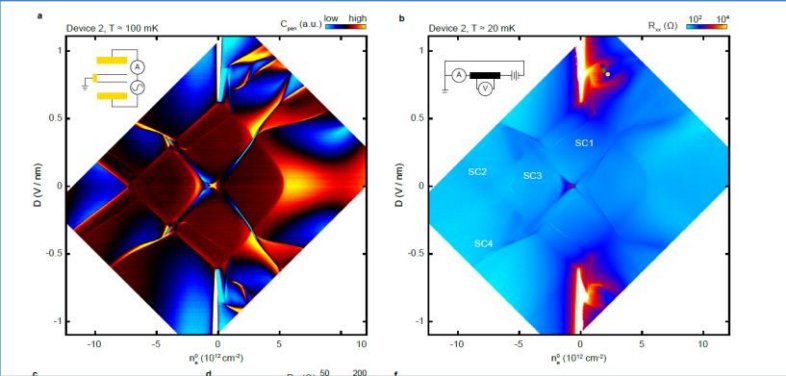
5 layers



## Flat band surface state superconductivity in thick rhombohedral graphene

Yi Guo,<sup>1</sup> Owen I. Sheekey,<sup>1</sup> Trevor Arp,<sup>1</sup> Kryštof Kolář,<sup>2</sup> Thibault Charpentier,<sup>1</sup> Ludwig Holleis,<sup>1</sup> Ben Fouty,<sup>1</sup> Aidan Keough,<sup>1</sup> Maya Kang-Chou,<sup>1</sup> Martin E. Huber,<sup>3</sup> Takashi Taniguchi,<sup>4</sup> Kenji Watanabe,<sup>5</sup> Cyprian Lewandowski,<sup>6,7</sup> and Andrea F. Young<sup>1,\*</sup>

arXiv:2511.17423

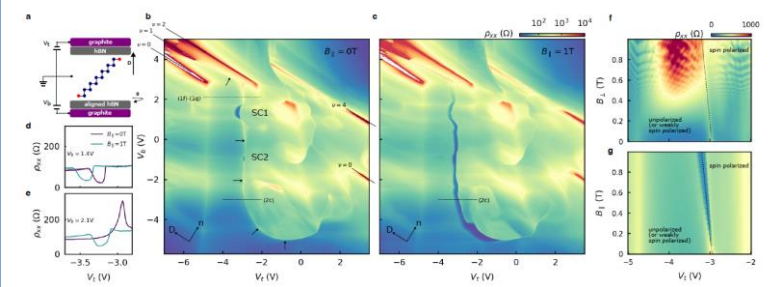


13 layers

## Pervasive spin-triplet superconductivity in rhombohedral graphene

Manish Kumar<sup>1\*</sup>, Derek Waleffe<sup>1\*</sup>, Anna Okounkova<sup>1\*</sup>, Raveel Tejani<sup>2,3</sup>, Kenji Watanabe<sup>6</sup>, Takashi Taniguchi<sup>7</sup>, Étienne Lantagne-Hurtubise<sup>8</sup>, Joshua Folk<sup>2,3†</sup>, and Matthew Yankowitz<sup>1,9†</sup>

arXiv:2511.16578



7 layers

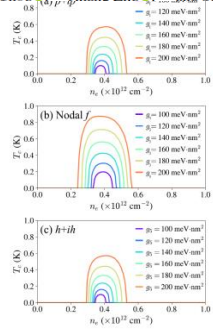


# Some calculations in a fully polarized tetralayer

PHYSICAL REVIEW B **111**, 174523 (2025)

## Intravalley spin-polarized superconductivity in rhombohedral tetralayer graphene

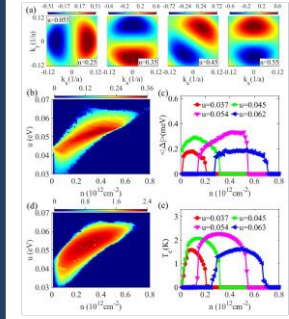
Yang-Zhi Chou<sup>1,\*</sup>, Jihane Zhu<sup>1,\*</sup> and Sankar Das Sarma<sup>1,2</sup>



## Chiral finite-momentum superconductivity in the tetralayer graphene

Qiong Qin<sup>1</sup> and Congjun Wu<sup>1, 2, 3, 4, \*</sup>

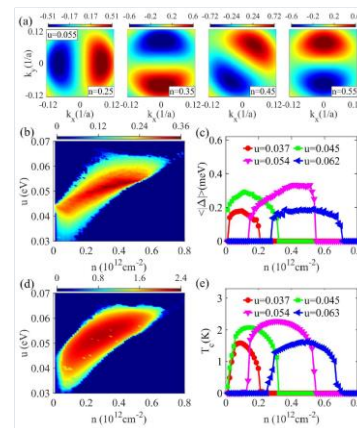
arXiv:2412.07145,  
Chinese Physics Lett., in  
press



## Quarter Metal Superconductivity

Chiho Yoon,<sup>1</sup> Tianyi Xu,<sup>1</sup> Yafis Barlas,<sup>2</sup> and Fan Zhang<sup>1</sup>

Phys. Rev. Lett. **136**, 026603 (2026)

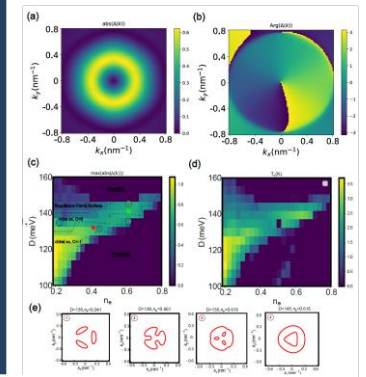
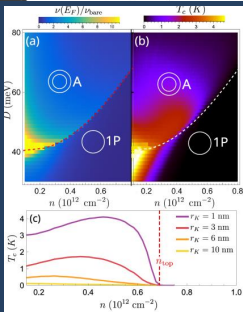


nature communications

## Chiral and topological superconductivity in isospin polarized multilayer graphene

Received: 27 September 2024

Max Geier<sup>1</sup>, Margarita Davydova<sup>1,2</sup> & Liang Fu<sup>1,3</sup>



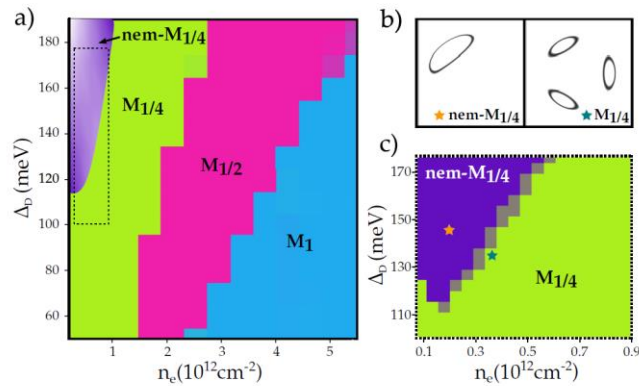
PHYSICAL REVIEW B **112**, L020506 (2025)

## Topological incommensurate Fulde-Ferrell-Larkin-Ovchinnikov superconductor and Bogoliubov Fermi surface in rhombohedral tetralayer graphene

Hui Yang<sup>1</sup> and Ya-Hui Zhang

# Band Renormalization, Quarter Metals, and Chiral Superconductivity in Rhombohedral Tetralayer Graphene

Guillermo Parra-Martínez<sup>1,\*</sup> Alejandro Jimeno-Pozo<sup>1,\*</sup> Võ Tiến Phong,<sup>2,3</sup> Héctor Sainz-Cruz<sup>1</sup> Daniel Kaplan<sup>4</sup>  
 Peleg Emanuel<sup>5</sup> Yuval Oreg<sup>5</sup> Pierre A. Pantaleón<sup>1,†</sup> José Ángel Silva-Guillén<sup>1,‡</sup> and Francisco Guinea<sup>1,6</sup>



**Figure 1. Hartree-Fock phase diagram.** (a) Self-consistently calculated HF phase diagram, showing  $M_{1/4}$ ,  $M_{1/2}$  and  $M_1$  phases as a function of the external displacement field ( $\Delta_D$ ) and electronic density ( $n_e$ ). The purple region at low densities and high displacement fields highlights schematically where the nem- $M_{1/4}$  is expected to emerge. In (c), we show the calculated phase boundary between  $M_{1/4}$  and nem- $M_{1/4}$  in the region indicated by a dashed rectangle in (a). (b) Representative Fermi surfaces at the same point in parameter space indicated by the yellow and blue stars in (c) solutions with and without  $C_3$  symmetry breaking.

