Unconventional 2D Ferromagnetism and Superconductivity within a ferromagnetic phase

Andrey Chubukov, UMN






Zach Raines
UMN

Leonid Glazman
Yale

Vladimir Calvera
UMN

Hequi Li,
San Sebastian

Andrei Bernevig
Princeton

Yijie Wang,
Beijing

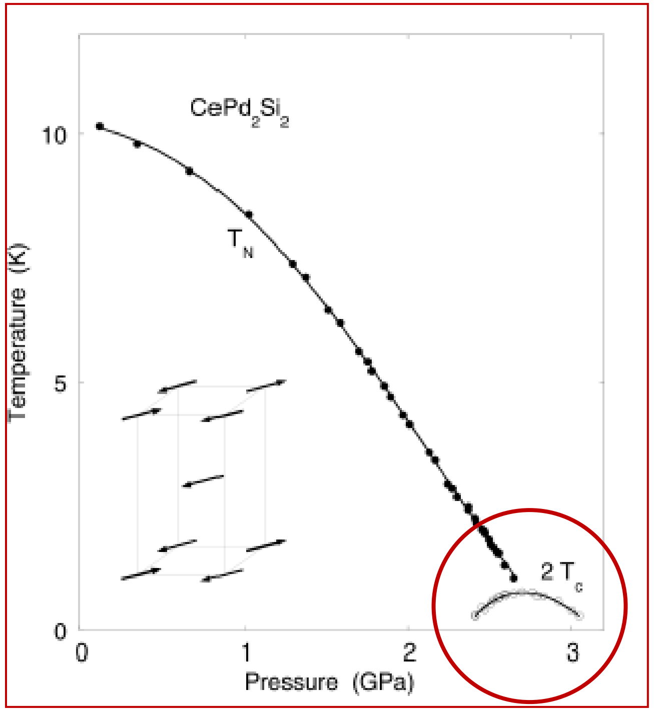
SPICE Workshop on Unconventional Superconductors and Magnets, May 12, 2026

This talk will be about unconventional Stoner ferromagnetism in 2D metals and superconductivity, co-existing with ferromagnetism.

Motivation: I

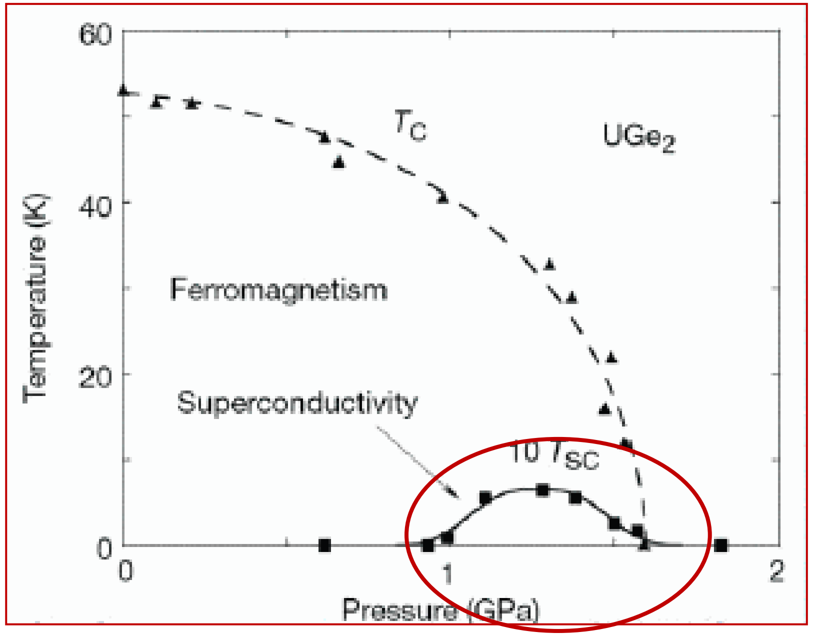
Superconductivity near the onset of a magnetic order

SC at the onset of antiferromagnetism

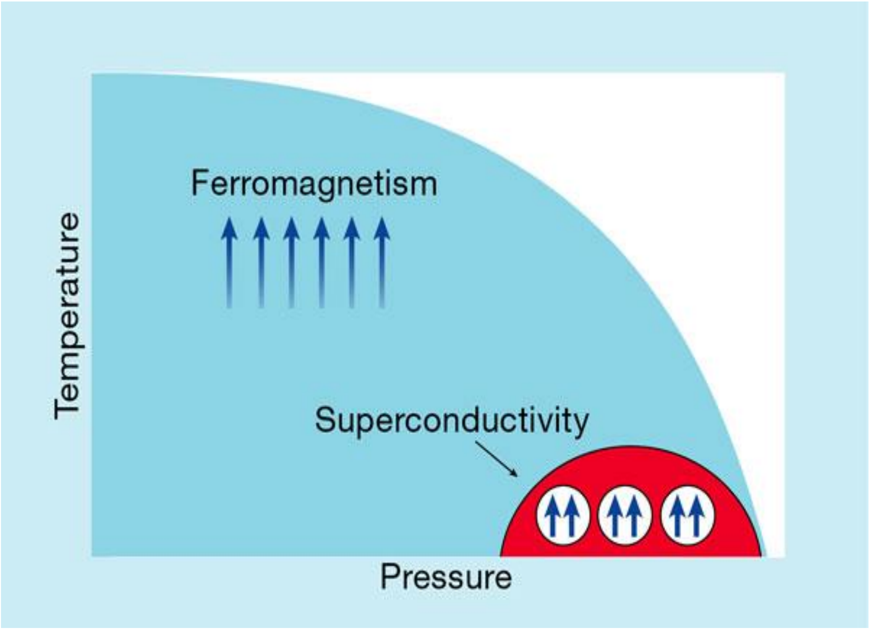


N.D. Marthur et al

SC at the onset of ferromagnetism



S.S. Saxena et al



Kirkpatrick, Belitz, Vojta & Narayan

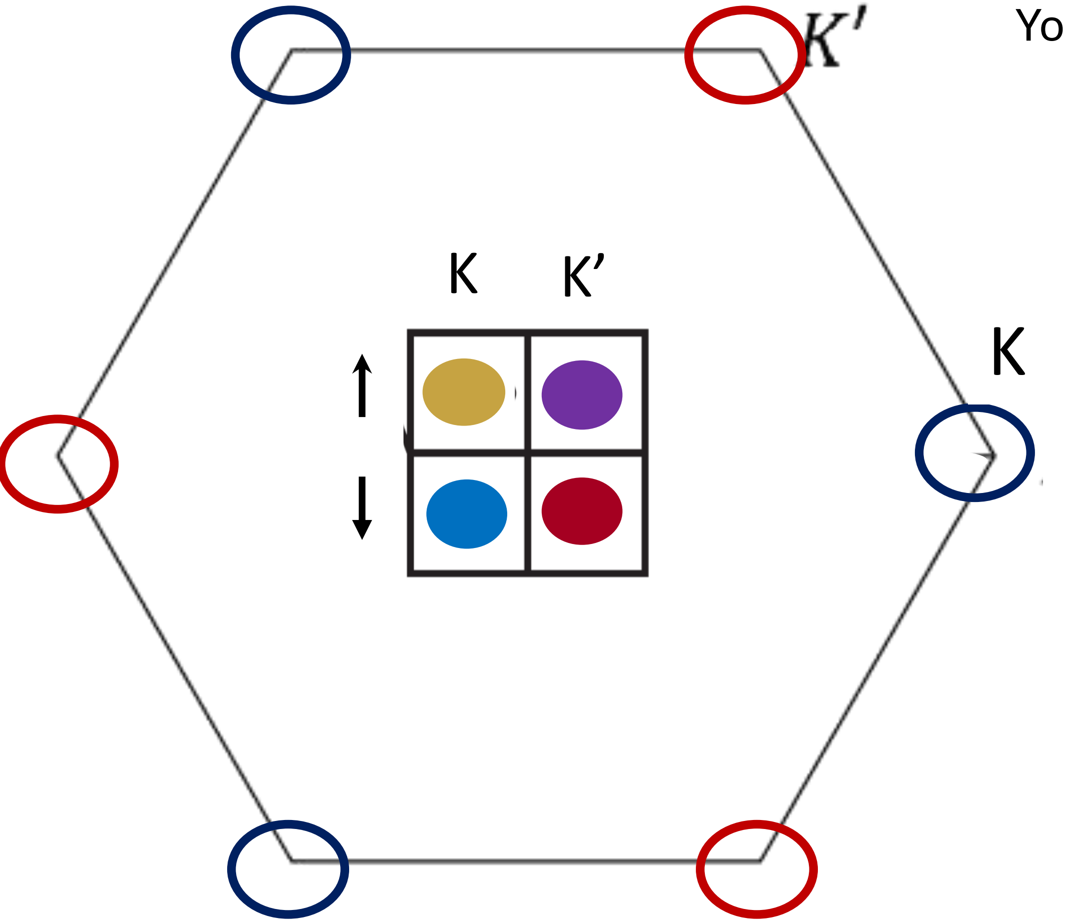
URhGe (Paulsen et al) UCoGe (Lohneysen et al)

