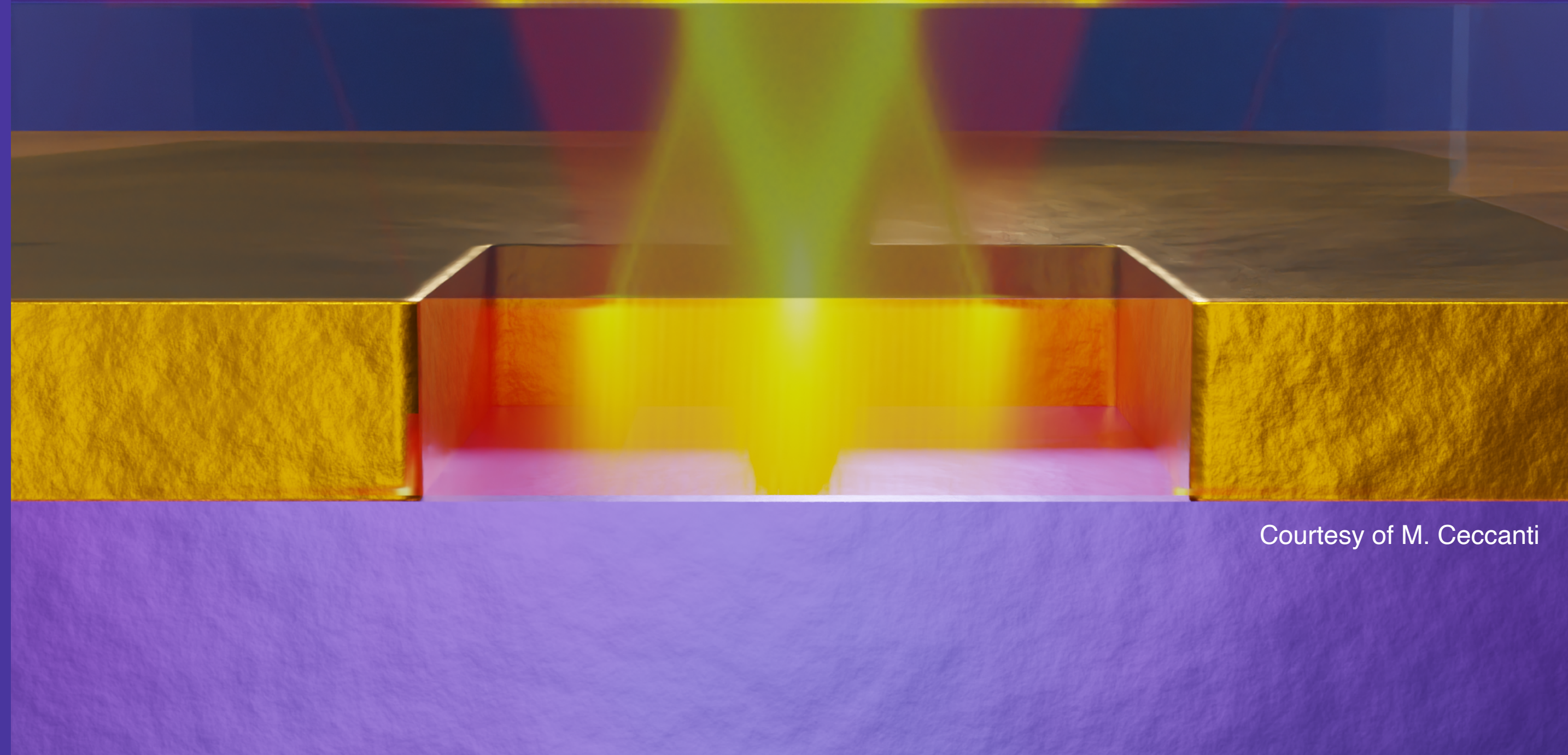


Polariton-induced superconductivity in 2D metals



Courtesy of M. Ceccanti

Iacopo Torre



Gian Marcello Andolina



Lorenzo Cavicchi



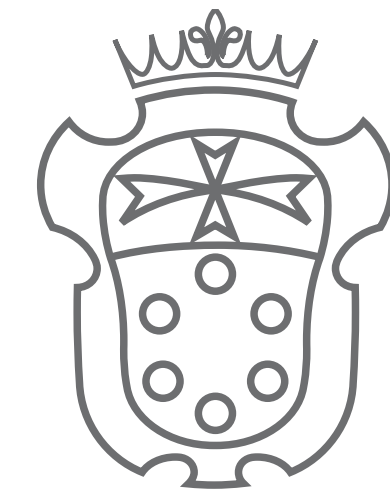
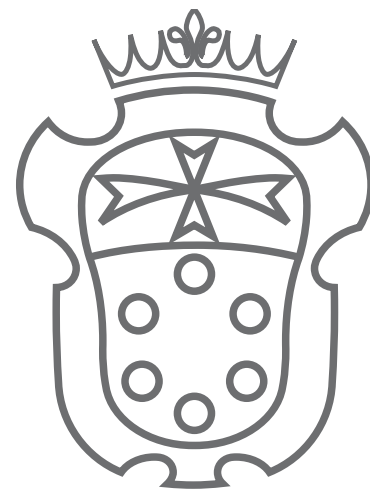
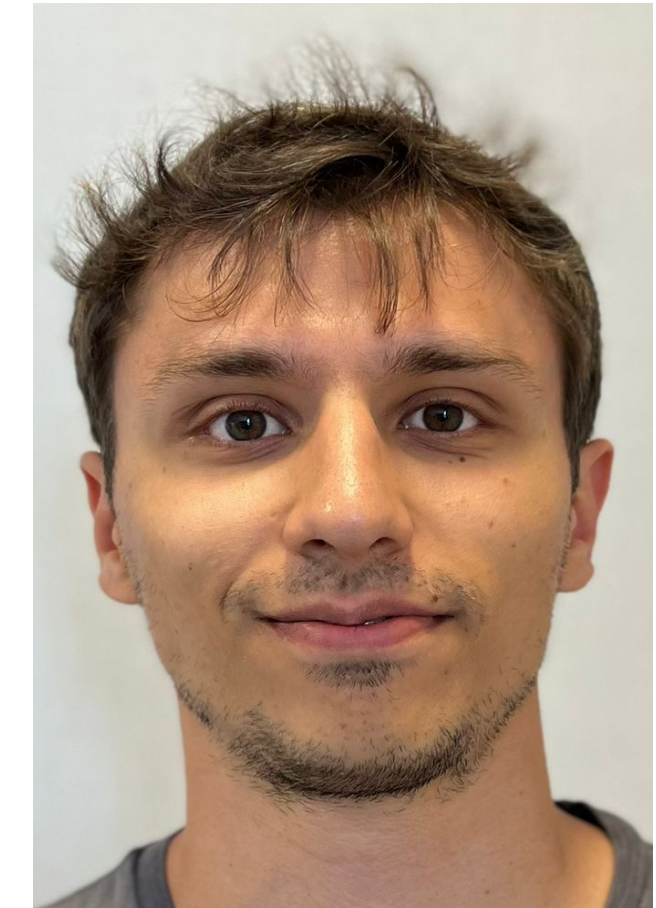
Riccardo Riolo



Giovanni Citeroni



Francesco Cioni



Moving to SISSA Trieste Moving to UT Austin

With Frank Koppens, Pablo Jarillo-Herrero, Giacomo Mazza, and Allan MacDonald
(many thanks to Andrey Chubukov for many interesting discussions)

Outline

cQED of quantum materials: A super-brief introduction

Our take on cQED of quantum materials: dyadic Green's function approach

Theory of effective electron-electron interactions (EEEs)

Polaritons as key enablers of 2D plasmon-mediated superconductivity

1) **R. Riolo**, A. Tomadin, G. Mazza, R. Asgari, A. H. MacDonald, and M. P., Proc. Natl. Acad. Sci. (USA) **122**, e2407995122 (2025)

2) **R. Riolo**, F. H. L. Koppens, P. Jarillo-Herrero, G. Mazza, A. H. MacDonald, and M. P., arXiv:2511.00608

cQED of quantum materials

Can the interaction between electrons and photons yield something interesting from the point of view of many-body physics?

The answer is of course yes.

Textbook example: interaction between electronic currents via the exchange of transverse photons. Unscreened (i.e. long-ranged) even in the case of metals! Despite being ever present, the coupling constant is the square of the ratio between Fermi velocity and speed of light. The resulting non-FL states, first discovered by Holstein et al. (1973) and prominently studied by Reizer (1989), manifest at prohibitively low temperatures and in ultra-pure metals

The holy grail of cQED is to make things happen in less extreme regimes. Honest transverse photons won't work. There is only one trick I know: squeeze light in a sub-wavelength fashion by using polaritons!

P. Forn-Díaz et al., Rev. Mod. Phys. 91, 025005 (2019)

F. Schlawin, D. M. Kennes, and M. A. Sentef, Appl. Phys. Rev. 9, 011312 (2022)

J. Bloch et al., Nature 606, 41 (2022)

F. J Garcia-Vidal, C. Ciuti, and T. W. Ebbesen, Science 373, 6551 (2021)

T. Low et al., Nature Mater. 16, 182 (2017)

D. N. Basov et al., Nanophoton. 10, 549 (2021)

Quantum Field Theory
in Condensed Matter
Physics

Second edition

CAMBRIDGE

Experimental results

Magneto-transport controlled by Landau polariton states

Gian L. Paravicini-Bagliani^{1*}, Felice Appugliese¹, Eli Richter¹, Federico Valmorra¹, Janine Keller¹, Mattias Beck¹, Nicola Bartolo², Clemens Rössler³, Thomas Ihn³, Klaus Ensslin³, Cristiano Ciuti², Giacomo Scalari^{1*} and Jérôme Faist¹

Hybrid excitations, called polaritons, emerge in systems with strong light-matter coupling. Usually, they dominate the linear and nonlinear optical properties with applications in quantum optics. Here, we show the crucial role of the electronic component of polaritons in the magneto-transport of a cavity-embedded two-dimensional electron gas in the ultrastrong coupling regime. We show that the linear direct-current resistivity is substantially modified by the coupling to the cavity even without external irradiation. Our observations confirm recent predictions of vacuum-induced modification of the resistivity. Furthermore, photo-assisted transport in the presence of a weak irradiation field at sub-terahertz frequencies highlights the different roles of localized and delocalized states.