- A variety of polarized phases
- Multiple Lifschitz transitions.
- Significant trigonal warping

The HF density of states shows an unphysical dip at the Fermi energy.

The bands are not suited for the calculation of superconductivity.



## Superconductivity. Critical temperature

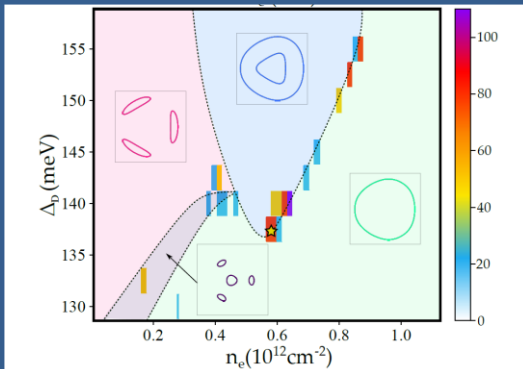


Figure 3. **Superconductivity critical temperature.** Superconducting critical temperature,  $T_c$ , as a function of displacement and electron filling in the  $M_{1/4}$  phase. The colored regions indicate different Fermi surface shapes. Black dashed lines mark Lifshitz transitions, between different shapes of the Fermi surfaces (see section IV SM [47]). A representative Fermi surface is displayed within each region. The yellow star indicates the parameters used for the calculation of the superconducting order parameter (OP) shown in Fig. 4(a-b).

- Calculations carried out using the RPA screened interaction.
- Trigonal warping suppresses the Cooper instability.
- Narrow slice in phase space where superconductivity appears.

## Superconductivity. Order parameter

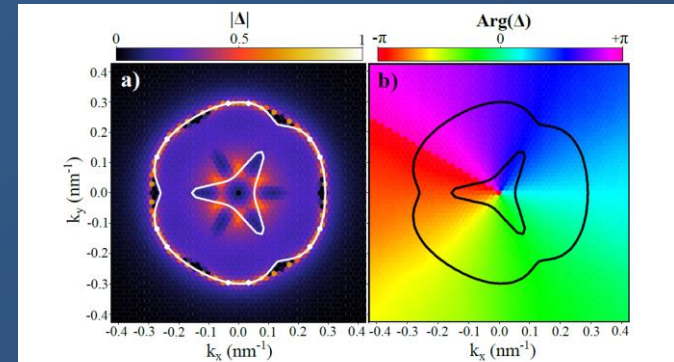


Figure 4. **Superconducting order parameter.** Order parameter of the leading eigenvalue in the screened Hartree-Fock superconductivity calculations. (a) Normalized magnitude of the order parameters and (b) its corresponding phase. The Fermi surface is indicated by solid white and black lines in each panel, respectively. Calculations were made at values  $\Delta_D = 137.5$  meV and  $n_e = 6.01 \times 10^{11} \text{ cm}^{-2}$  with  $T_c = 80$  mK, marked in Fig. 3 as a yellow star.

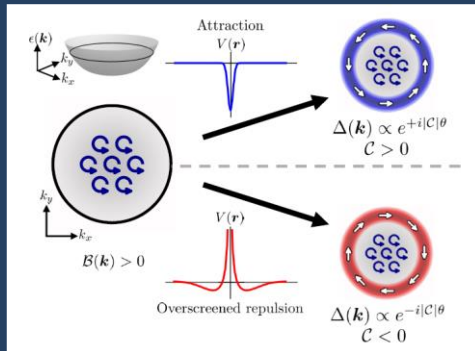
- p-wave superconductivity
- No degeneracy between  $p_+$  and  $p_-$ .
- The gap is not pinned to the Fermi surface
- Gapless superconductivity, due to the parabolic breaking effects of trigonal warping. Similar to s-wave superconductors with magnetic impurities.

# Other works on quarter metal superconductivity

How pairing mechanism dictates topology in valley-polarized superconductors with Berry curvature

Julian May-Mann,<sup>\*</sup> Tobias Helbig, and Trithip Devakul<sup>†</sup>

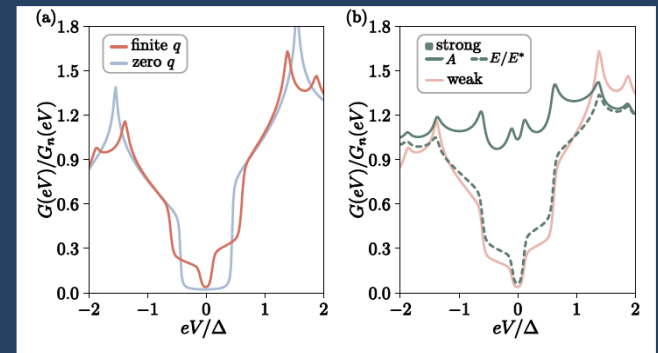
arXiv.2503.05697



Probing superconductivity with tunneling spectroscopy in rhombohedral graphene

Denis Sedov<sup>1</sup> and Mathias S. Scheurer<sup>1</sup>

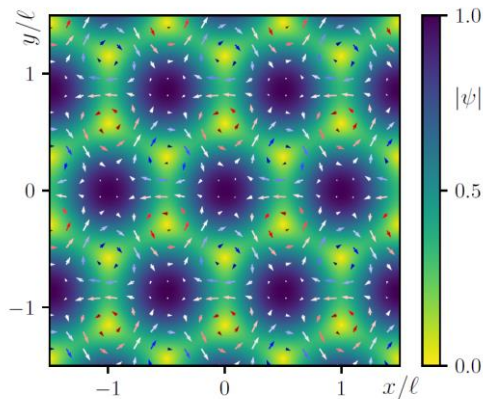
arXiv.2503.12650



PHYSICAL REVIEW LETTERS **135**, 116001 (2025)

Spontaneous Vortex-Antivortex Lattice and Majorana Fermions in Rhombohedral Graphene

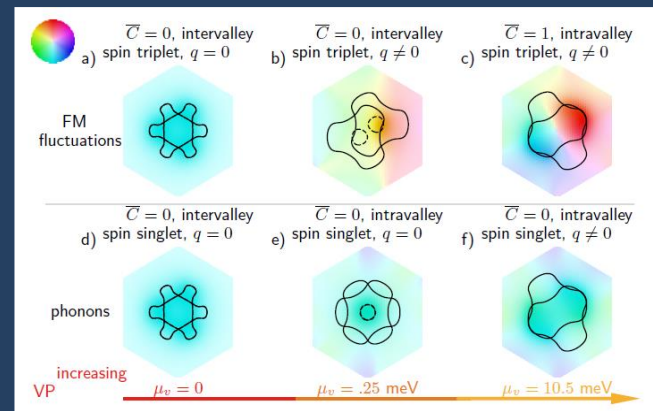
Filippo Gaggioli<sup>✉</sup>, Daniele Guerci<sup>✉</sup>, and Liang Fu<sup>✉</sup>