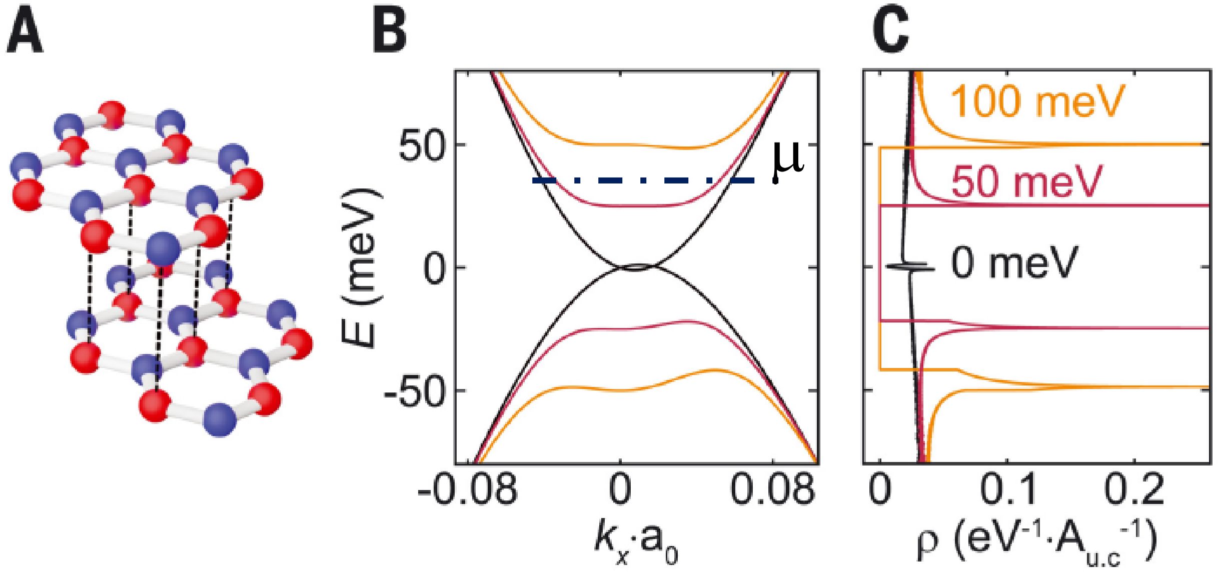
Motivation: II

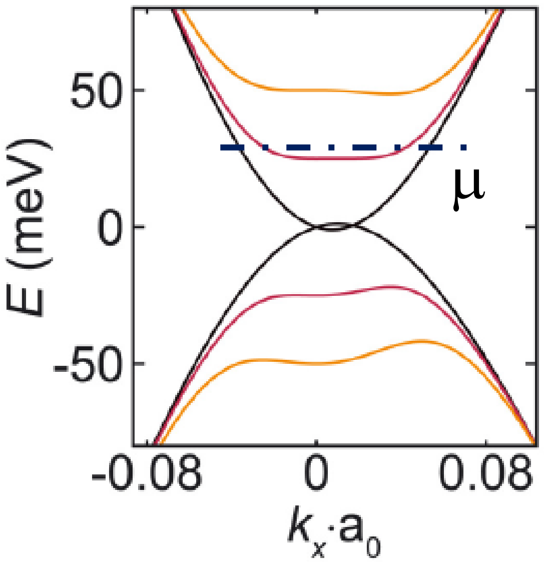
Recent works on superconductivity in non-twisted graphene multi-layers

Bernal bilayer graphene, rhombohedral tri-layer graphene, 4-5 layer graphene in a displacement field

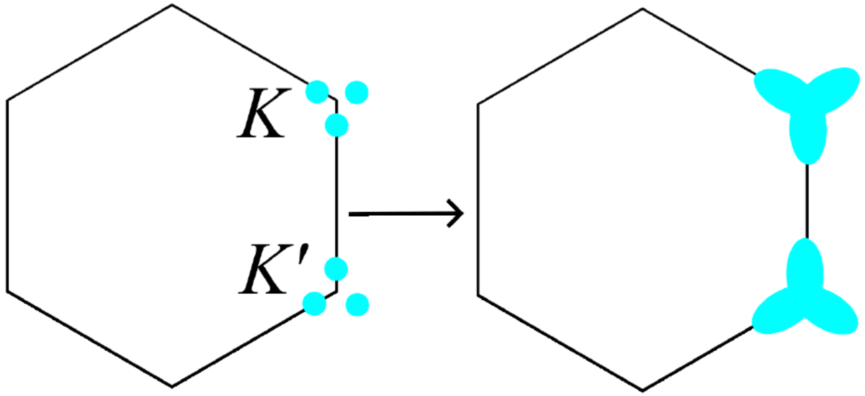
Zhou, Berg, Young et al, Nature (2021), Zhou, Young et al, Science (2022), Seiler, Weitz et al, Nature (2022), Hollies, Young, Nadj-Perge et al, Nat. Phys. (2023), Zhang, Nadj-Perge et al, Nature, (2023), Patterson, Young, et al (2024) Nature, T, Han, Long Ju et al, Nature (2025)……