Nat. Phys. **15**, 186 (2019)

QUANTUM OPTICS

Breakdown of topological protection by cavity vacuum fields in the integer quantum Hall effect

Felice Appugliese^{1*}, Josefine Enkner¹, Gian Lorenzo Paravicini-Bagliani^{1†}, Mattias Beck¹, Christian Reichl², Werner Wegscheider², Giacomo Scalari¹, Cristiano Ciuti³, Jérôme Faist^{1*}

Science **375**, 6584 (2022)

Article

Tunable vacuum-field control of fractional and integer quantum Hall phases

<https://doi.org/10.1038/s41586-025-08894-3>

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Nature **641**, 884 (2025)

Article

Cavity-altered superconductivity

<https://doi.org/10.1038/s41586-025-10062-6>

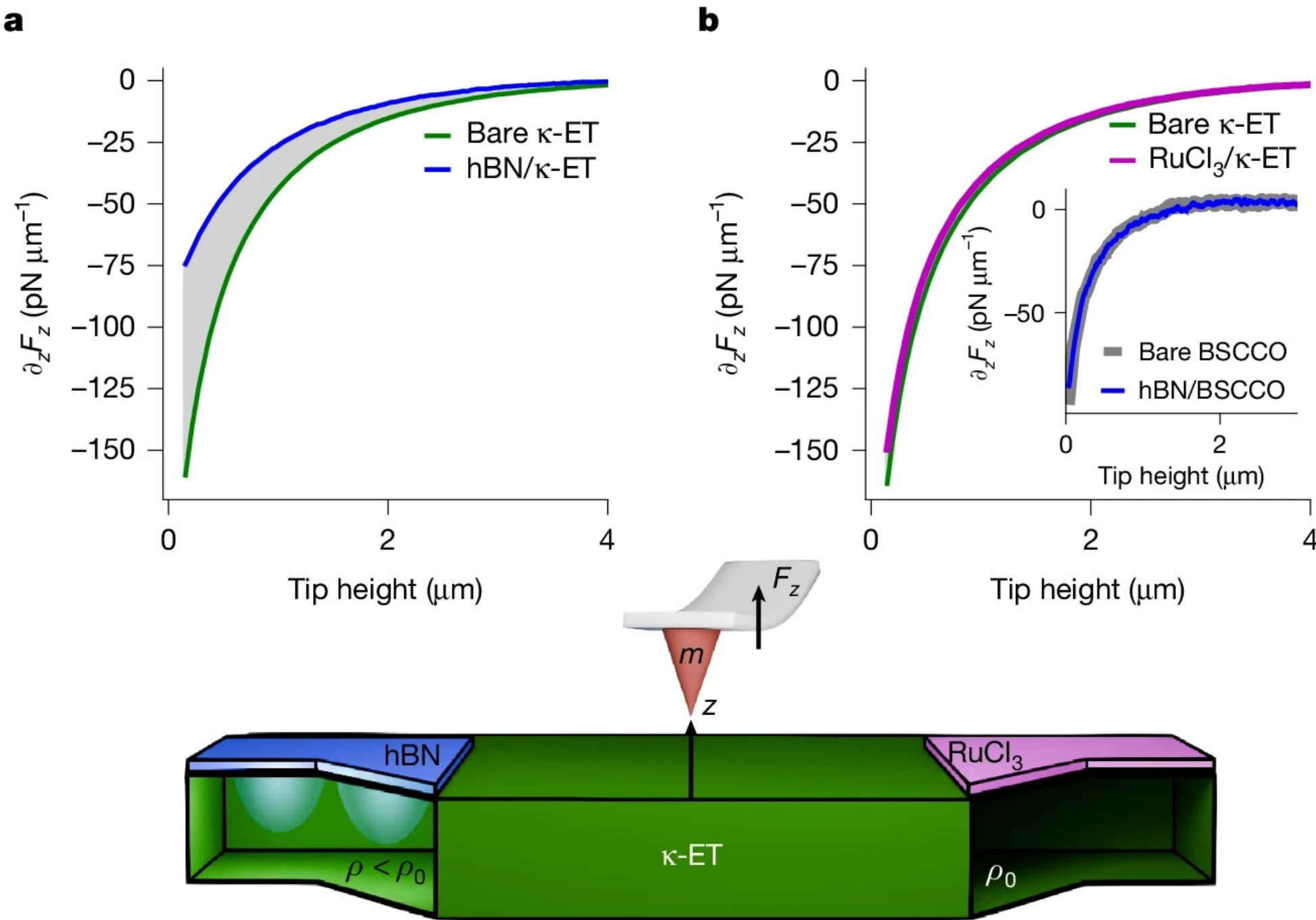
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Open access

Itai Keren^{1,18}, Tatiana A. Webb^{1,18}, Shuai Zhang^{1,18}, Jikai Xu¹, Dihao Sun¹, Brian S. Y. Kim¹, Dongbin Shin^{2,3}, Songtian S. Zhang¹, Junhe Zhang¹, Giancarlo Pereira¹, Juntao Yao^{4,5}, Takuya Okugawa^{1,2}, Marios H. Michael², Emil Viñas Boström², James H. Edgar⁶, Stuart Wolf⁷, Matthew Julian⁷, Rohit P. Prasankumar⁷, Kazuya Miyagawa⁸, Kazushi Kanoda^{8,9,10}, Genda Gu⁴, Matthew Cothrine¹¹, David Mandrus¹¹, Michele Buzzi², Andrea Cavalleri^{2,12}, Cory R. Dean¹, Dante M. Kennes^{2,13}, Andrew J. Millis^{1,14}, Qiang Li^{4,15}, Michael A. Sentef^{2,16}, Angel Rubio^{2,17}, Abhay N. Pasupathy^{1,4} & D. N. Basov¹



Nature **650**, 864 (2026)

Tricky theoretical issues

Simple commandments to be respected when coupling light to matter degrees of freedom:

- 1) Honour gauge invariance
- 2) Thou shalt not use light-matter coupling as an arbitrary parameter to get what you want

Different types of cavities lead to different physics. Macroscopic difference between diffraction limited cavities (such as Fabry-Pérot cavities) and sub-wavelength cavities (such as polaritonic cavities)

$$g = \frac{g_{\text{true}}}{A} \gg g_{\text{true}}$$

$$A = \text{mode volume confinement} \equiv \left(\frac{\lambda_p}{\lambda_d} \right)^3 \ll 1$$

The simplest theory you can cook: Dyadic Green's function approach

Maxwell equations in a local medium

Maxwell equations in a material:

$$\partial_\alpha D_\alpha(\mathbf{r}, \omega) = 4\pi \rho^{\text{ext}}(\mathbf{r}, \omega)$$

$$\partial_\alpha B_\alpha(\mathbf{r}, \omega) = 0$$

$$\varepsilon_{\alpha\beta\gamma} \partial_\beta E_\gamma(\mathbf{r}, \omega) = \frac{i\omega}{c} B_\alpha(\mathbf{r}, \omega)$$

$$\varepsilon_{\alpha\beta\gamma} \partial_\beta H_\gamma(\mathbf{r}, \omega) = \frac{4\pi}{c} J_\alpha^{\text{ext}}(\mathbf{r}, \omega) - \frac{i\omega}{c} D_\alpha(\mathbf{r}, \omega)$$

In order to keep the math simple, we assume spatial locality (generalization needed for chiral cavities):

$$D_\alpha(\mathbf{r}, \omega) = \epsilon_{\alpha\beta}(\mathbf{r}, \omega) E_\beta(\mathbf{r}, \omega)$$

$$B_\alpha(\mathbf{r}, \omega) = \mu_{\alpha\beta}(\mathbf{r}, \omega) H_\beta(\mathbf{r}, \omega)$$

Dyadic Green's function

The (causal) Green's function associated with the Helmholtz equation for the electric field is known as “dyadic Green's function” and obeys the following equation:

$$\mathcal{D}'_{\beta\gamma}(^T\mu^{-1})G_{\alpha\gamma}(\mathbf{r}, \mathbf{r}', \omega) - \frac{\omega^2}{c^2}{}^T\epsilon_{\beta\gamma}(\mathbf{r}', \omega)G_{\alpha\gamma}(\mathbf{r}, \mathbf{r}', \omega) = 4\pi\delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}')$$

The dyadic Green's function directly connects the total electric field with the external source and is therefore gauge invariant:

$$E_{\alpha}(\mathbf{r}, \omega) = E_{\alpha}^{(0)}(\mathbf{r}, \omega) + \frac{i\omega}{c^2} \int d\mathbf{r}' G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega) J_{\beta}^{\text{ext}}(\mathbf{r}', \omega)$$

Gauge freedom

We now introduce scalar and vector potentials:

$$B_\alpha(\mathbf{r}, \omega) = \varepsilon_{\alpha\beta\gamma} \partial_\beta A_\gamma(\mathbf{r}, \omega)$$

$$E_\alpha(\mathbf{r}, \omega) = -\partial_\alpha \phi(\mathbf{r}, \omega) + \frac{i\omega}{c} A_\alpha(\mathbf{r}, \omega)$$

The first ME and the equation for the magnetic field become:

$$\partial_\alpha \left(\epsilon_{\alpha\beta}(\mathbf{r}, \omega) \left[-\partial_\beta \phi(\mathbf{r}, \omega) + \frac{i\omega}{c} A_\beta(\mathbf{r}, \omega) \right] \right) = 4\pi \rho^{\text{ext}}(\mathbf{r}, \omega)$$

$$\mathcal{D}_{\alpha\beta}(\mu^{-1}) A_\beta(\mathbf{r}, \omega) - \frac{\omega^2}{c^2} \epsilon_{\alpha\beta}(\mathbf{r}, \omega) A_\beta(\mathbf{r}, \omega) = \frac{4\pi}{c} J_\alpha^{\text{ext}}(\mathbf{r}, \omega) + \frac{i\omega}{c} \epsilon_{\alpha\beta}(\mathbf{r}, \omega) \partial_\beta \phi(\mathbf{r}, \omega)$$

Gauge freedom. In the generalized Coulomb gauge*:

$$\partial_\alpha [\epsilon_{\alpha\beta}(\mathbf{r}, \omega) A_\beta(\mathbf{r}, \omega)] = 0$$

*R. J. Glauber and M. Lewenstein, Phys. Rev. A **43**, 467 (1991)

“Scalar” and “vector” decomposition

The gauge-fixing condition leads immediately to the Poisson equation:

$$-\partial_\alpha [\epsilon_{\alpha\beta}(\mathbf{r}, \omega) \partial_\beta \phi(\mathbf{r}, \omega)] = 4\pi \rho^{\text{ext}}(\mathbf{r}, \omega)$$

What is the equivalent of the standard longitudinal-transverse decoupling of the Maxwell equations in vacuum?