Finite-momentum pairing and superlattice superconductivity in valley-imbalanced rhombohedral graphene

Maine Christos,<sup>1</sup> Pietro M. Bonetti,<sup>1</sup> and Mathias S. Scheurer<sup>2</sup>

arXiv.2503.15471



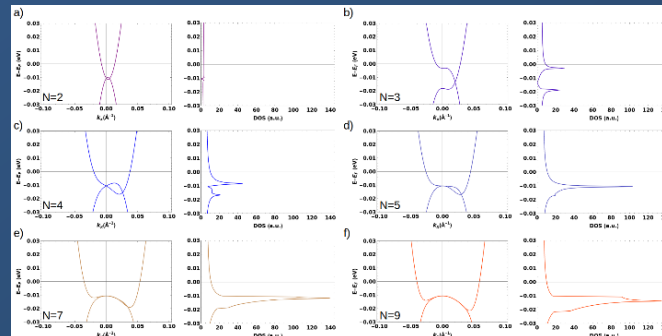
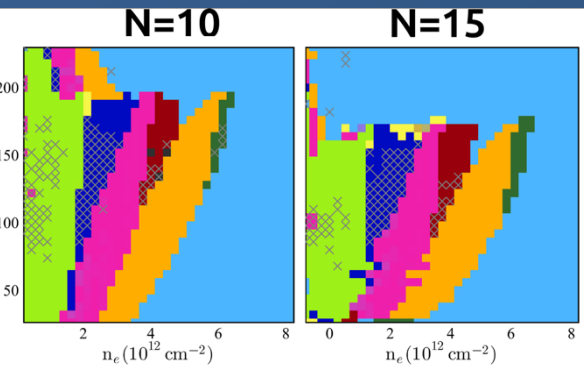
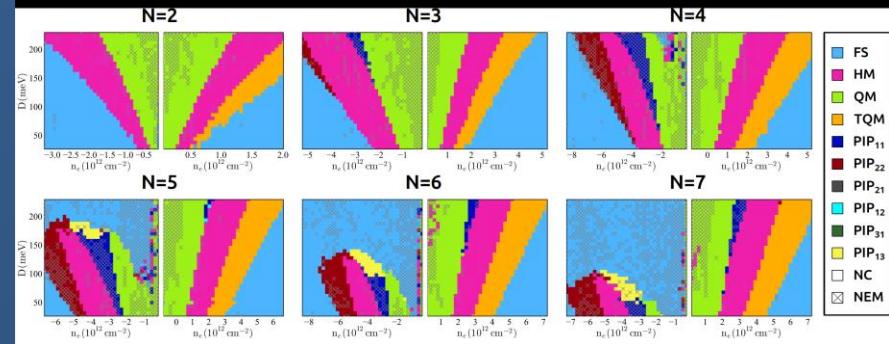
# Phase diagrams of rhombohedral multilayers

## RESEARCH ARTICLE

ADVANCED  
MATERIALS  
www.advmat.de

## Nematic and Partially Polarized Phases in Rhombohedral Graphene with Varying Number of Layers: An Extensive Hartree–Fock Study

Guillermo Parra-Martínez, Alejandro Jimeno-Pozo, Jose Angel Silva-Guillén,\*  
and Francisco Guinea\*



| Name                               | Symbol            | $\beta$                            | Fermi surface | Legend |
|------------------------------------|-------------------|------------------------------------|---------------|--------|
| Isospin polarized states           |                   |                                    |               |        |
| Fully symmetric                    | FS                | $(\alpha, \alpha, \alpha, \alpha)$ |               |        |
| Half metal                         | HM                | $(\alpha, 0, \alpha, 0)$           |               |        |
| Quarter metal                      | QM                | $(\alpha, 0, 0, 0)$                |               |        |
| Three-quarter metal                | TQM               | $(\alpha, \alpha, \alpha, 0)$      |               |        |
| Partially isospin polarized states |                   |                                    |               |        |
| QM to HM                           | PIP <sub>11</sub> | $(\alpha, \beta, 0, 0)$            |               |        |
| HM to FS                           | PIP <sub>22</sub> | $(\alpha, \alpha, \beta, \beta)$   |               |        |
| HM to TQM                          | PIP <sub>21</sub> | $(\alpha, \alpha, \beta, 0)$       |               |        |
| QM to TQM                          | PIP <sub>12</sub> | $(\alpha, \beta, \beta, 0)$        |               |        |
| TQM to FS                          | PIP <sub>31</sub> | $(\alpha, \alpha, \alpha, \beta)$  |               |        |
| QM to FS                           | PIP <sub>13</sub> | $(\alpha, \beta, \beta, \beta)$    |               |        |

- Same succession of unpolarized, half metal, and quarter metal phases in all systems
- Polarized phases stronger on the electron side
- Nematic phases, which break  $C_3$ , also possible
- Polarized phases favored by increased band splittings induced by the exchange term
- Trigonal warping leads to three hot spots at the Fermi surface, leading to p-wave SC

# "Giant" spin-orbit coupling in rhombohedral graphite

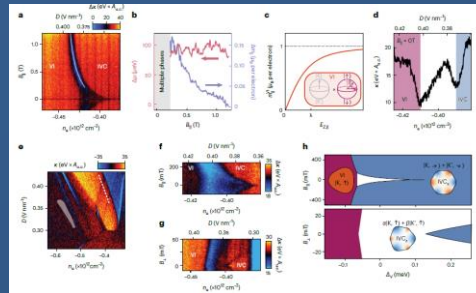
nature physics

Article <https://doi.org/10.1038/s41567-024-02960-7>

## Intervalley coherence and intrinsic spin-orbit coupling in rhombohedral trilayer graphene

Received: 20 November 2023  
Accepted: 22 May 2024  
Published online: 1 July 2024

Trevor Arp<sup>1</sup>, Owen Sheekey<sup>1,2</sup>, Haoxin Zhou<sup>1,2</sup>, C. L. Tschirhart<sup>1,2</sup>, Caitlin L. Patterson<sup>1</sup>, H. M. Yoo<sup>1</sup>, Ludwig Holleis<sup>1</sup>, Evgeny Bedekov<sup>1</sup>, Grigory Babikyan<sup>1</sup>, Tian Xie<sup>1</sup>, Jiewen Xiao<sup>1</sup>, Yasar Viturli<sup>1</sup>, Tobias Holder<sup>1,2</sup>, Takashi Taniguchi<sup>1</sup>, Kenji Watanabe<sup>1</sup>, Martin E. Huber<sup>1</sup>, Erez Berg<sup>1,2</sup> & Andrea F. Young<sup>1,2</sup>✉



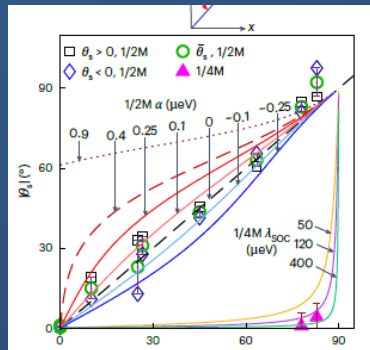
nature physics

Article <https://doi.org/10.1038/s41567-025-03035-z>

## Isospin magnetic texture and intervalley exchange interaction in rhombohedral tetralayer graphene

Received: 5 October 2024  
Accepted: 10 August 2025  
Published online: 1 October 2025