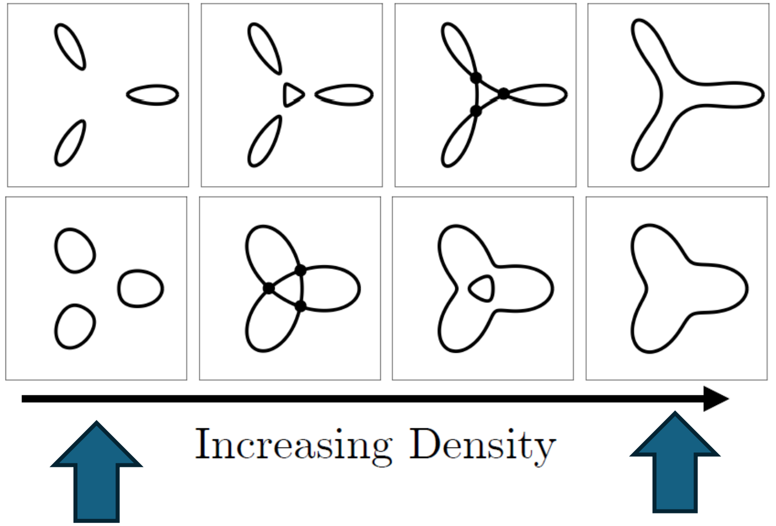




Free fermions: evolution with doping



Near each K or K' point



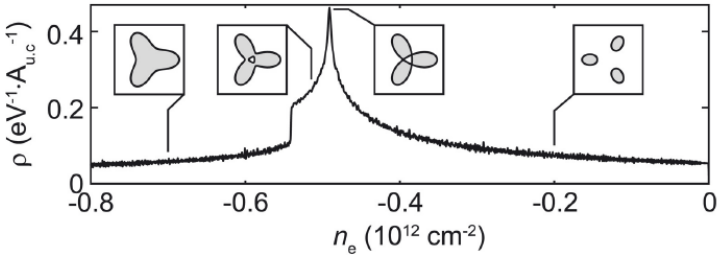
Smaller displacement field

Larger displacement field

Three smaller pockets

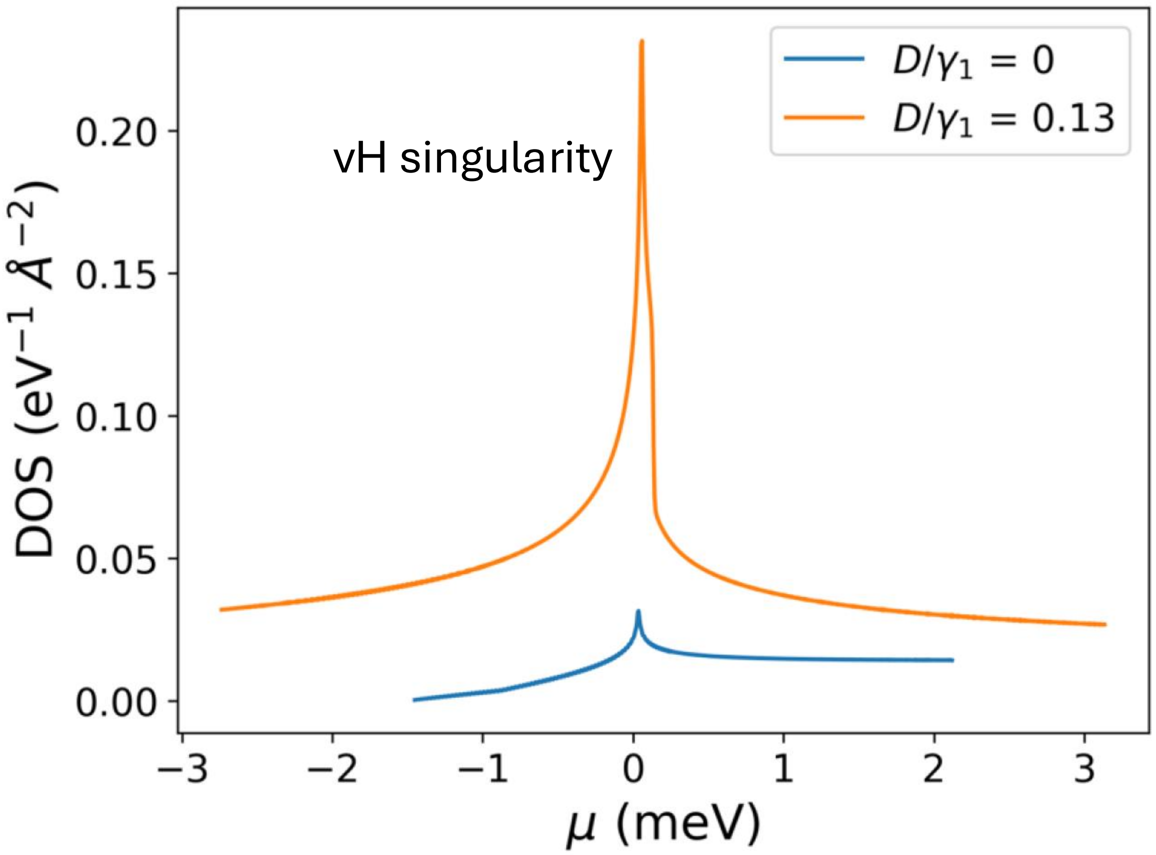
One larger pocket

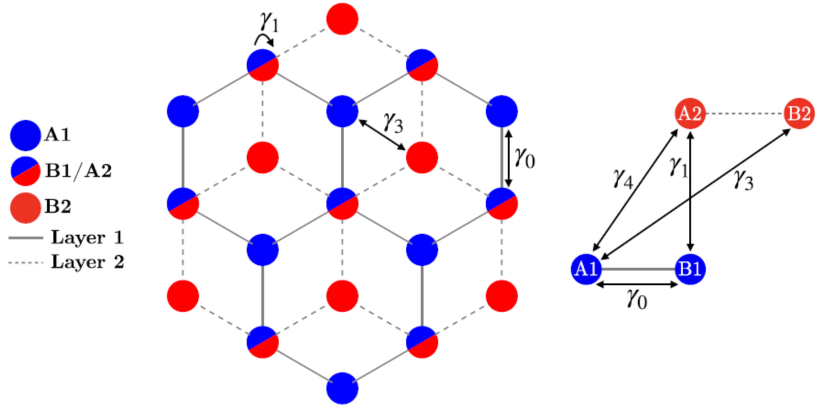
At a certain density, there is a van Hove point



Berg, Guinea, Young, Seiler, Weitz, ...A.C.