$$J_\alpha^{\text{ext}}(\mathbf{r}, \omega) = J_\alpha^{(\text{v})}(\mathbf{r}, \omega) + J_\alpha^{(\text{s})}(\mathbf{r}, \omega)$$

$$J_\alpha^{(\text{v})}(\mathbf{r}, \omega) \equiv J_\alpha^{\text{ext}}(\mathbf{r}, \omega) + \frac{i\omega}{4\pi} \epsilon_{\alpha\beta}(\mathbf{r}, \omega) \partial_\beta \phi(\mathbf{r}, \omega)$$

$$J_\alpha^{(\text{s})}(\mathbf{r}, \omega) \equiv -\frac{i\omega}{4\pi} \epsilon_{\alpha\beta}(\mathbf{r}, \omega) \partial_\beta \phi(\mathbf{r}, \omega)$$

The vectorial current is solenoidal (i.e. transverse) but the scalar current is not irrotational:

$$\partial_\alpha J_\alpha^{(\text{v})}(\mathbf{r}, \omega) = 0 \quad \epsilon_{\alpha\beta\gamma} \partial_\beta J_\gamma^{(\text{s})}(\mathbf{r}, \omega) \neq 0$$

“Scalar” and “vector” Green’s functions

The equation for the vector potential then reads:

$$\mathcal{D}_{\alpha\beta}(\mu^{-1})A_{\beta}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\epsilon_{\alpha\beta}(\mathbf{r},\omega)A_{\beta}(\mathbf{r},\omega) = \frac{4\pi}{c}J_{\alpha}^{(\mathbf{v})}(\mathbf{r},\omega)$$

We introduce the “scalar” Green’s function as the Green’s function for the Poisson equation:

$$-\partial'_{\beta}[\text{T}\epsilon_{\beta\alpha}(\mathbf{r}',\omega)\partial'_{\alpha}g^{(\text{s})}(\mathbf{r},\mathbf{r}',\omega)] = 4\pi\delta(\mathbf{r} - \mathbf{r}')$$

The “vector” Green’s function is the Green’s function for the Helmholtz equation for the vector potential:

$$\begin{aligned} \epsilon_{\beta\delta\zeta}\epsilon_{\eta\lambda\gamma}\partial'_{\delta}[\text{T}\mu_{\zeta\eta}^{-1}(\mathbf{r}',\omega)\partial'_{\lambda}g_{\alpha\gamma}^{(\mathbf{v})}(\mathbf{r},\mathbf{r}',\omega)] - \frac{\omega^2}{c^2}\text{T}\epsilon_{\beta\gamma}(\mathbf{r}',\omega)g_{\alpha\gamma}^{(\mathbf{v})}(\mathbf{r},\mathbf{r}',\omega) = \\ = 4\pi \left[\delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}') - \frac{1}{4\pi}\epsilon_{\gamma\beta}(\mathbf{r}',\omega)\partial'_{\gamma}\partial_{\alpha}g^{(\text{s})}(\mathbf{r},\mathbf{r}',\omega) \right] \end{aligned}$$

“Scalar” and “vector” Green’s functions

Scalar and vector potentials can be found from the Green’s functions:

$$\phi(\mathbf{r}, \omega) = \phi_0(\mathbf{r}, \omega) + \int d\mathbf{r}' g^{(s)}(\mathbf{r}, \mathbf{r}', \omega) \rho_{\text{ext}}(\mathbf{r}', \omega)$$

$$A_\alpha(\mathbf{r}, \omega) = A_{0,\alpha}(\mathbf{r}, \omega) + \frac{1}{c} \int d\mathbf{r}' g_{\alpha\beta}^{(v)}(\mathbf{r}, \mathbf{r}', \omega) J_\beta^{(v)}(\mathbf{r}', \omega)$$

Expressing everything in terms of the external current we find:

$$\phi(\mathbf{r}, \omega) = \phi_0(\mathbf{r}, \omega) - \frac{1}{i\omega} \int d\mathbf{r}' \partial'_\alpha g^{(s)}(\mathbf{r}, \mathbf{r}', \omega) J_\alpha^{\text{ext}}(\mathbf{r}', \omega)$$

$$A_\alpha(\mathbf{r}, \omega) = A_{0,\alpha}(\mathbf{r}, \omega) + \frac{1}{c} \int d\mathbf{r}' g_{\alpha\beta}^{(v)}(\mathbf{r}, \mathbf{r}', \omega) J_\beta^{\text{ext}}(\mathbf{r}', \omega)$$

Finally, we compare these equations with the dyadic Green’s function

Dyadic Green's function decomposition

In the generalized Coulomb gauge the Green's function can be separated as:

$$G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega) = -\frac{c^2}{\omega^2} \partial_\alpha \partial'_\beta g^{(s)}(\mathbf{r}, \mathbf{r}', \omega) + g_{\alpha\beta}^{(v)}(\mathbf{r}, \mathbf{r}', \omega)$$

where the “scalar” Green's function is simply the Green's function for the Poisson equation

$$-\partial'_\beta [\text{T} \epsilon_{\beta\alpha}(\mathbf{r}', \omega) \partial'_\alpha g^{(s)}(\mathbf{r}, \mathbf{r}', \omega)] = 4\pi \delta(\mathbf{r} - \mathbf{r}')$$

while the “vector” Green's function is the Green's function for the Helmolzt equation

$$\begin{aligned} \epsilon_{\beta\delta\zeta} \epsilon_{\eta\lambda\gamma} \partial'_\delta [\text{T} \mu_{\zeta\eta}^{-1}(\mathbf{r}', \omega) \partial'_\lambda g_{\alpha\gamma}^{(v)}(\mathbf{r}, \mathbf{r}', \omega)] - \frac{\omega^2}{c^2} \text{T} \epsilon_{\beta\gamma}(\mathbf{r}', \omega) g_{\alpha\gamma}^{(v)}(\mathbf{r}, \mathbf{r}', \omega) = \\ = 4\pi \left[\delta_{\alpha\beta} \delta(\mathbf{r} - \mathbf{r}') - \frac{1}{4\pi} \epsilon_{\gamma\beta}(\mathbf{r}', \omega) \partial'_\gamma \partial_\alpha g^{(s)}(\mathbf{r}, \mathbf{r}', \omega) \right] \end{aligned}$$

Dyadic Green's function: physical implications

Massive difference between sub-wavelength cavities (e.g. polaritonic cavities) and diffraction-limited cavities (e.g. Fabry-Pérot cavities). In the former ones, the scalar (i.e. electrical-like) term dominates because the spatial variation of the field occurs on the polariton wavelength λ_p

$$-\frac{c^2}{\omega^2} \partial_\alpha \partial'_\beta g^{(s)}(\mathbf{r}, \mathbf{r}', \omega) \propto \left(\frac{\lambda_d}{\lambda_p} \right)^2 \gg 1$$

where λ_d is the free-space wavelength:

$$\lambda_d = \pi(c/\omega)$$

$$G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega) = \underbrace{-\frac{c^2}{\omega^2} \partial_\alpha \partial'_\beta g^{(s)}(\mathbf{r}, \mathbf{r}', \omega)}_{\text{much bigger effects}} + g_{\alpha\beta}^{(v)}(\mathbf{r}, \mathbf{r}', \omega)$$

Theory of effective electron-electron interactions (EEEs) in sub-wavelength cavities

Coulomb interactions in a sub-wavelength cavity

As we have just demonstrated, the density-density EEEI in a sub-wavelength cavity is given by

$$V(\mathbf{r}, \mathbf{r}', \omega) = e^2 g^{(s)}(\mathbf{r}, \mathbf{r}', \omega)$$

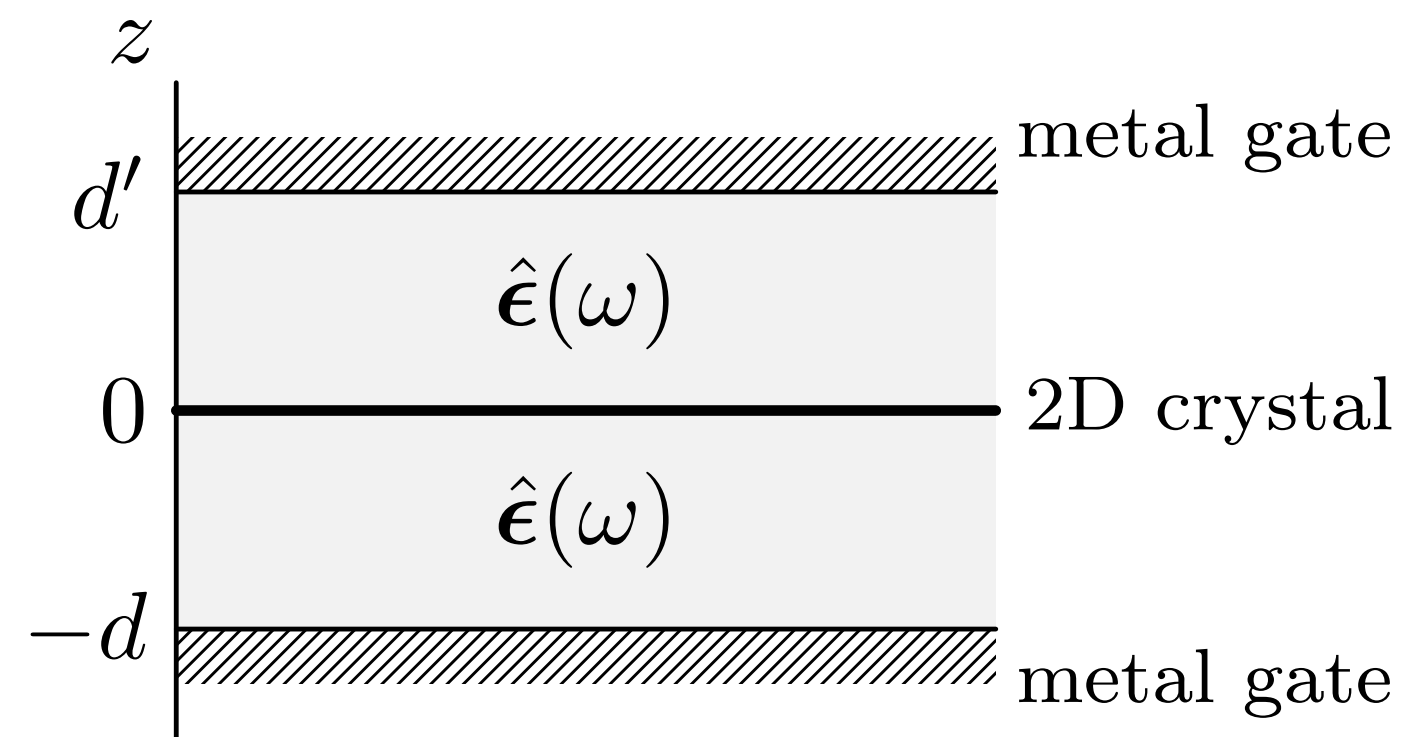
A few observations:

- In free space this clearly reduces to the instantaneous (i.e. non-retarded) Coulomb interaction
- In the static limit it encodes screening due to polarization charges and image charges in the metallo-dielectric environment
- At finite frequency, it encodes the polariton modes of the latter (including losses!), which appear as poles in the lower half of the complex frequency plane
- It is therefore extremely clear how to formulate quantum many-body theory in a cQED setup. You just need to derive generalized Hedin's equation for this retarded interaction!