Nadav Auerbach<sup>1,2</sup>, Surajit Dutta<sup>1,2</sup>, Matan Utan<sup>1,2</sup>, Yasar Viturli<sup>1</sup>, Yaosheng Zhou<sup>1</sup>, Alexander T. Meltzer<sup>1</sup>, Sameer Grover<sup>1</sup>, Tobias Holder<sup>1</sup>, Peleg Emmanuel<sup>1</sup>, Martin E. Huber<sup>1</sup>, Yuri Mysoskov<sup>1</sup>, Kenji Watanabe<sup>1</sup>, Takashi Taniguchi<sup>1</sup>, Yuval Oreg<sup>1</sup>, Erez Berg<sup>1,2</sup> & Eli Zeldov<sup>1,2</sup>✉



## New Journal of Physics

The open-access journal for physics

## Spin-orbit coupling in a graphene bilayer and in graphite

F Guinea

Instituto de Ciencia de Materiales de Madrid, CSIC, Sor Juana Inés de la Cruz 3, E28049 Madrid, Spain  
E-mail: [paco.guinea@icmm.csic.es](mailto:paco.guinea@icmm.csic.es)

New Journal of Physics **12** (2010) 083063 (9pp)  
Received 8 March 2010  
Published 31 August 2010  
Online at <http://www.njp.org/>  
doi:10.1088/1367-2630/12/8/083063

## Spin Orbit Interaction in Graphite

G. Dresselhaus and M. S. Dresselhaus  
Physical Review **140**, A401 (1965)

PRL **95**, 226801 (2005)

PHYSICAL REVIEW LETTERS

week ending  
25 NOVEMBER 2005

## Quantum Spin Hall Effect in Graphene

C.L. Kane and E.J. Mele

PHYSICAL REVIEW B **82**, 245412 (2010)

## Tight-binding theory of the spin-orbit coupling in graphene

Sergej Konschuh, Martin Gmitra, and Jaroslav Fabian

PHYSICAL REVIEW B **81**, 241409(R) (2010)



## Spin-orbit coupling and broken spin degeneracy in multilayer graphene

Edward McCann<sup>1</sup> and Mikito Koshino<sup>2</sup>

## Giant Intrinsic Spin-Orbit Coupling in Bilayer Graphene

Hai-Wen Liu<sup>1</sup>, X.C. Xie<sup>1,2</sup>, and Qing-feng Sun<sup>1\*</sup>

arXiv:1004.0881



# Spin-orbit in rhombohedral multilayers

Calculation including all  $p$  and  $s$  orbitals in the valence shell  
Projection onto the  $p_z$  orbitals

In collaboration with A. Stern, G. Parra-Martínez, A. Herando-Pozo, J. Silva

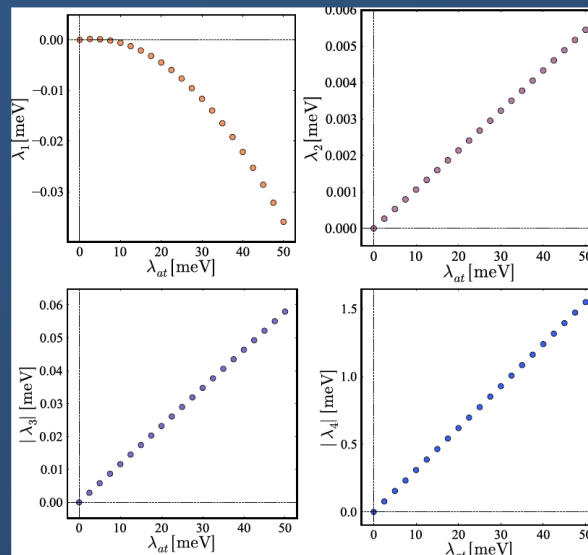
Atomic spin-orbit coupling  $\mathcal{H}_{soc}^{at} = \lambda_{at} \sum_n \vec{L}_n \vec{S}_n$

Symmetry allowed terms

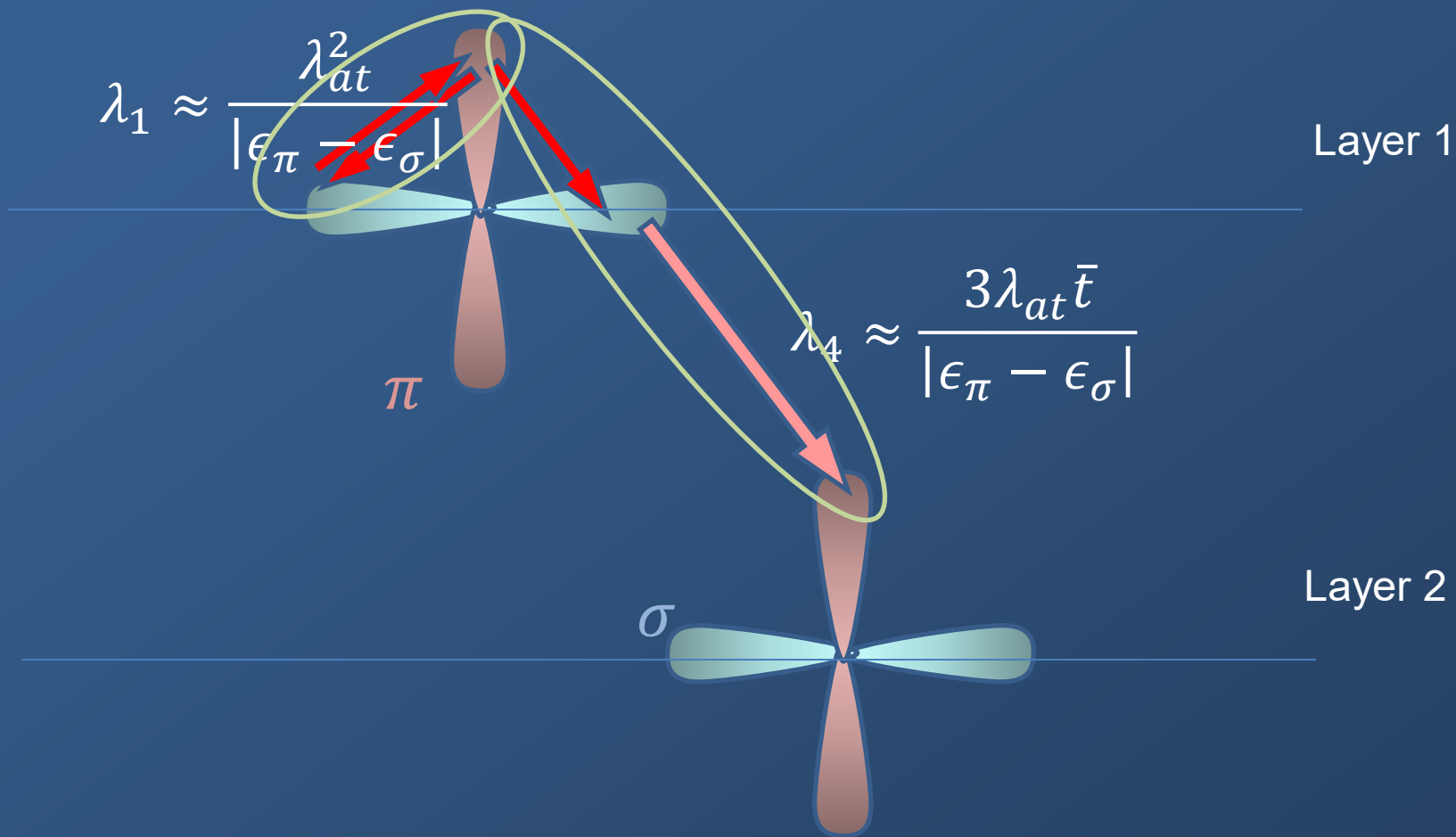
$$\mathcal{H}_{soc} = \sum_{n=1}^{N_L} [\lambda_1 \tau_z^n \sigma_z^n s_z^n + \lambda_2 (-1)^n \tau_z^n s_z^n + \lambda_3 (-1)^n (\sigma_y^n s_x^n - \tau_z^n \sigma_x^n s_y^n)] + \sum_{n=1}^{N_L-1} \lambda_4 \sigma_z^n (\mu_y^{n,n+1} s_x^n - \tau_z^n \mu_x^{n,n+1} s_y^n)$$