Displacement field D increases the DOS in a wide range of densities



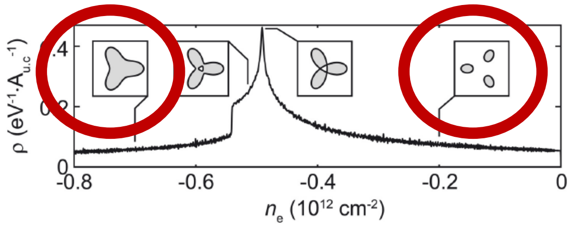


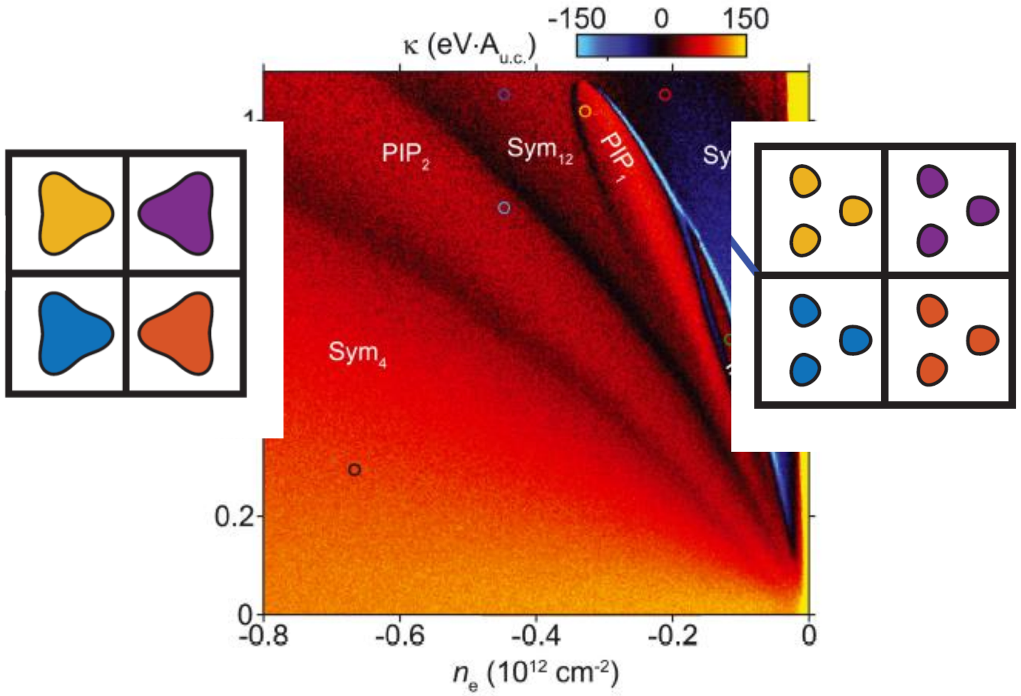
David Mayrhofer

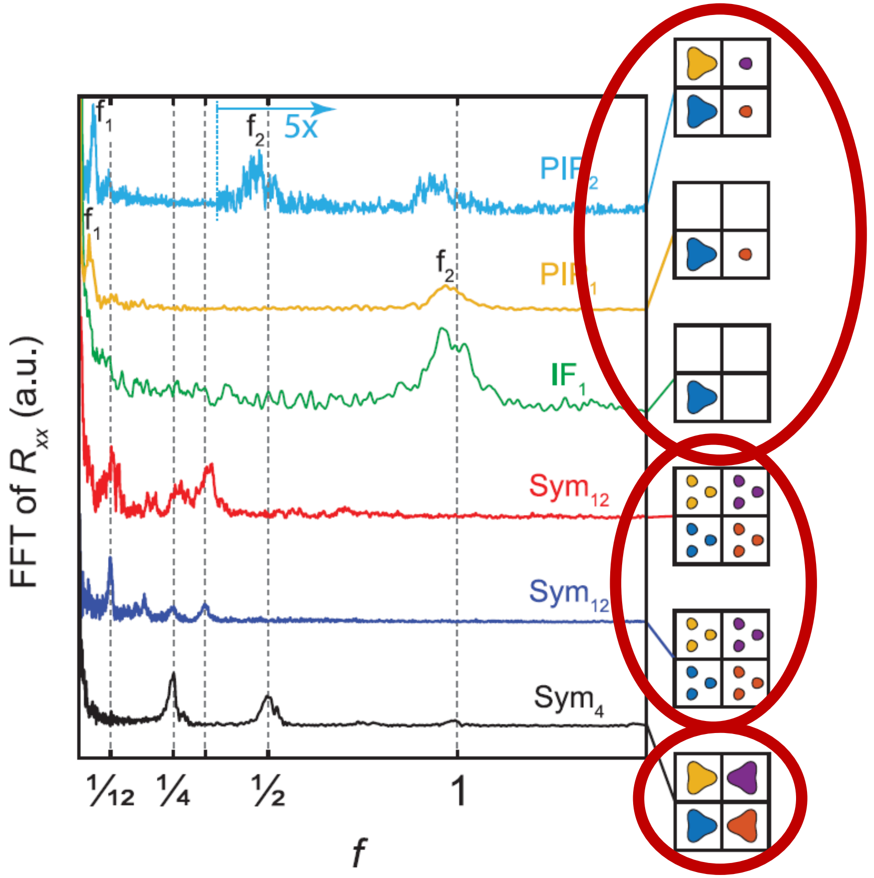
$$N_F^D = N_F^{D=0}\ \gamma_0/\gamma_3\ \sim 10\ N_F^{D=0}$$

One may expect Stoner-type instabilities towards spin and valley orders

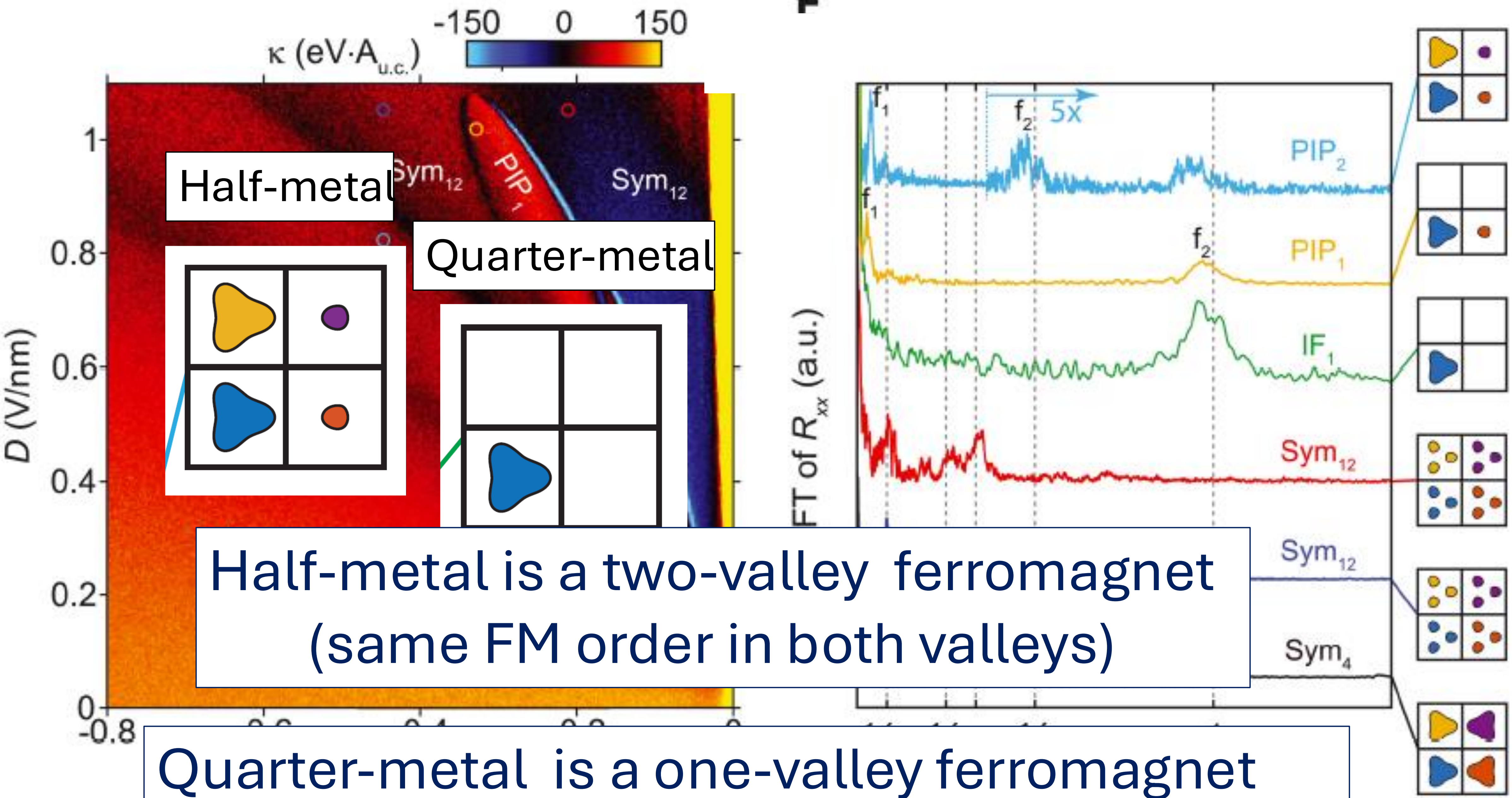
Experiments (BBG): evolution with doping involves several phase transitions