The case of van der Waals planar cavities

Polariton mediated propagator

In order to illustrate in a *fully analytical* fashion our approach, we focus on a planar sub-wavelength cavity based on a vdW heterostructure



$$\hat{\epsilon}(\omega) = \text{diag}(\epsilon_x(\omega), \epsilon_x(\omega), \epsilon_z(\omega))$$

See e.g. S. Dai et al., Science **343**, 1125 (2014)
 J. D. Caldwell et al., Nat. Commun. **5**, 5221 (2014)
 T. Low et al., Nature Mater. **16**, 182 (2017)
 D. N. Basov et al., Nanophoton. **10**, 549 (2021)

The polariton mediated propagator can be found analytically

$$V(q, \omega) = \frac{2\pi e^2}{q} \frac{2 \sinh \left[q \sqrt{\frac{\epsilon_x(\omega)}{\epsilon_z(\omega)}} d \right] \sinh \left[q \sqrt{\frac{\epsilon_x(\omega)}{\epsilon_z(\omega)}} d' \right]}{\epsilon_z(\omega) \sqrt{\frac{\epsilon_x(\omega)}{\epsilon_z(\omega)}} \sinh \left[q \sqrt{\frac{\epsilon_x(\omega)}{\epsilon_z(\omega)}} (d + d') \right]}$$

Inside the reststrahlen bands, it displays poles (sub-diffractive polaritons)

Plasmon polaritons and spectral weight transfer

We introduce the retarded (i.e. causal) density-density response function:

$$\chi_{nn}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - V(q, \omega)\chi_0(q, \omega)}$$

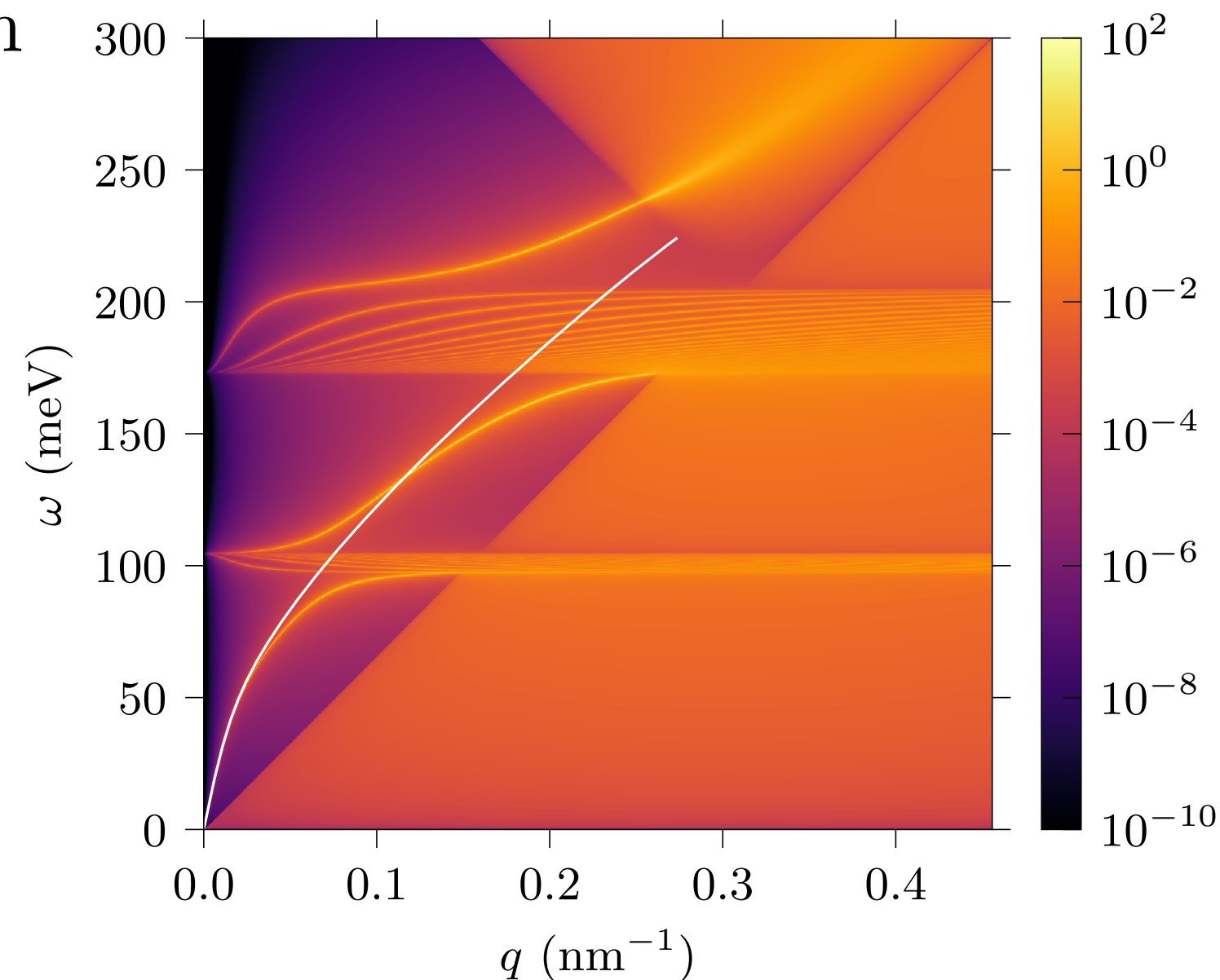
The fluctuation dissipation theorem yields the dynamical structure factor:

$$S(q, \omega) = -(\hbar/\pi)\Theta(\omega)\text{Im}\chi_{nn}(q, \omega)$$

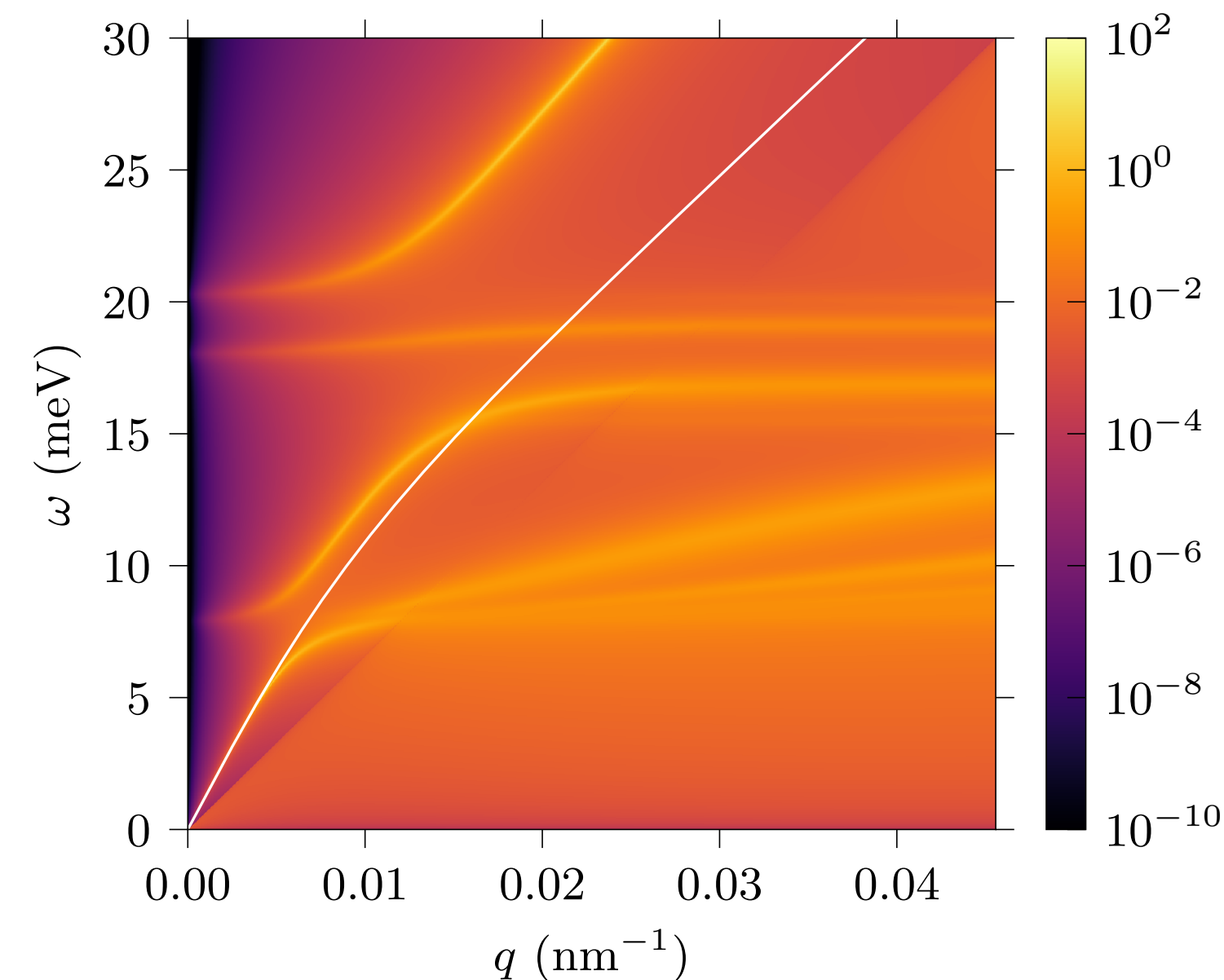
$$n_0 = 3 \times 10^{12} \text{ cm}^{-2}$$

$$d = d' = 50 \text{ nm}$$

hBN



Bi₂Se₃



Polariton-induced superconductivity in 2D metals

Superconductivity in systems with repulsive interactions

Electron systems can display instabilities towards superconductivity, also in the case of purely repulsive interactions. Two textbook examples are known:

- **Kohn-Luttinger superconductivity (already discussed in this workshop by Andrey and Paco).** Strong repulsive interactions lead to strong Friedel oscillations. A partial-wave analysis reveals that channels with high and odd angular momentum can become unstable (attractive interactions come from over-screening) - lots of work in TBG from Paco and Tommaso Cea
- **McMillan-type superconductivity.** The key here is to introduce frequency structure in the EEEI. Interactions that are less repulsive at low frequencies than at high frequencies (on the imaginary-frequency axis) lead to a superconducting instability. That's exactly what polaritons do!