Kane Mele term

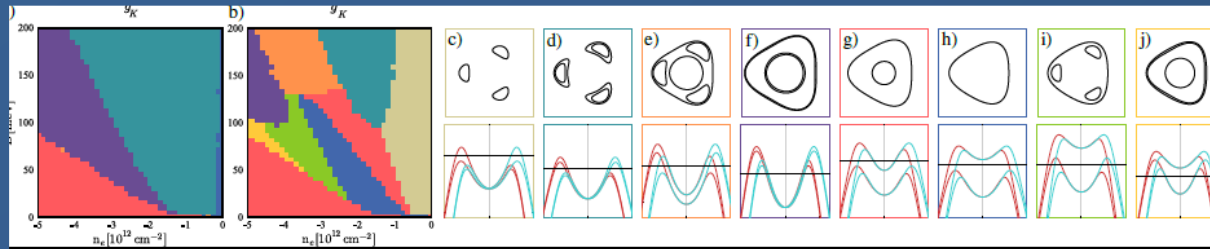
Spin flip+interlayer hopping



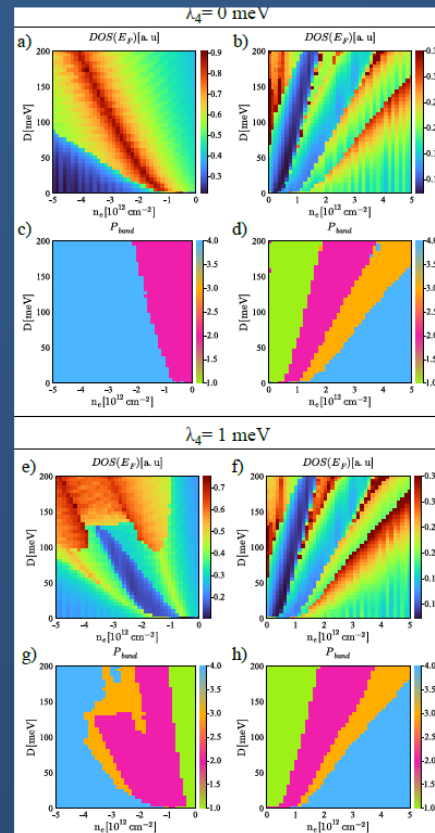
Spin orbit couplings: Kane-Mele, interlayer jump + spin flip



# Fermi Surface topology, R4G



$$\lambda_4 = 0 \quad \lambda_4 = 1 \text{ meV}$$



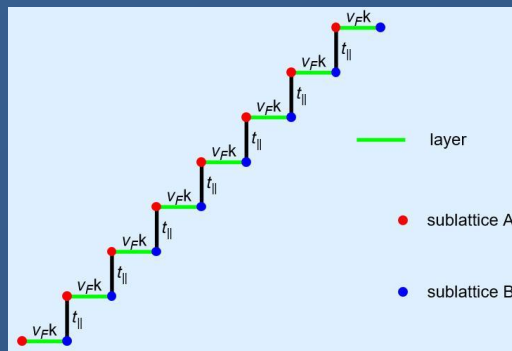
Hartree Fock  
phase  
diagram

# Rhombohedral graphene. A toy model.

In collaboration with A. Bernevig and Y. Sao

Consider only the flat band of a semiinfinite stack (Berry trashcan approximation),

SSH approximation



Tight binding hamiltonian

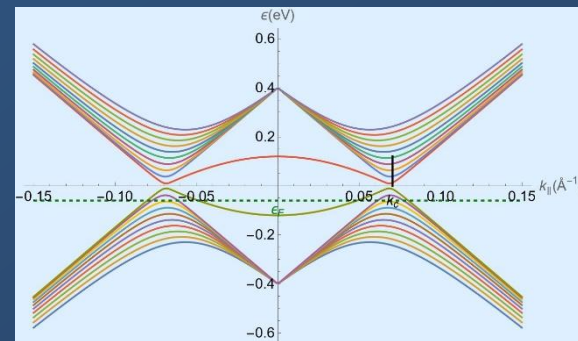
$$|\vec{k}\rangle = N^{-1/2} \sum_n \left( \frac{v_F k e^{i\phi_k}}{t_{\parallel}} \right)^n |n, A\rangle$$

Flat band dispersion

“Berry Trashcan” Model of Interacting Electrons in Rhombohedral Graphene

B. Andrei Bernevig<sup>1,2,3</sup> and Yves H. Kwan<sup>4</sup>

arXiv:2503.09692



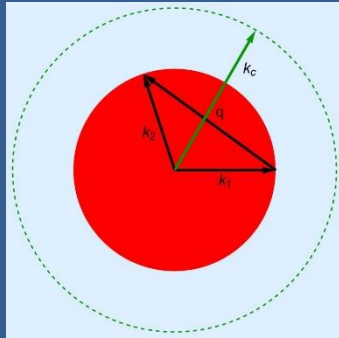
Bands with shift due to displacement field.  
Approximate quadratic dispersion.

$$\epsilon_{\vec{k}} = \frac{\vec{k}^2}{2m^*}$$



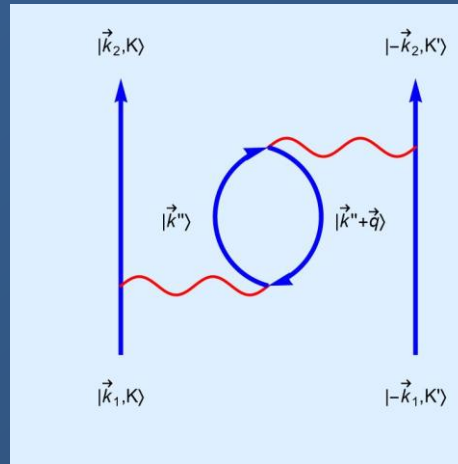
# Pairing due to long range interactions (Kohn-Luttinger)

## Fermi surface



Radius of the flat band:

$$k_c = \frac{v_F}{t_{\parallel}}$$



$$\text{screened potential} \\ v_{scr}(q) = \frac{v_0(q)}{1 + n_f v_0(q) \chi_0(q)} \approx \frac{1}{n_f \chi_0(q)}$$

$$\text{susceptibility} \\ \chi_0(q) = \sum_{\vec{k}''} |\langle \vec{k}'' | \vec{k}'' + \vec{q} \rangle|^2 \frac{n_{\vec{k}'' + \vec{q}} - n_{\vec{k}''}}{\epsilon_{\vec{k}'' + \vec{q}} - \epsilon_{\vec{k}''}}$$

$$\text{Linear equation for the gap} \\ \Delta_l(T_c) \approx -\log\left(\frac{W}{T_c}\right) \lambda_l \Delta_l(T_c)$$

$$\text{Gap at zero temperature} \\ \Delta_l \approx W e^{-1/|\lambda_l|}$$

See also:

RESEARCH ARTICLE | AUGUST 13 2013  
**Superconductivity from repulsive interaction** FREE  
 Saurabh Maiti, Andrey V. Chubukov  
*AIP Conf. Proc.* 1550, 3–73 (2013)  
<https://doi.org/10.1063/1.4818400>