4-fold spin/valley degeneracy is broken at intermediate dopings

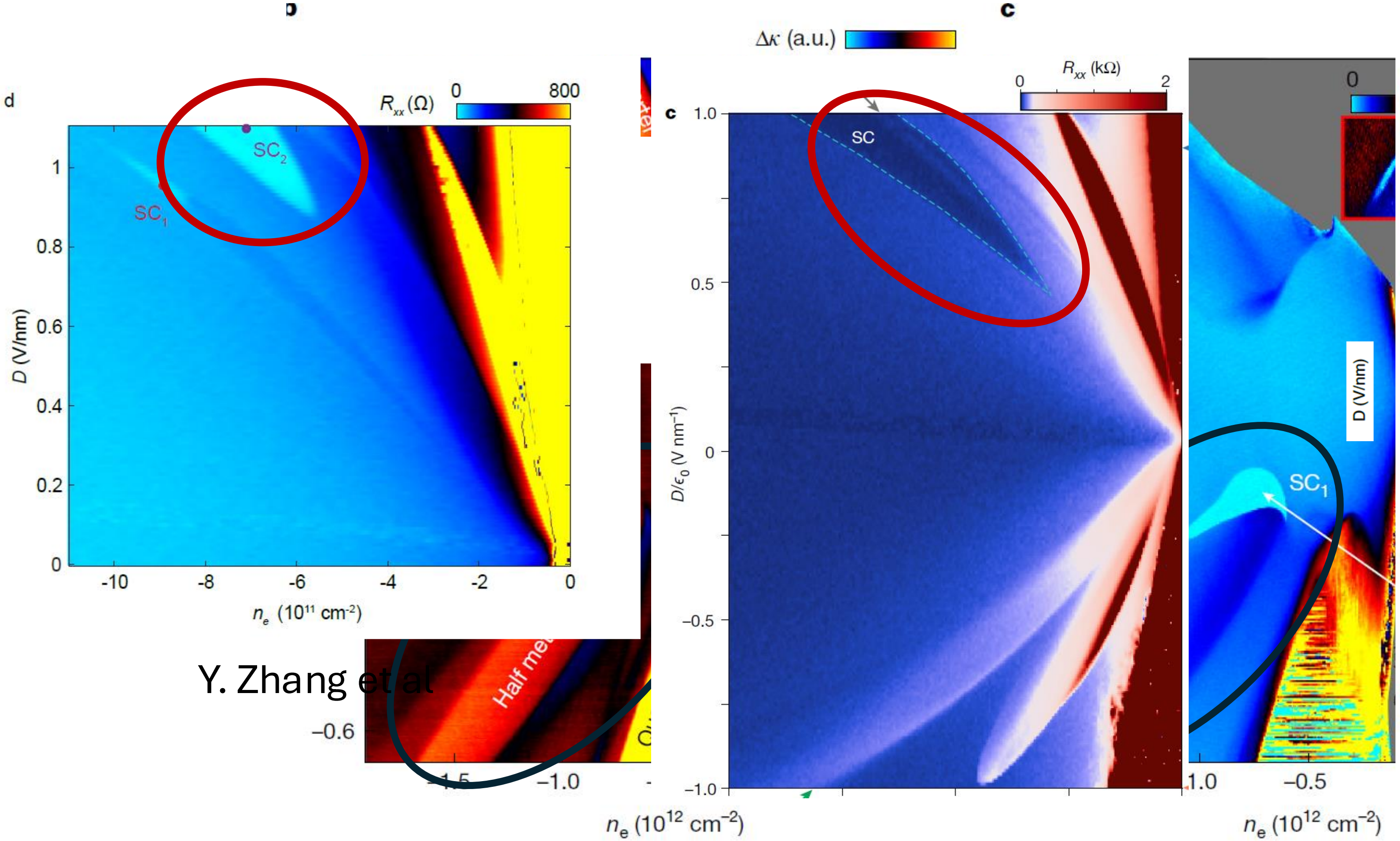


Half-metal is a two-valley ferromagnet
(same FM order in both valleys)

Quarter-metal is a one-valley ferromagnet
(spin ferromagnetism + valley polarization)

Islands of superconductivity near the onset of a half-metal

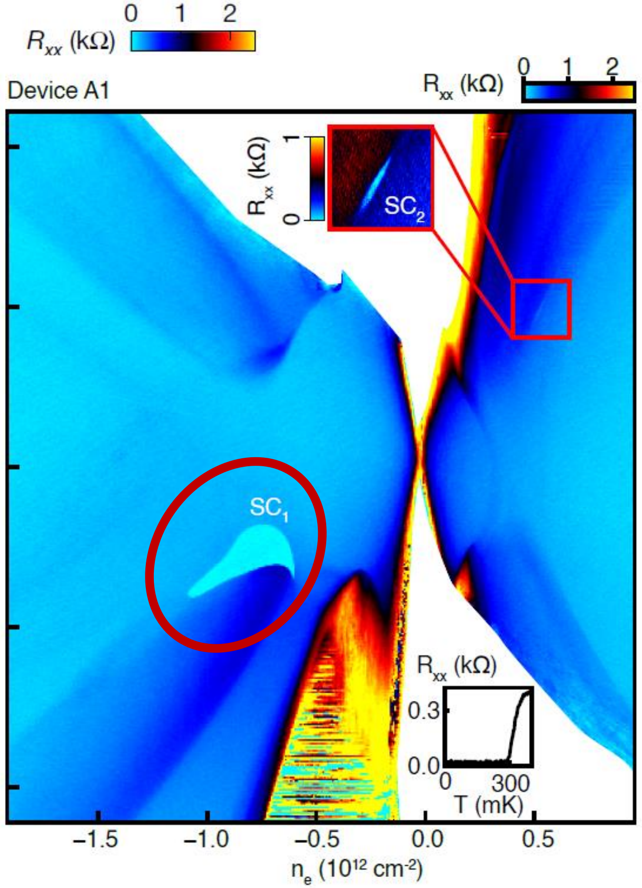
BBG



Y. Zhang et al

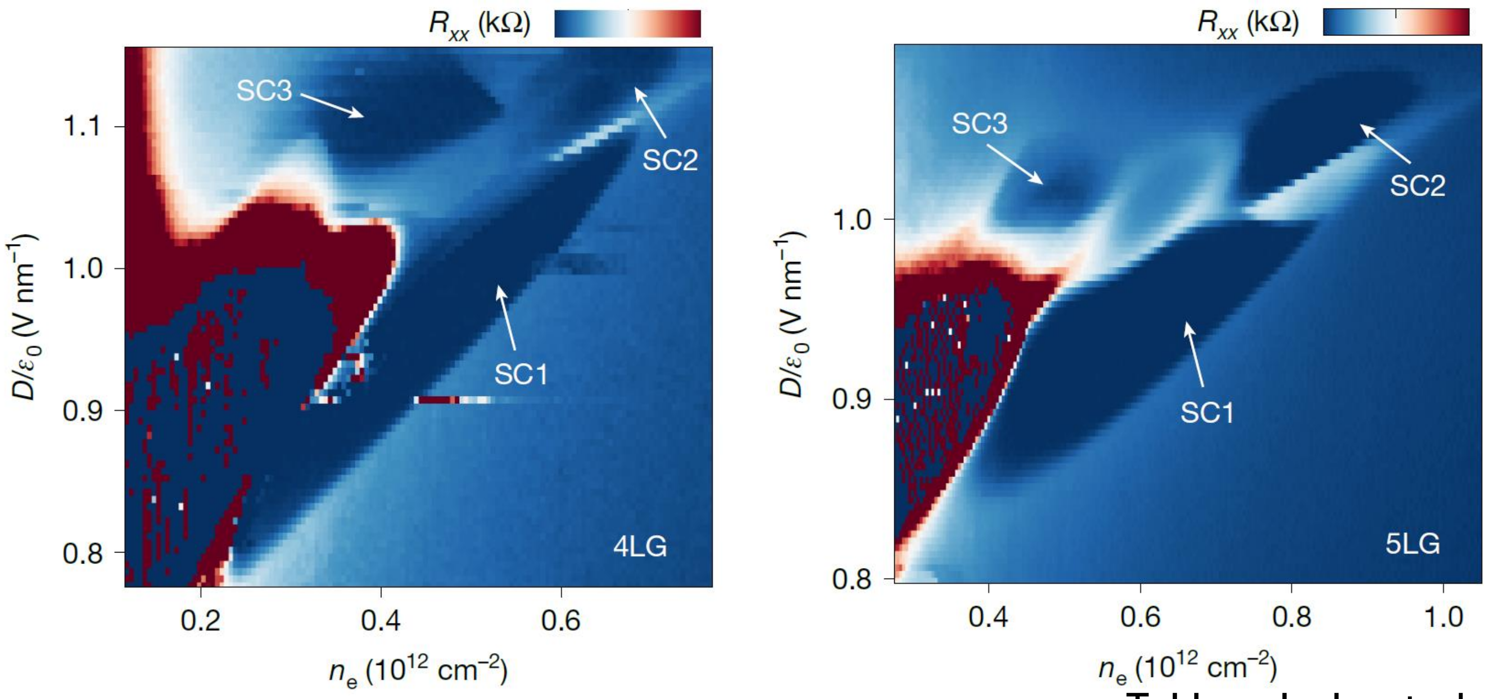
Holleis et al

RTG



Patterson et al

Islands of superconductivity inside a quarter metal state (ferromagnetism and valley polarization)

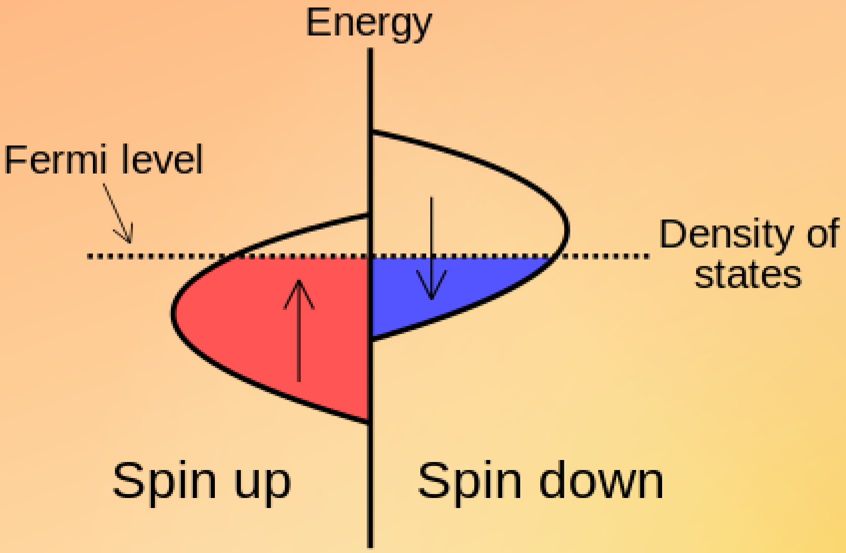


T. Han, L. Ju et al

Let’s start with ferromagnetism

E.C. Stoner (1899-1968)

How a Stoner instability develops in a 2D metal?