Elementary considerations

1) Frequency dependence of the EEEI is natural when dynamical screening is taken into account. Strong frequency dependence associated with the plasmon pole. Difficult to treat. 3D plasmon-mediated superconductivity never observed (to the best of my knowledge). In 3D no Migdal smallness:

$$\text{3D Migdal parameter} \equiv \frac{\hbar\Omega_p}{E_F} = \mathcal{O}(1)$$

2) In 2D an effective small Migdal parameter emerges at long-wavelengths:

$$\text{2D Migdal parameter} \equiv \frac{\hbar\omega_p(q^*)}{E_F} \ll 1$$

3) Key to superconductivity is spectral weight flow from high to low energies. Hybridization between 2D plasmons and polaritons can do that!

Migdal-Eliashberg theory

Linearized gap equation (neglecting here self-energy corrections, which can be taken into account):

$$\Delta^{(\ell)}(k, i\omega_n) = -k_B T \sum_{n'=-\infty}^{\infty} \int_0^{\infty} dk' \frac{k'}{2\pi} \frac{I^{(\ell)}(k, k', i\omega_n - i\omega_{n'}) \Delta^{(\ell)}(k', i\omega_{n'})}{(\hbar\omega_{n'})^2 + \xi^2(k')}$$

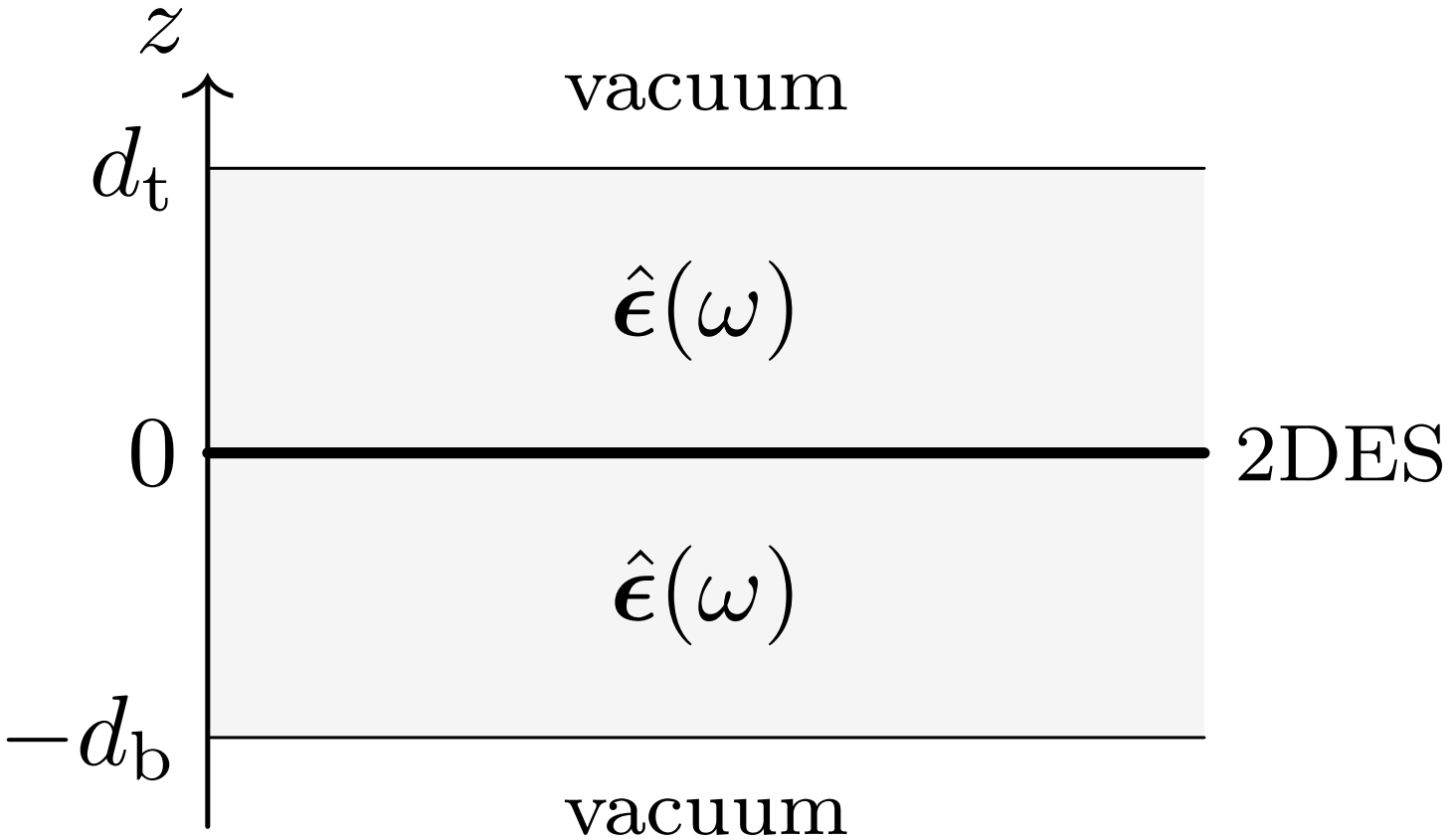
$$I^{(\ell)}(k, k', i\Omega) \equiv \int_0^{2\pi} \frac{d\theta}{2\pi} \cos(\ell\theta) \frac{1 + \cos(\theta)}{2} W(\sqrt{k^2 + k'^2 - 2kk' \cos \theta}, i\Omega)$$

Dynamically-screened interaction in the RPA:

$$W(q, \omega) = \frac{V(q, \omega)}{1 - V(q, \omega)\chi_0(q, \omega)}$$

(particle-hole susceptibility with appropriate number of fermion flavors)

Device geometry

$$V(q, \omega) = \frac{2\pi e^2}{\epsilon(\omega)q} \frac{\tanh(qd) + \epsilon(\omega)}{1 + \epsilon(\omega) \tanh(qd)}$$


The diagram illustrates the device geometry in the z -direction. A central layer, labeled "2DES", is located at $z=0$. This layer is sandwiched between two regions of dielectric material, both labeled $\hat{\epsilon}(\omega)$. The top dielectric region extends from $z=0$ to $z=d_t$, and the bottom region extends from $z=0$ to $z=-d_b$. The regions above $z=d_t$ and below $z=-d_b$ are labeled "vacuum". The z -axis is indicated by an upward arrow on the left.

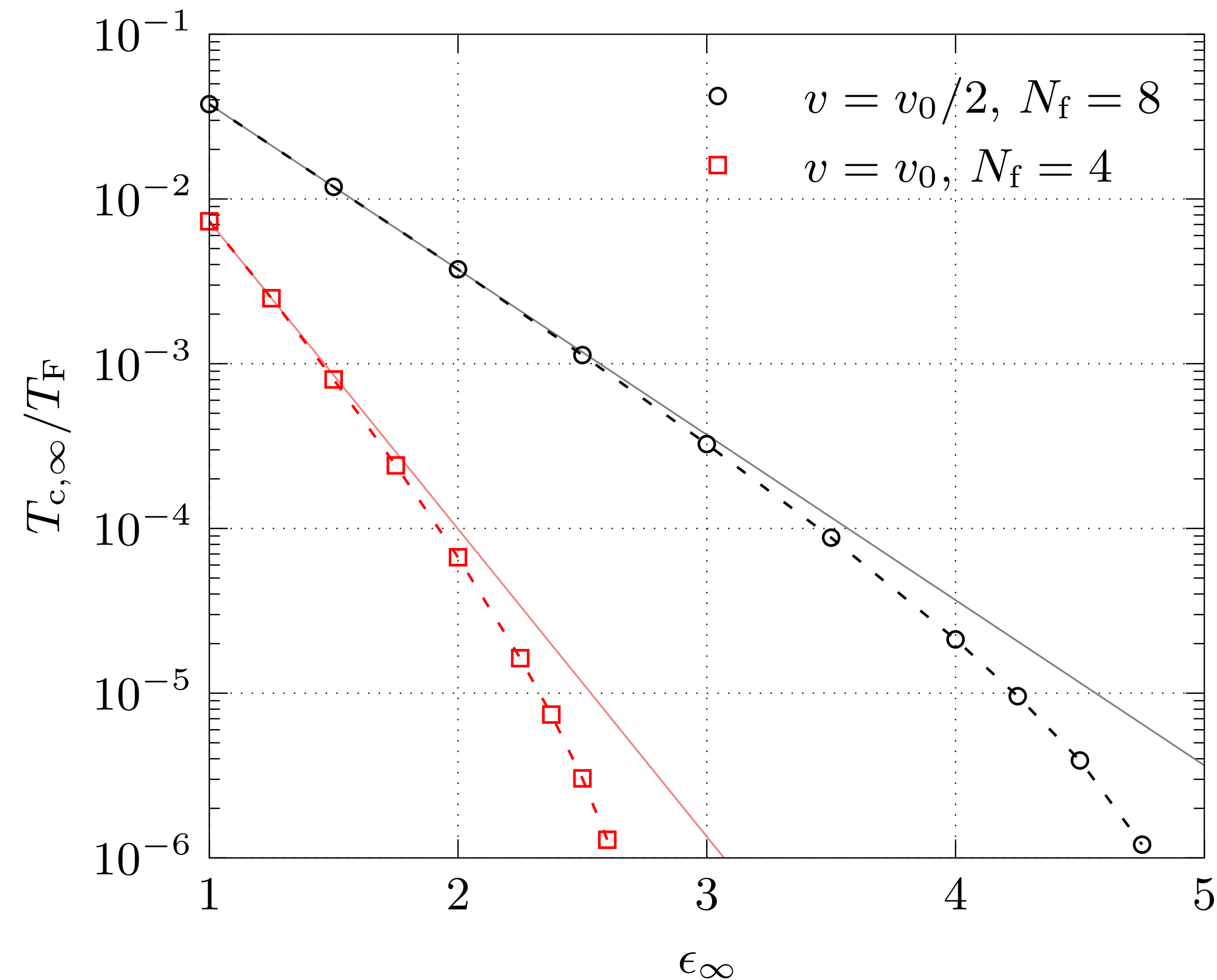
Simple parametrization of the frequency dependence of permittivity:

$$\frac{1}{\epsilon(\omega)} = \frac{1}{\epsilon_\infty} + \left(\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_0} \right) \frac{\omega_L^2}{\omega^2 - \omega_L^2}$$

As far as the 2ES is concerned, we have picked up massless Dirac fermions with a reduced Fermi velocity (TBG, Bistritzer-MacDonald model):