How pairing mechanism dictates topology in valley-polarized superconductors with Berry curvature

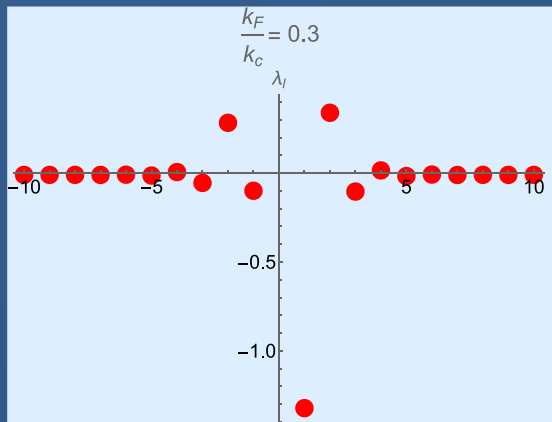
Julian May-Mann,\* Tobias Helbig, and Trithep Devakul†

Dimensionless coupling constant

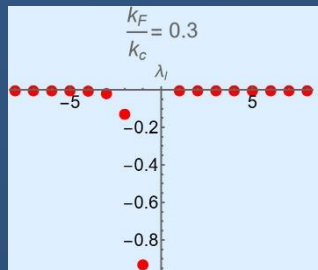
$$\lambda_l = \frac{\rho(\epsilon_F)}{2\pi} \int_0^{2\pi} d\theta e^{il\theta} \times \left\{ \frac{M_{K,K'}(\vec{k}_1, \vec{k}_2)}{M_{K,K}(\vec{k}_1, \vec{k}_2)} \right\} \times v_{scr}(q) \approx \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{il\theta} \times \left\{ \frac{M_{K,K'}(\vec{k}_1, \vec{k}_2)}{M_{K,K}(\vec{k}_1, \vec{k}_2)} \right\} \times \frac{\chi_0(0)}{n_f \chi_0(q)}$$

System dependent parameters,  $e^2$ ,  $m^*$ , drop out, only dimensionless form factors remain

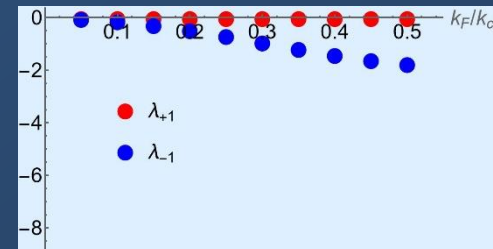
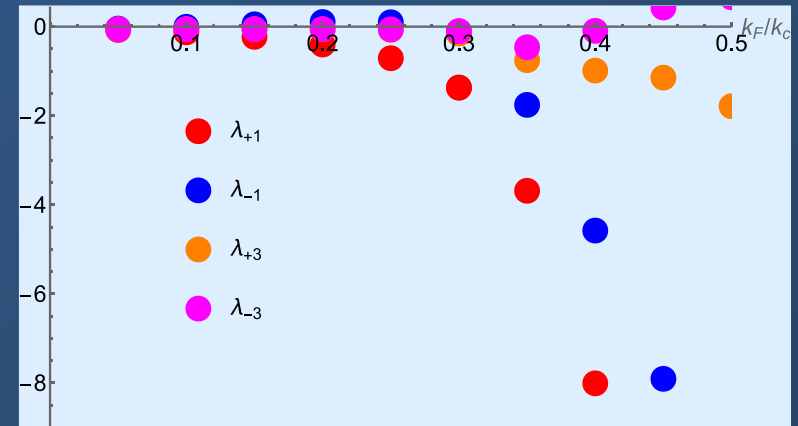
# Quarter metal. Superconducting coupling constant



Screened  
Coulomb  
interaction



Negative  
Hubbard U



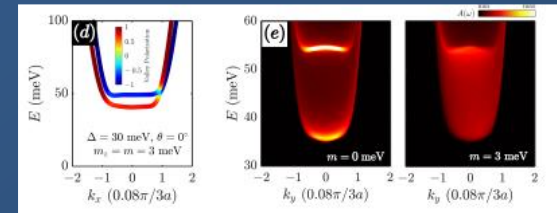
# Domain walls in quarter metals

## Reconfigurable chiral superconductivity

Surajit Dutta<sup>1†</sup>, Nadav Auerbach<sup>1†</sup>, Tonghang Han<sup>2†</sup>, Yaozhang Zhou<sup>1</sup>, Gal Shavit<sup>3</sup>,  
Niladri-Sekhar Kander<sup>1</sup>, Yuri Myasoedov<sup>1</sup>, Martin E. Huber<sup>4</sup>, Kenji Watanabe<sup>5</sup>, Takashi Taniguchi<sup>6</sup>,  
Long Ju<sup>2#</sup>, and Eli Zeldov<sup>1\*</sup>

arXiv:2605.13303

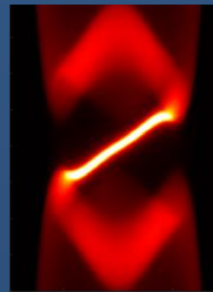
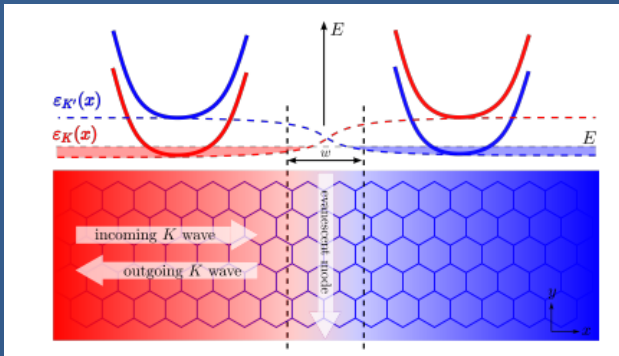
In collaboration with V. T. Phong, E. Mele, E. Prada, P. San-Jose, F. Lobo



Electronic structure of the domain wall in the normal state.

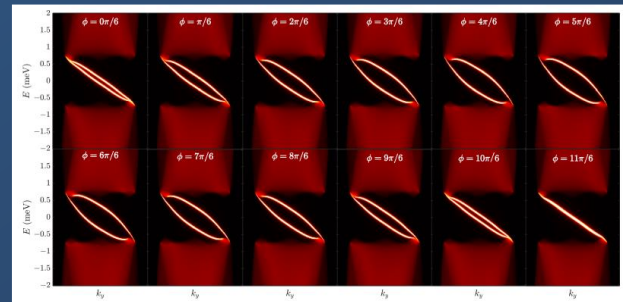
The transmission vanishes unless:

- The barrier has the armchair orientation.
- There is intervalley coherence at the barrier.

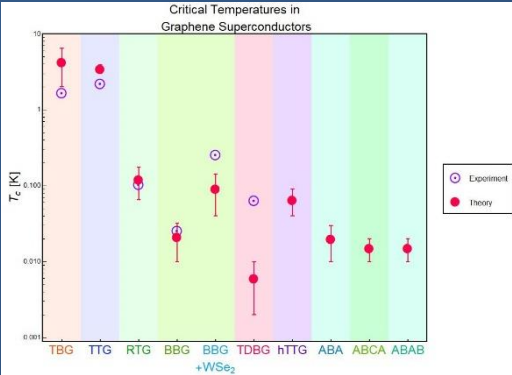


Electronic structure of the domain wall in the superconducting state.

- Topological superconductivity.
- Existence of Majorana states.