Recall how a Stoner ferromagnetic transition happens in 3D

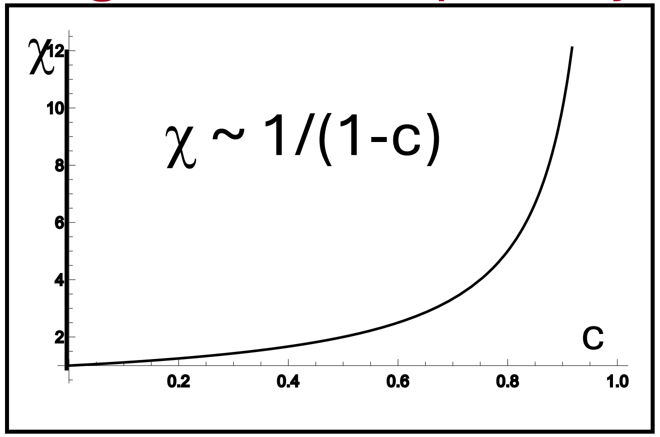
One-valley system of fermions with $k^2/2m$ dispersion and Hubbard interaction U

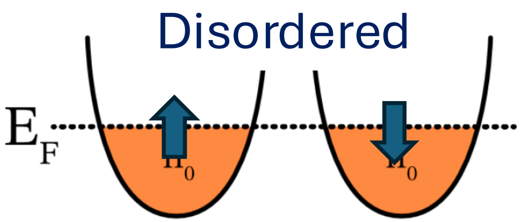
Compute the ground state energy in Hartree-Fock (mean-field) approximation by expanding in magnetization ζ.

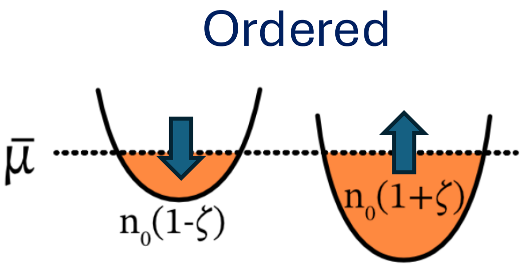
Stoner instability is a second order, continuous transition

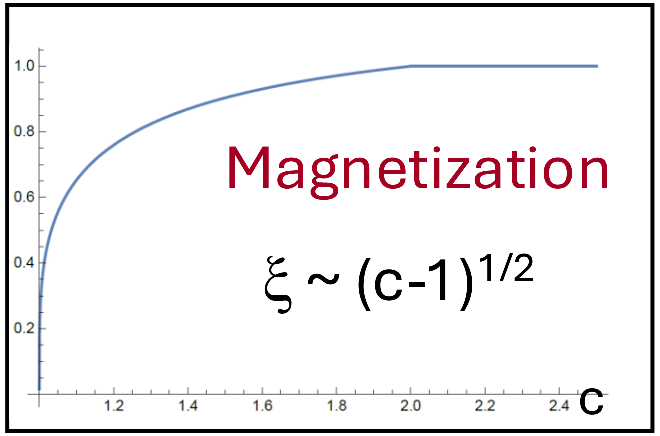
b >0

Magnetic susceptibility









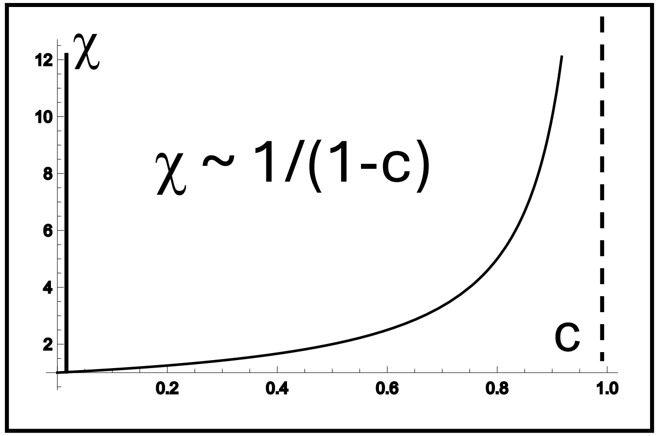
Stoner transition in 2D

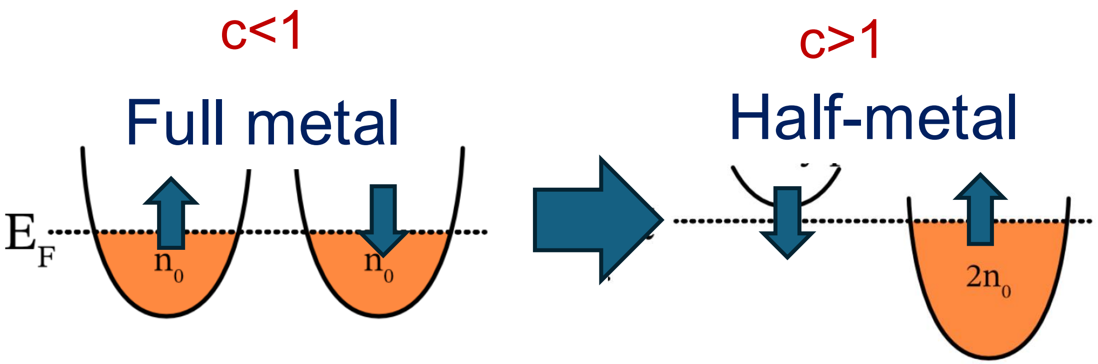
Same conditions, parabolic dispersion

Hartree -Fock: instability towards ferromagnetism at c=1

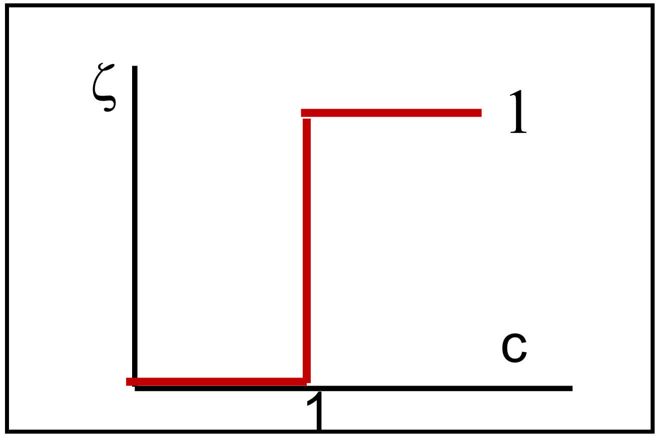
$$E = (1-c)\,\zeta^2 + 0\,\zeta^4 + 0\,\zeta^6 + 0^*\ldots$$