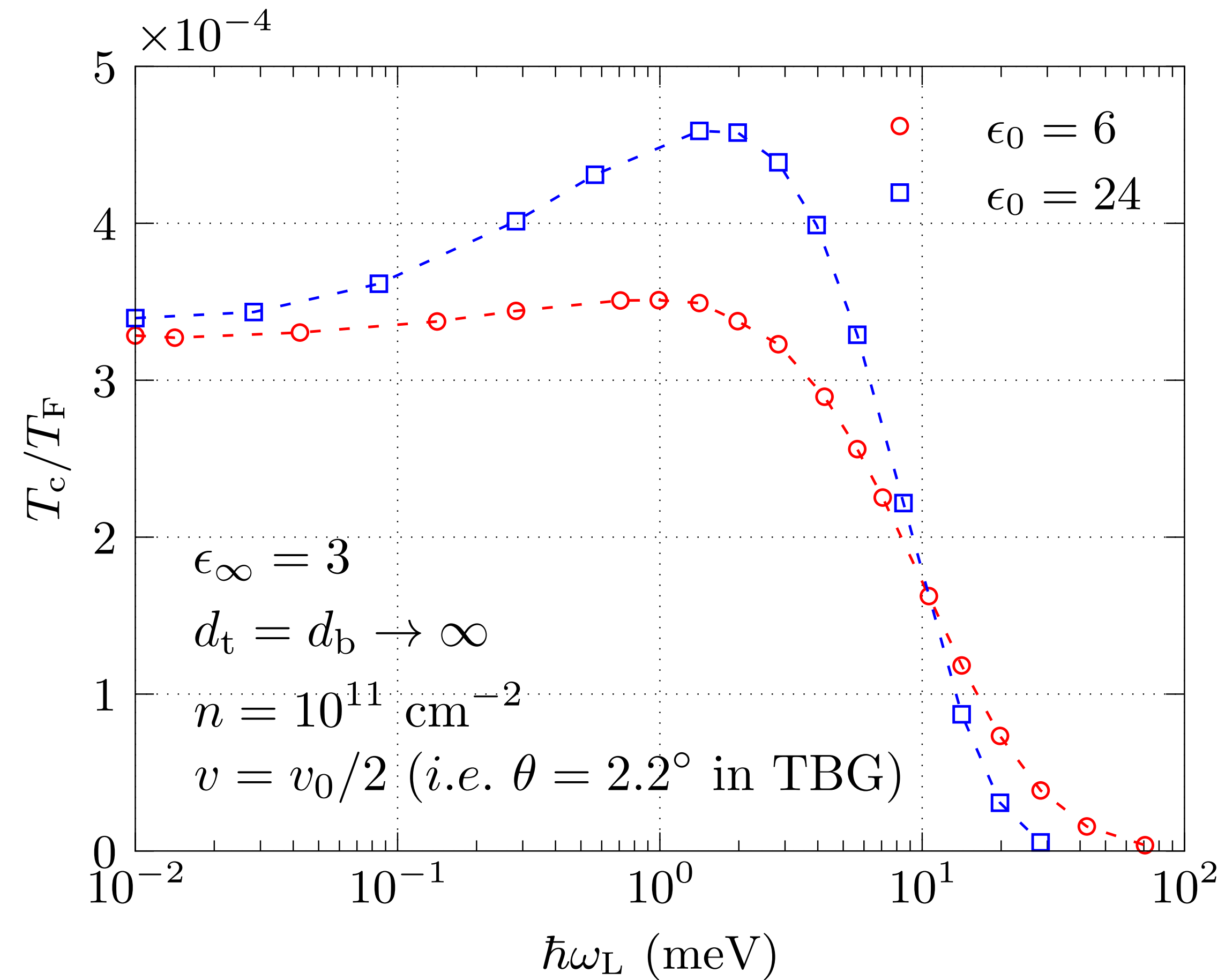
$$v = v_0 \frac{1 - 3\alpha^2(\theta)}{1 + 6\alpha^2(\theta)}$$

Placing limits on the vertex-free theory



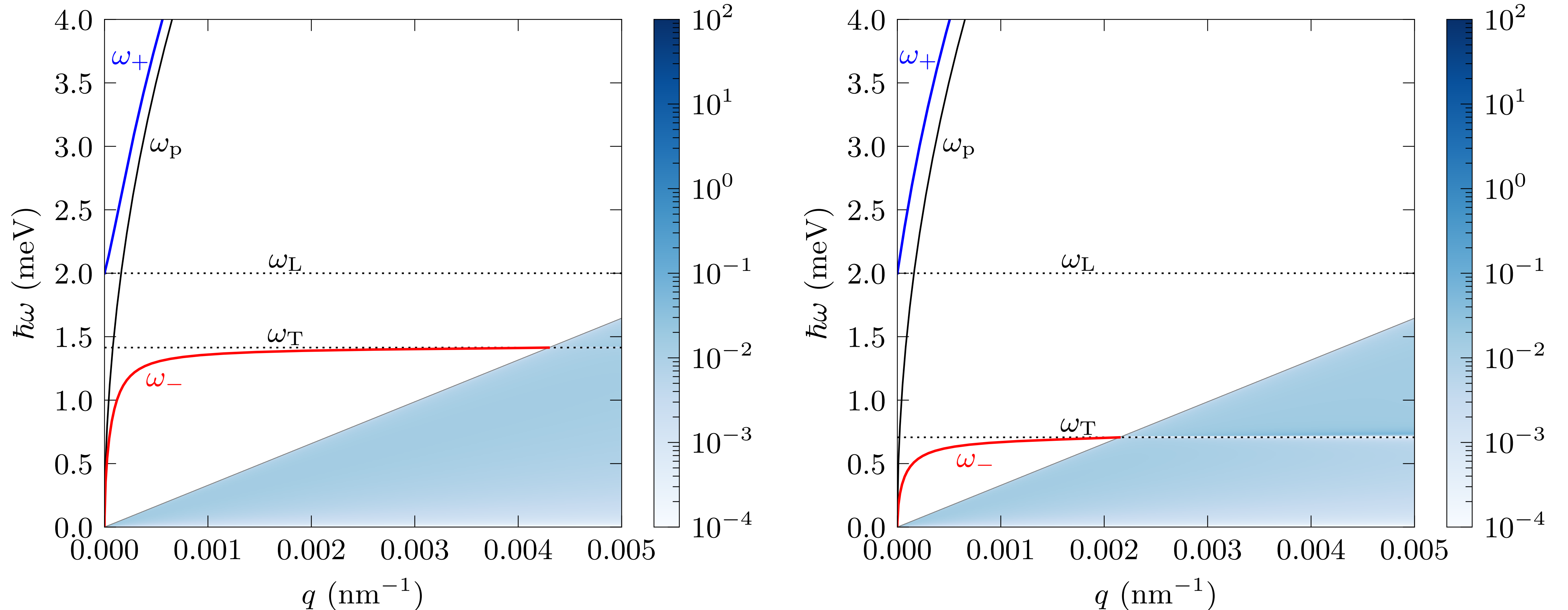
Validity of the vertex-free theory can be checked by comparing theory predictions with available experimental results (on single-layer graphene)

Numerical results



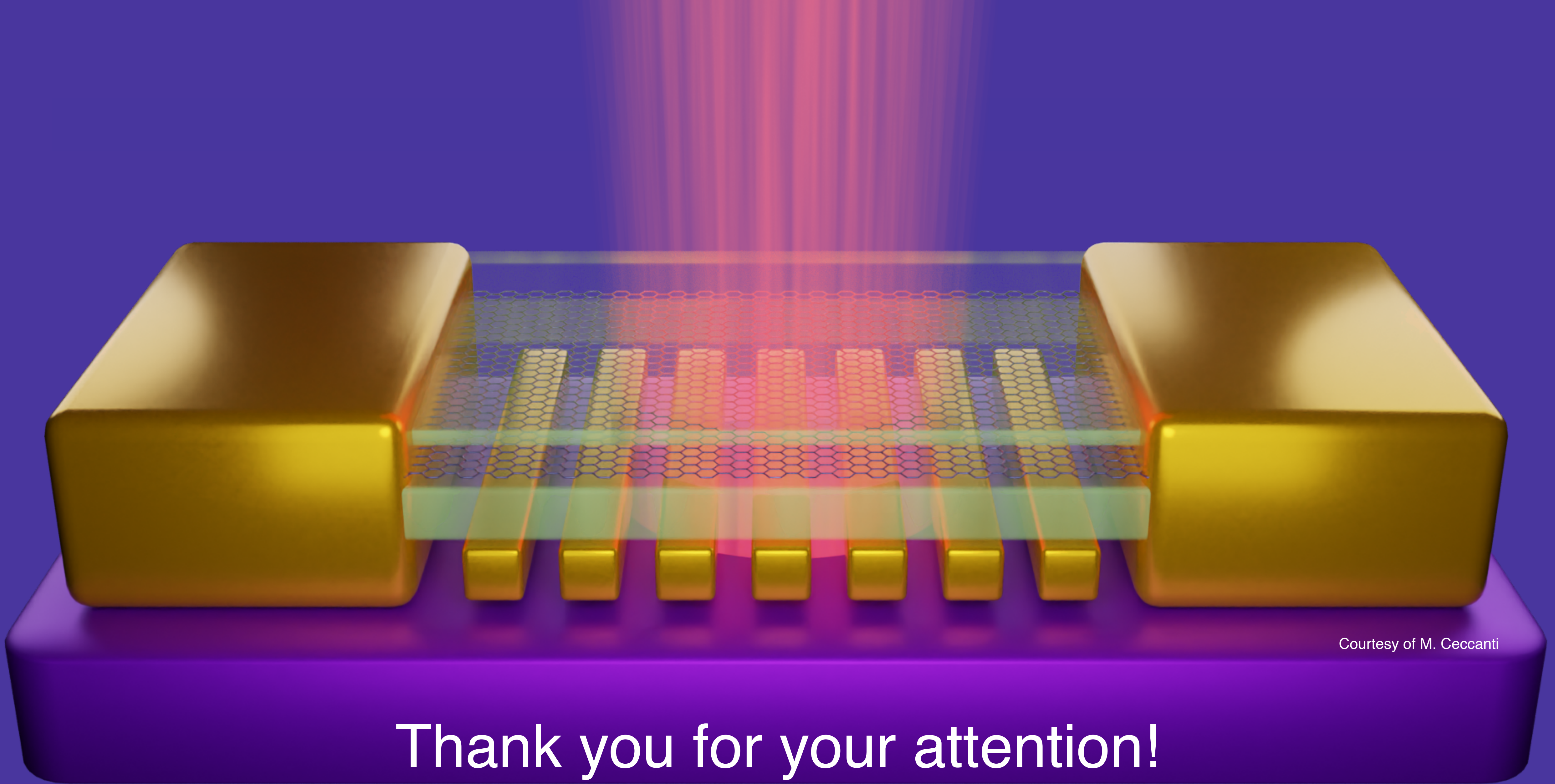
Provided phonon oscillator strength of the polar material is large enough, a large boost in critical temperature occurs under resonant conditions

Plasmon-phonon polaritons



Peak in critical temperature occurs when the 2DES plasmon is resonant with the phonon, creating lower and upper phonon polaritons

- A theory of cQED that applies to realistic sub-wavelength cavities and relies on the gauge-invariant dyadic Green's function is available. Green's function is decomposed into a scalar- and vector-like component, i.e. cavities modify both density-density and current-current interactions in materials
- Sub-wavelength polaritonic cavities tend to be dominated by density-density interactions. In these cavities, mod-vol confinement arguments can be used. The cavity “environment” can substantially alter interactions with respect to free space, in a frequency-dependent fashion
- Sub-wavelength cavities can alter transport (e.g. SdH oscillations) and many-body phases (e.g. they can induce superconductivity in 2DESs. In this respect, theory appears to be under control in 2D, when polaritonic medium is resonant)



Courtesy of M. Ceccanti

Thank you for your attention!