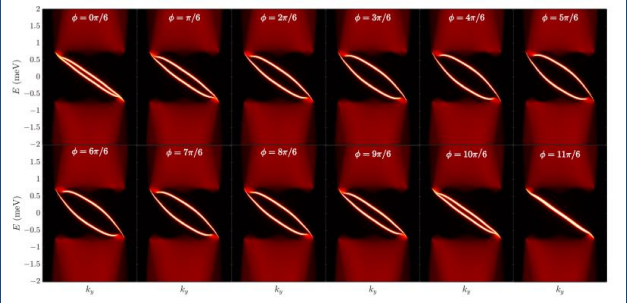
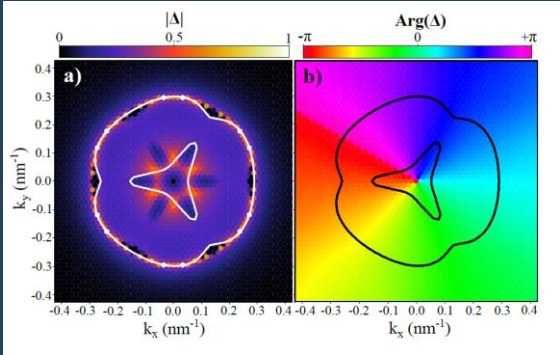


- The long range Coulomb interaction plays a major role in the electronic properties of graphene stacks.
- The main trends of the superconducting properties are reasonably described using standard parameters,



| Electronic bands |                     |                |                |                |               |              |              | Coulomb potential |            | Deformation potential and elastic constants |                              |                          |  |
|------------------|---------------------|----------------|----------------|----------------|---------------|--------------|--------------|-------------------|------------|---------------------------------------------|------------------------------|--------------------------|--|
| $a(\text{\AA})$  | $v_F(eV\text{\AA})$ | $\gamma_1(eV)$ | $\gamma_3(eV)$ | $\gamma_4(eV)$ | $\Delta'(eV)$ | $g_{AA}(eV)$ | $g_{AB}(eV)$ | $d_G(nm)$         | $\epsilon$ | $D(eV)$                                     | $\lambda(eV\text{\AA}^{-2})$ | $\mu(eV\text{\AA}^{-2})$ |  |
| 2.46             | 5.253               | 0.4            | 0.32           | 0.044          | 0.05          | 0.797        | 0.0975       | 40                | 10         | 20                                          | 3.25                         | 9.44                     |  |

Rhombohedral graphene stacks show a rich electronic behavior all the way to the 3D limit







Héctor Sainz-Cruz (Imdea)



Tien Phong (Penn, FSU)



S. R. Walet (Manchester)



Guillermo Parra  
 Federico Escudero (UNS, Argentina)  
 Min Long (Wuhan U)  
 Alejandro Jimeno-Pozo  
 Zhen Zhan  
 Saul Herrera (UNAM, Mexico)  
 Adrian Ceferino  
 Pierre Pantaleon  
 Jose Silva  
 Hector Sainz-Cruz

Also:

L. Levitov (MIT)  
 L. Fu (MIT)  
 V. Crépel (MIT)  
 S. Yuan (Wuhan U.)  
 J. Lischner, (Imperial C.)  
 Z. Goodwin, (Imperial C.)  
 H. Rostami (Bath U.),  
 E. Cappelluti (Trieste)  
 D. Loo (NUS)  
 M. M. E. Alezzi (NUS)  
 S. Adam (NUS)  
 G. Vignale (NUS)  
 A. Stern (Weizmann)  
 R. Makov (Weizmann)  
 Y. Oreg (Weizmann)  
 P. Emanuel (Weizmann)  
 L. Glazman (Yale)  
 F. Von Oppen (Berlin)  
 E. Mele (U. Penn)  
 A. Bernevig (Princeton and DIPC)  
 Nemin Wei (Yale)





- Electronic charge fluctuations describe a pairing mechanism which is not suppressed by repulsive interactions. It is described by few adjustable parameters.
- The role of charge fluctuations is enhanced in systems with complex electronic wavefunctions, like twisted bilayer graphene. The strength of Umklapp processes correlates with the critical temperature.
- f-wave symmetry is a possible order parameter in twisted bilayer graphene.
- Superconductivity in a fully polarized is influenced by trigonal warping, and can be gapless.

# Superconducting stiffness: recent results

## Superfluid stiffness of magic-angle twisted bilayer graphene

<https://doi.org/10.1038/s41586-024-08494-7>

Received: 17 June 2024

Accepted: 5 December 2024

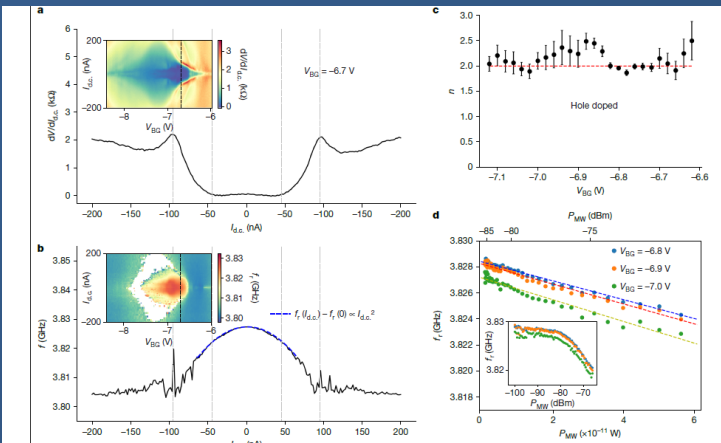
Published online: 5 February 2025

Check for updates

Miuko Tanaka<sup>1,2,3</sup>, Joel Li, Wang<sup>1,2,3</sup>, Thao H. Dinh<sup>2</sup>, Daniel Rodan-Legrain<sup>1</sup>, Samia Zaman<sup>1,4</sup>, Max Hays<sup>1</sup>, Aziza Alamanakly<sup>1,4</sup>, Bharath Kannan<sup>1,4</sup>, David K. Kim<sup>2</sup>, Bethany M. Niedzielski<sup>1</sup>, Kyle Serniak<sup>1,5</sup>, Mollie E. Schwartz<sup>2</sup>, Kenji Watanabe<sup>6</sup>, Takashi Taniguchi<sup>7</sup>, Terry P. Orlando<sup>1,4</sup>, Simon Gustavsson<sup>1,8</sup>, Jeffrey A. Grover<sup>1</sup>, Pablo Jarillo-Herrero<sup>1,2,3</sup> & William D. Oliver<sup>1,3,4,2</sup>

The physics of superconductivity in magic-angle twisted bilayer graphene

Nature **638**, 99 (2025)



## Article

## Superfluid stiffness of twisted trilayer graphene superconductors

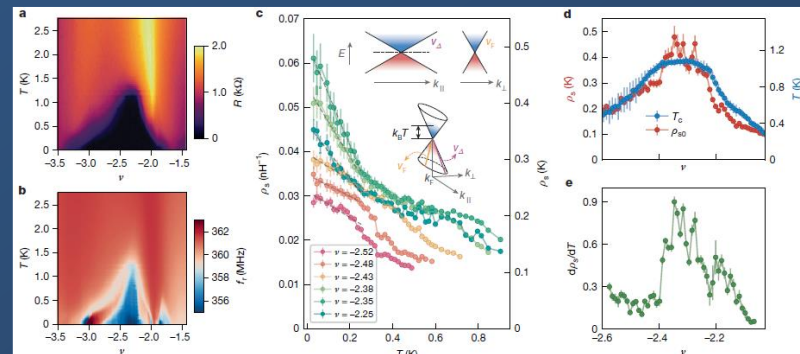
<https://doi.org/10.1038/s41586-024-08444-3>

Received: 19 June 2024

Accepted: 22 November 2024

Abhishek Banerjee<sup>1,2</sup>, Zeyu Hao<sup>1,2</sup>, Mary Kreidel<sup>1,2</sup>, Patrick Ledwith<sup>1</sup>, Isabelle Phinney<sup>1</sup>, Jeong Min Park<sup>1</sup>, Andrew Zimmerman<sup>1</sup>, Marie E. Wesson<sup>1</sup>, Kenji Watanabe<sup>3</sup>, Takashi Taniguchi<sup>3</sup>, Robert M. Westervelt<sup>1</sup>, Amir Yacoby<sup>1</sup>, Pablo Jarillo-Herrero<sup>1</sup>, Pavel A. Volkov<sup>1,4</sup>, Ashvin Vishwanath<sup>1</sup>, Kin Chung Fong<sup>1,2,3</sup> & Philip Kim<sup>1,2</sup>

Nature **638**, 93 (2025)



Superconducting stiffness: power law dependence on temperature

# Pair breaking due to interlayer scattering

- Pair breaking due to intervalley scattering in a nodeless f-wave superconductor is equivalent to scattering by magnetic impurities in a conventional s-wave superconductor.