Susceptibility diverges at c->1





Magnetization jumps at c = 1+0

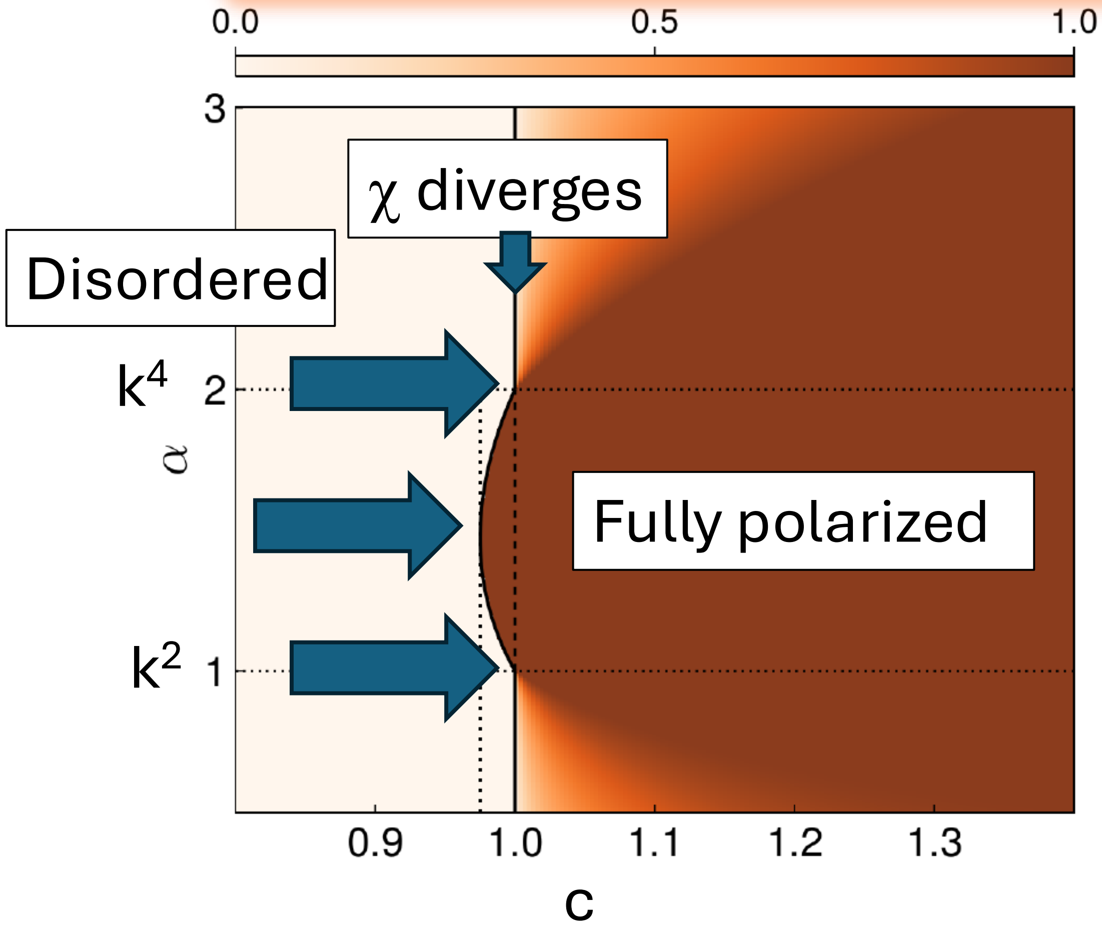


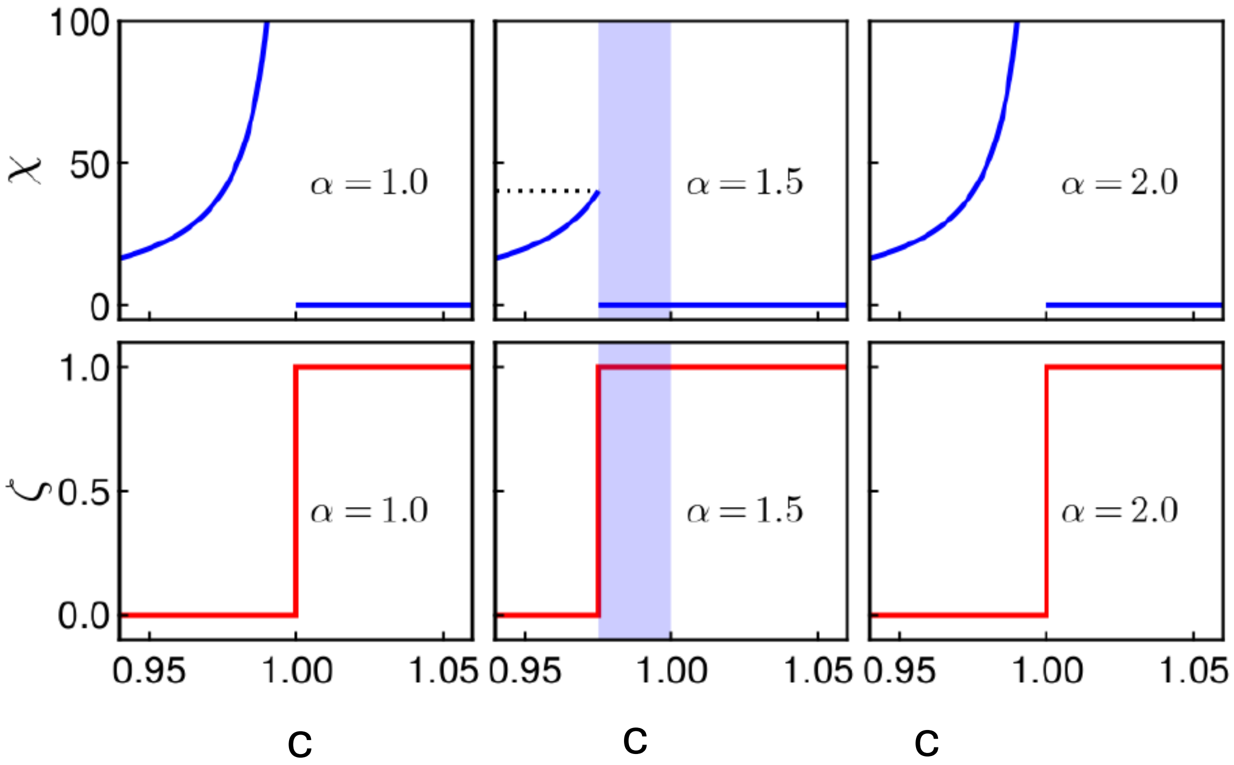
An instant transition to a half-metal

D=2

And what if the dispersion is not k^2?

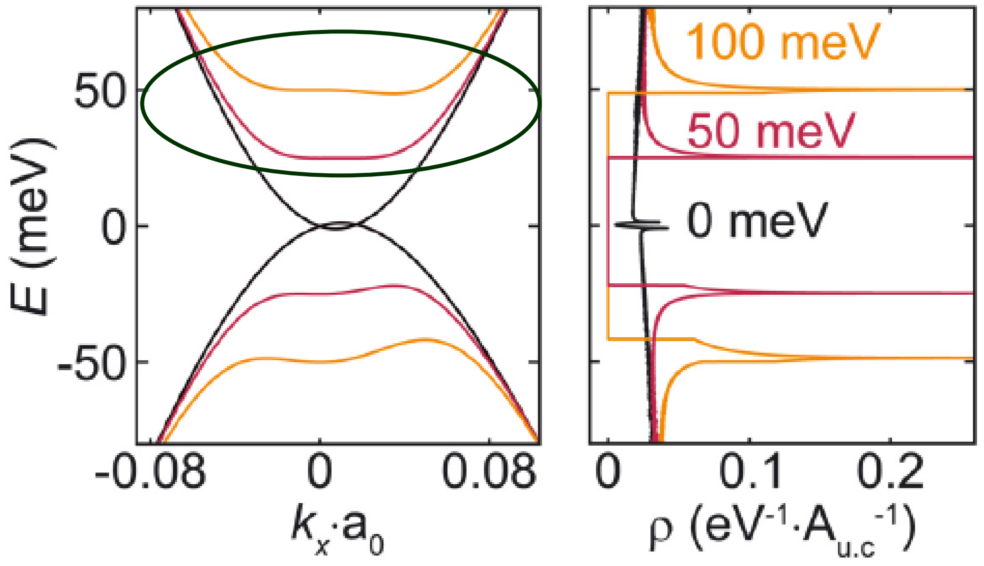
For all practical purposes, for exponent α between 1 and 2, the transition is 1st order into full polarization (half-metal), yet the susceptibility diverges at c -> 1





Why $k^{2\alpha}$ with α between 1 and 2 are relevant?