Backup slides

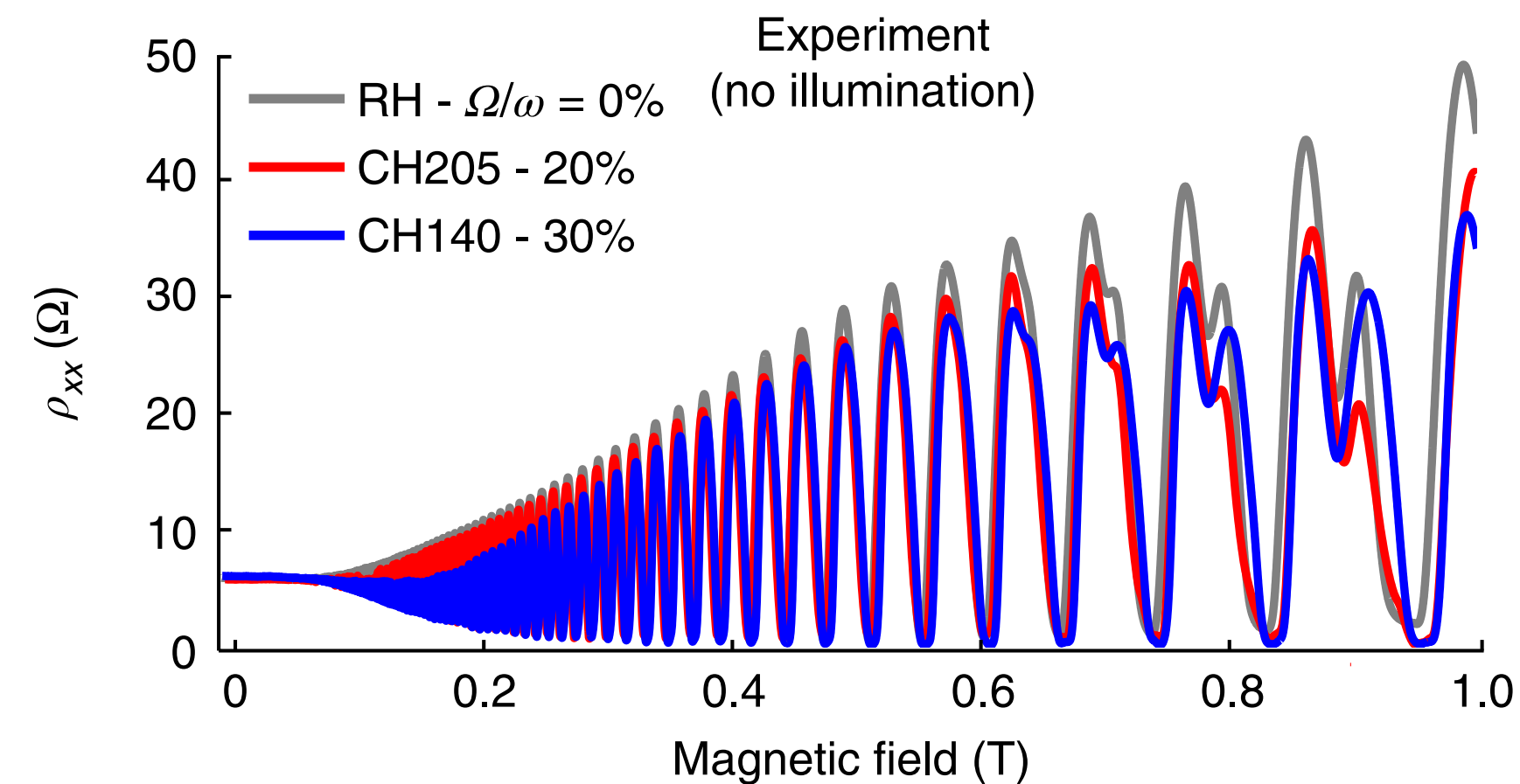
Altering DC magneto-transport properties

Longitudinal magneto-transport

Magneto-transport controlled by Landau polariton states

Gian L. Paravicini-Bagliani^{1*}, Felice Appugliese¹, Eli Richter¹, Federico Valmorra¹, Janine Keller¹, Mattias Beck¹, Nicola Bartolo², Clemens Rössler³, Thomas Ihn³, Klaus Ensslin³, Cristiano Ciuti², Giacomo Scalari^{1*} and Jérôme Faist¹

Hybrid excitations, called polaritons, emerge in systems with strong light-matter coupling. Usually, they dominate the linear and nonlinear optical properties with applications in quantum optics. Here, we show the crucial role of the electronic component of polaritons in the magneto-transport of a cavity-embedded two-dimensional electron gas in the ultrastrong coupling regime. We show that the linear direct-current resistivity is substantially modified by the coupling to the cavity even without external irradiation. Our observations confirm recent predictions of vacuum-induced modification of the resistivity. Furthermore, photo-assisted transport in the presence of a weak irradiation field at sub-terahertz frequencies highlights the different roles of localized and delocalized states.



Longitudinal magneto-transport

Any 2D metal in an applied perpendicular field will display SdH oscillations:

$$\frac{\delta\rho_{xx}(B)}{\rho_0} = R(B, T) \cos \left(2\pi \left(\frac{B_F}{B} + \frac{1}{2} + \gamma \right) \right)$$

Inelastic effects (i.e. electron-electron interactions) control the amplitude:

$$R(B, T) = R_0 e^{-\pi c m_c / (e B \tau)} \frac{2\pi^2 c m_c k_B T / (\hbar e B)}{\sinh(2\pi^2 c m_c k_B T / (\hbar e B))}$$

The cyclotron mass is a normal Fermi liquid parameter:

Massive fermions: $m_c = m^*$

Massless fermions: $m_c = \hbar k_F / v_F^*$

D. Shoenberg, Magnetic Oscillations in Metals (Cambridge University Press, Cambridge, 1984)

L. D. Landau and E. M. Lifshitz, Course of Theoretical Physics: Statistical Physics, Part 2 (Pergamon, New York, 1980)

T. Ando, A. B. Fowler, and F. Stern, Electronic properties of two-dimensional systems, Rev. Mod. Phys. **54**, 437 (1982)

Fermi liquid theory in a graphene cQED setup

Feynman diagrams in the GW approximation:

$$\Sigma_{\lambda}(\mathbf{k}, i\omega_n) = \text{Diagram: A fermion line with momentum } \mathbf{k} \text{ and energy } i\omega_n \text{ emits a boson with momentum } \mathbf{q} \text{ and energy } i\Omega_m, \text{ and then reabsorbs it. The fermion line continues with momentum } \mathbf{k} - \mathbf{q} \text{ and energy } i\omega_n - i\Omega_m.$$

$$\text{Diagram: A wavy line labeled } W \text{ is equal to a wavy line labeled } V \text{ plus a diagram where a } V \text{ wavy line connects to a fermion loop (labeled } \chi_0 \text{) which then connects to a } W \text{ wavy line.} \quad W(q, i\Omega) = \frac{V(q, i\Omega)}{1 - V(q, i\Omega)\chi_0(q, i\Omega)}$$

Self-energy of massless Dirac fermions in graphene:

$$\Sigma_{\lambda}(\mathbf{k}, i\omega_n) = -k_{\text{B}}T \sum_{\lambda'} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \sum_{m=-\infty}^{+\infty} W(q, i\Omega_m) F_{\lambda\lambda'}(\theta_{\mathbf{k}, \mathbf{k}-\mathbf{q}}) G_{\lambda'}^{(0)}(\mathbf{k} - \mathbf{q}, i\omega_n - i\Omega_m)$$