PHYSICAL REVIEW

VOLUME 137, NUMBER 4A

15 FEBRUARY 1965

## Theory of the Thermal Conductivity of Superconducting Alloys with Paramagnetic Impurities

VINAY AMBEGAOKAR\* AND ALLAN GRIFFIN†

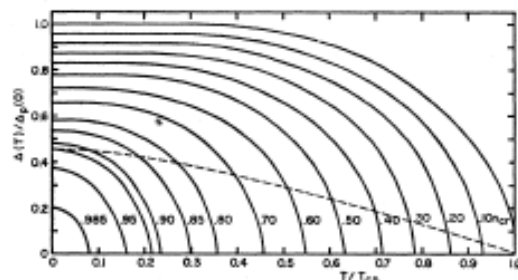


FIG. 4. The Gor'kov order parameter  $\Delta(T, \tau_S)$  for a paramagnetic alloy (relative to its value in a pure metal at  $T=0^\circ\text{K}$ ) as a function of the temperature (relative to the transition temperature of a pure metal). The dashed curve is defined by  $\Delta(T, \tau_S) \tau_S = 1$ ,  $\tau_S^{-1}$  being proportional to the impurity concentration  $n_i$  [see (2.26)]. For a given value of  $n_i$ , the gapless region occurs for temperatures  $T \leq T < T_c$ , where  $T_c$  is the transition temperature for that concentration and  $T_c$  is the temperature at which the dashed curve crosses  $\Delta(T, n_i)$ . The minimum value of  $n_i$  at which gaplessness occurs down to  $0^\circ\text{K}$  is  $\exp(-\pi/4) n_{cr} \approx 0.91 n_{cr}$ , assuming the relation  $\Delta_p(0) = 1.767 \tau_S$ . The curve for this concentration is not labeled.

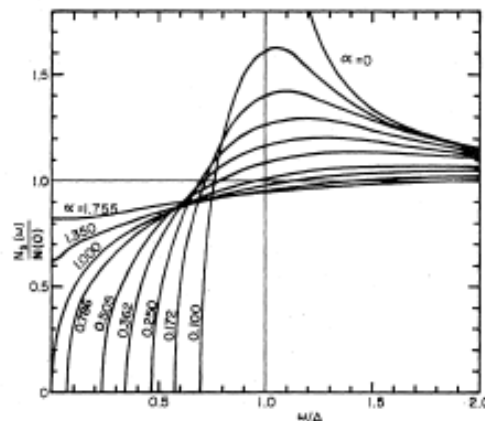


FIG. 6. The density of states  $N_s(\omega)$  in a paramagnetic alloy function of the excitation frequency  $\omega$  (we have normalized the former with respect to a normal metal and the latter with respect to the Gor'kov order parameter  $\Delta$ ). The density of states for nonmagnetic impurities is also shown [ $\alpha = (\tau_S \Delta)^{-1} = 0$ ].

- The relevant energy scales are the superconducting gap,  $\Delta$ , and the scattering time due to intervalley processes,  $\tau_{iv}$ .
- The superconducting phase is always gapless near the critical temperature.
- The superconductor is gapless at all temperatures if  $\Delta(T = 0) \leq \hbar / \tau_{iv}$ .

Estimates:  $\Delta(T = 0) \sim 0.5 \text{ meV}$ ,  $v_F \sim v_F^G / 50$ ,  $\ell_{iv} \sim 50 - 500 \text{ nm}$ ,  $\hbar / \tau_{iv} \sim 0.02 - 0.2 \text{ meV}$

# Dissipation in TBG Josephson junctions

nature communications



Article

<https://doi.org/10.1038/s41467-025-58325-0>

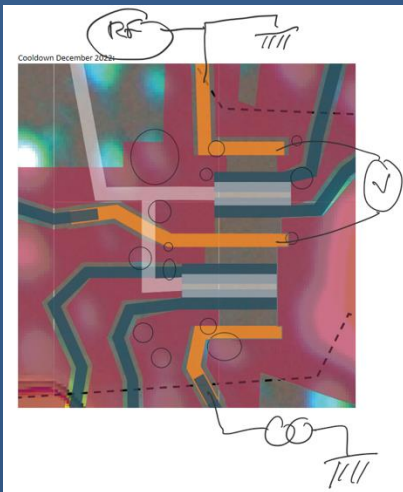
## Quasiparticle and superfluid dynamics in Magic-Angle Graphene

Received: 15 October 2024

Accepted: 19 March 2025

Published online: 08 May 2025

Elias Portolés<sup>1</sup>, Marta Perego<sup>1</sup>, Pavel A. Volkov<sup>2,3</sup>, Mathilde Toschini<sup>1</sup>, Yana Kemna<sup>1</sup>, Alexandra Mestre-Torà<sup>1</sup>, Giulia Zheng<sup>1</sup>, Artem O. Denisov<sup>1</sup>, Folkert K. de Vries<sup>1</sup>, Peter Rickhaus<sup>1</sup>, Takashi Taniguchi<sup>4</sup>, Kenji Watanabe<sup>5</sup>, J. H. Pixley<sup>6,7</sup>, Thomas Ihn<sup>1,8</sup> & Klaus Ensslin<sup>1,8</sup>



$$I_c \approx 10 \text{ nA}$$

$$C = 32 \text{ fF}$$

$$R = 2.5 \text{ k}\Omega$$

$$\omega_p/2\pi = 4.9 \text{ GHz}$$

$$T \approx 10 \text{ mK}$$

$$E_J \approx 0.021 \text{ meV}$$

$$E_C \approx 0.0025 \text{ meV}$$

$$\hbar\omega_p \approx 0.02 \text{ meV}$$

$$Q \approx 2.5$$

Quantum effects

$$H = E_C \hat{Q}^2 + E_J \frac{\hat{\phi}^2}{2}$$

$$[\hat{Q}, \hat{\phi}] = 2i$$

$$\hat{Q} = 2i\partial_\phi$$

$$\psi(\phi) = \frac{2}{\sqrt{\pi}} \left( \frac{2E_C}{E_J} \right)^{1/4} e^{-\frac{1}{4} \sqrt{\frac{E_J}{2E_C}} \phi^2}$$

$$\langle \phi^2 \rangle = \sqrt{\frac{8E_C}{E_J}}$$

$$\langle \phi^2 \rangle \approx 0.99$$



# “Berry Trashcan” Model of Interacting Electrons in Rhombohedral Graphene

B. Andrei Bernevig<sup>1,2,3</sup> and Yves H. Kwan<sup>4</sup>

arXiv:2503.09692