Quasiparticle renormalization factor and Fermi velocity

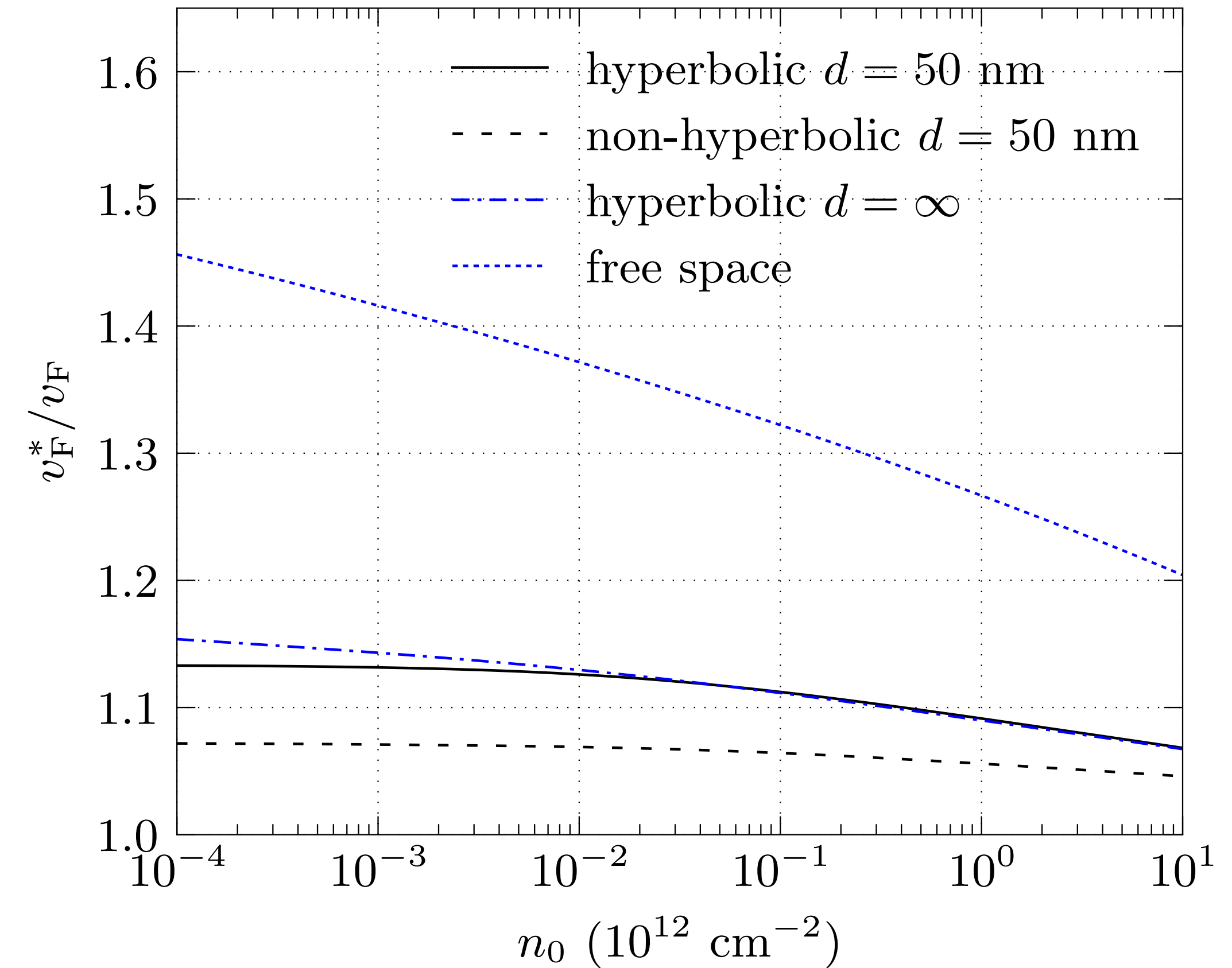
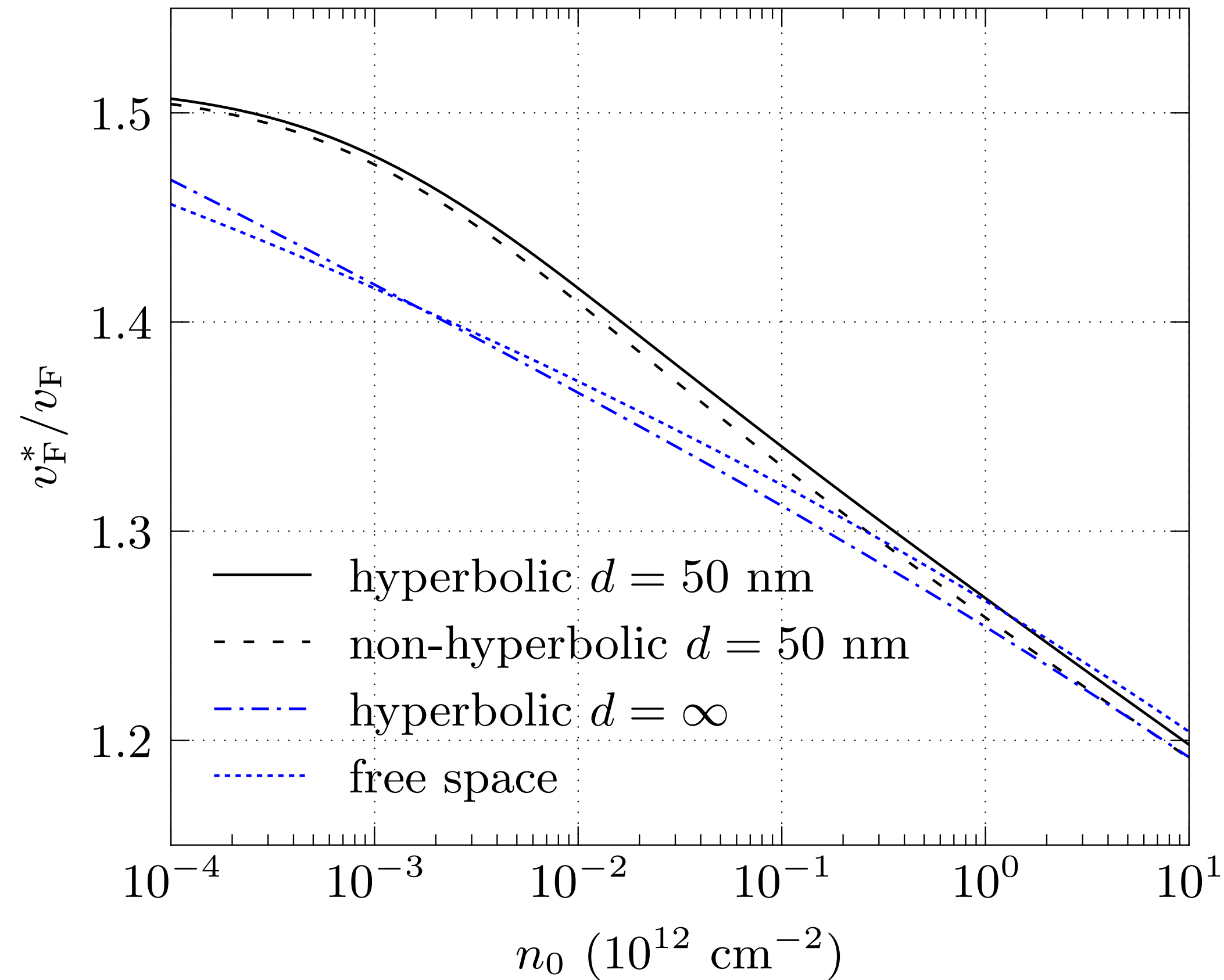
Landau normal Fermi liquid parameters can be calculated from derivatives of the self-energy:

$$Z = \frac{1}{1 - \partial_{\omega} \text{Re} \Sigma_{+}^{\text{ret}}(\mathbf{k}, \omega) \big|_{k=k_F, \omega=0}}$$
$$\frac{v_F^*}{v_F} = \frac{1 + (v_F)^{-1} \partial_k \text{Re} \Sigma_{+}^{\text{ret}}(\mathbf{k}, \omega) \big|_{k=k_F, \omega=0}}{1 - \partial_{\omega} \text{Re} \Sigma_{+}^{\text{ret}}(\mathbf{k}, \omega) \big|_{k=k_F, \omega=0}}$$

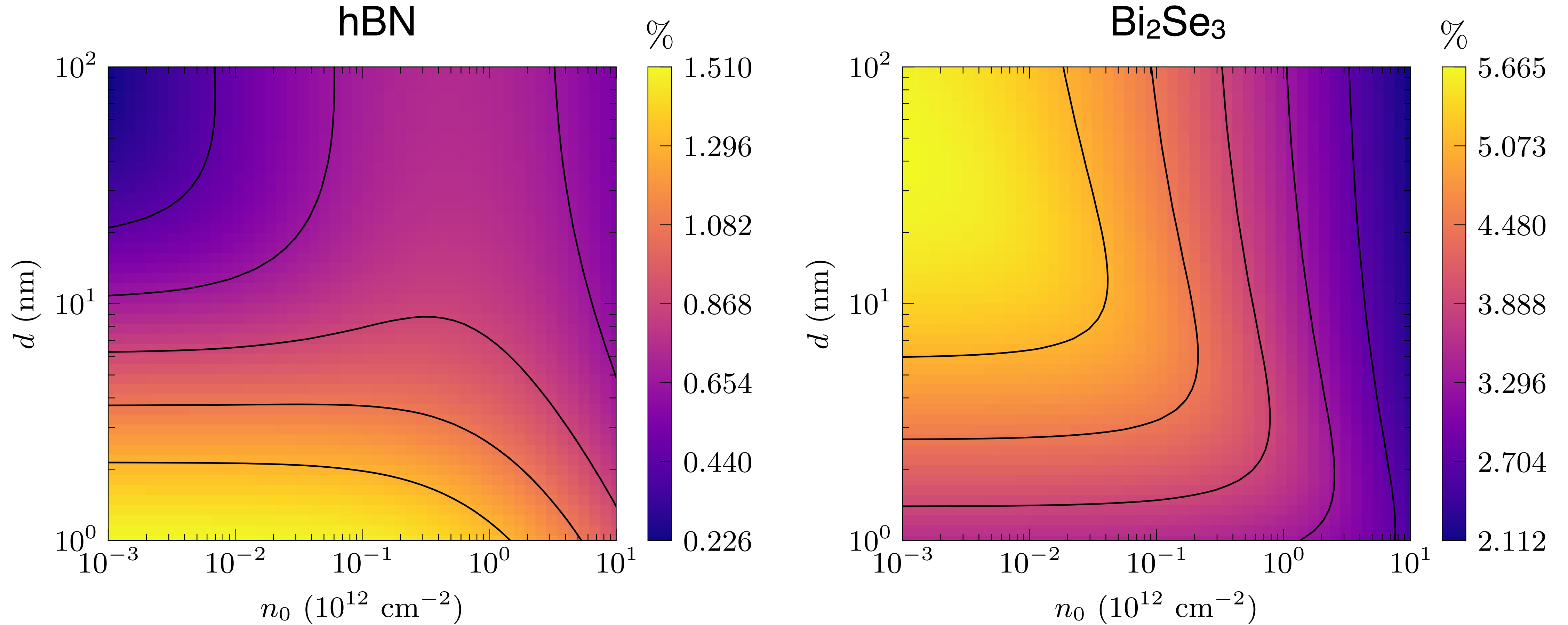
Important definitions:

$$\left. \frac{v_F^*}{v_F} \right|_{\text{free space}} \equiv \left. \frac{v_F^*}{v_F} \right|_{F(q, \omega) \rightarrow 1}$$
$$\left. \frac{v_F^*}{v_F} \right|_{\text{non-hyperbolic}} \equiv \left. \frac{v_F^*}{v_F} \right|_{F(q, \omega) \rightarrow F(q, 0)}$$
$$\left. \frac{v_F^*}{v_F} \right|_{\text{hyperbolic}} \equiv \left. \frac{v_F^*}{v_F} \right|_{F(q, \omega)}$$

Numerical results: quasiparticle velocity



Genuine cavity QED effects



Honest figure of merit:

$$\Delta_{\text{QED}} = \frac{v_{\text{F}}^*|_{\text{hyperbolic}} - v_{\text{F}}^*|_{\text{non-hyperbolic}}}{v_{\text{F}}^*|_{\text{non-hyperbolic}}}$$