BBG



Neglect trigonal warping

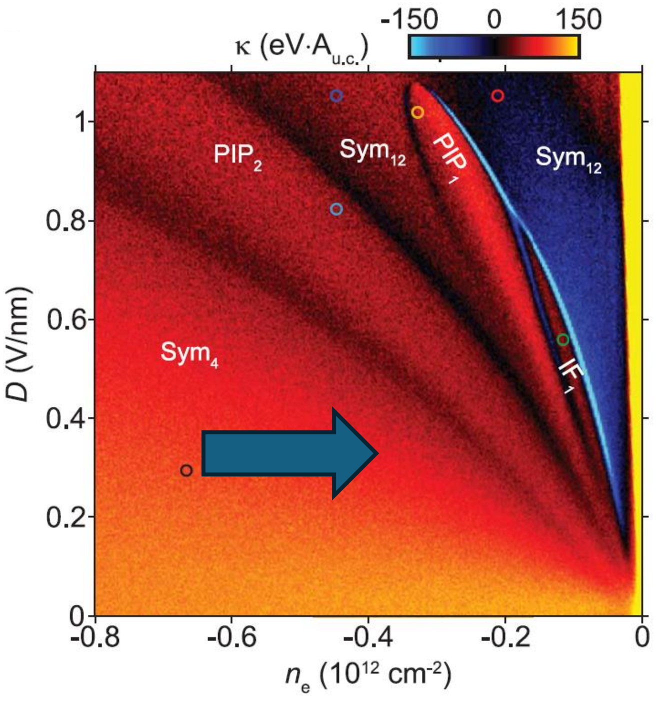
$$\epsilon \approx \sqrt{(k^2/2m)^2 + D^2}$$

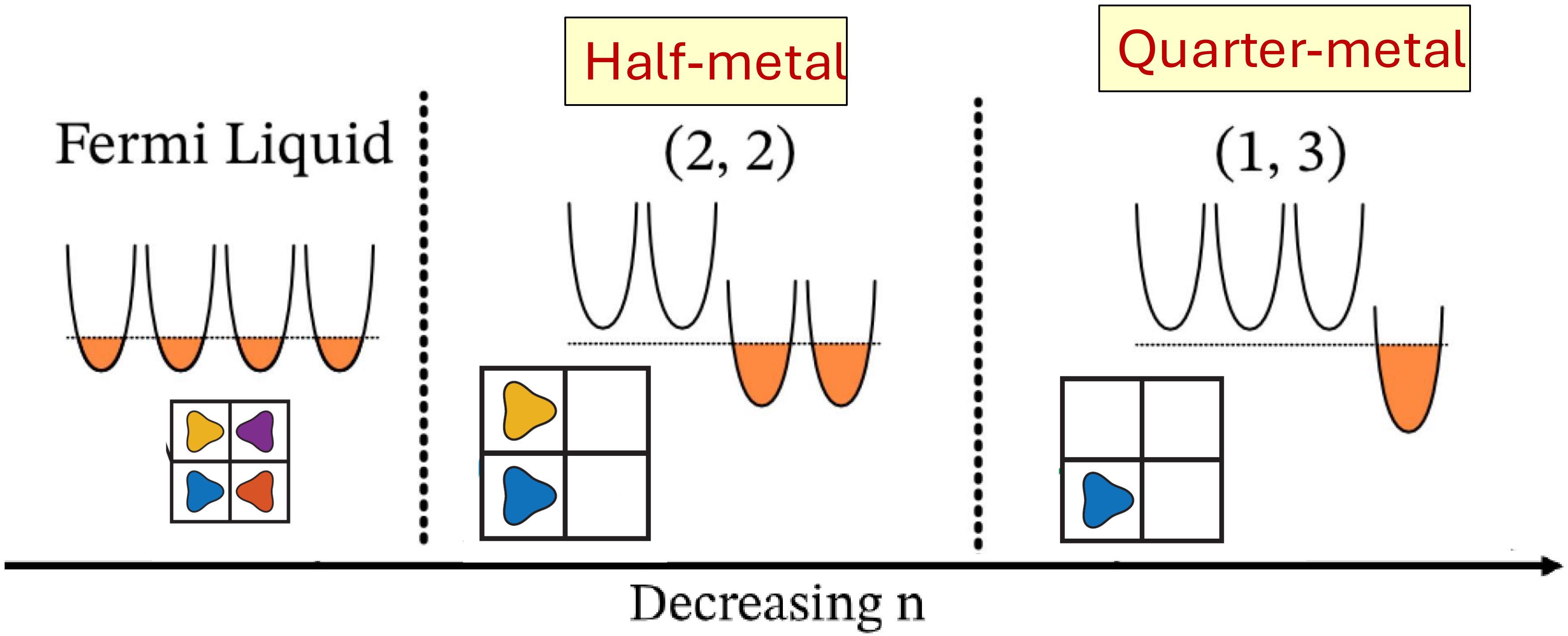
Can be interpreted as $\varepsilon(k) \sim k^{2\alpha}$

α interpolates between 1 and 2

A two-valley system (still Hartree-Fock)

One of iso-spin orders develops first

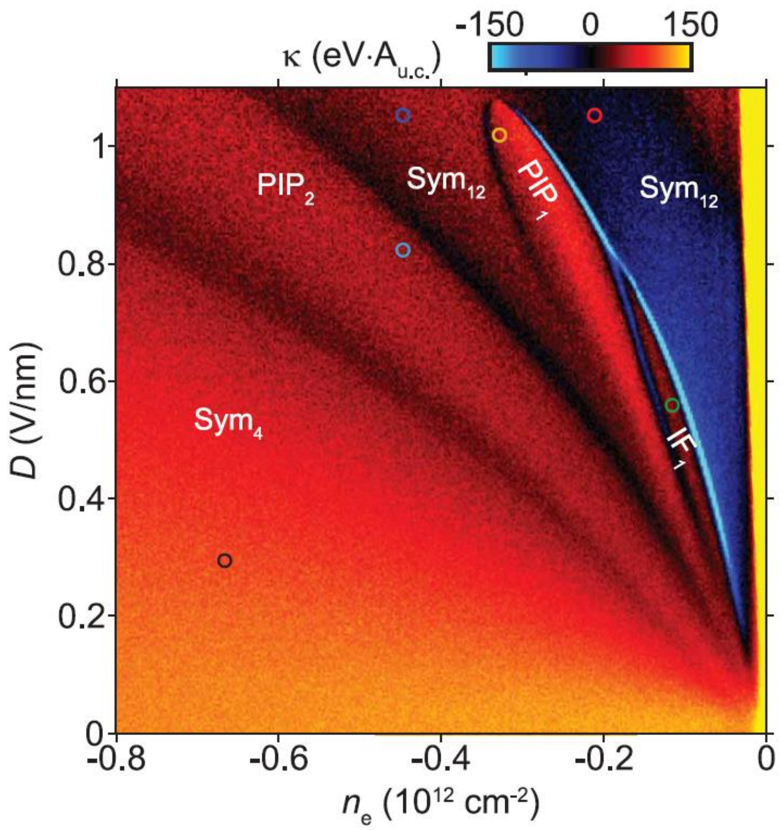




An instant transition to a half-metal followed by an instant transition to a quarter-metal

Theory: instant transitions into fully polarized states,
+ the divergence of the corresponding susceptibility

Experiments on BBG/RTG: transitions into fully polarized states

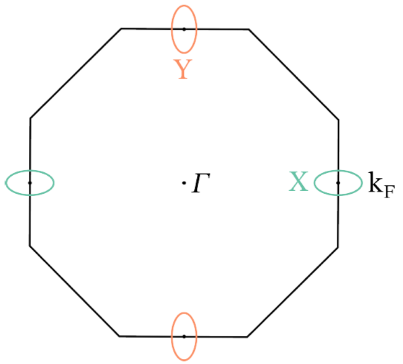


Dark lines: compressibility diverges
A cascade of transitions, like in TBG
A. Yazdani, S. Ilani, S. Nadj-Perge

No direct evidence for divergent susceptibility

Another spin-valley system: an ultra-clean AlAs quantum well (a 2D electron system)

Ultra-clean AlAs quantum well

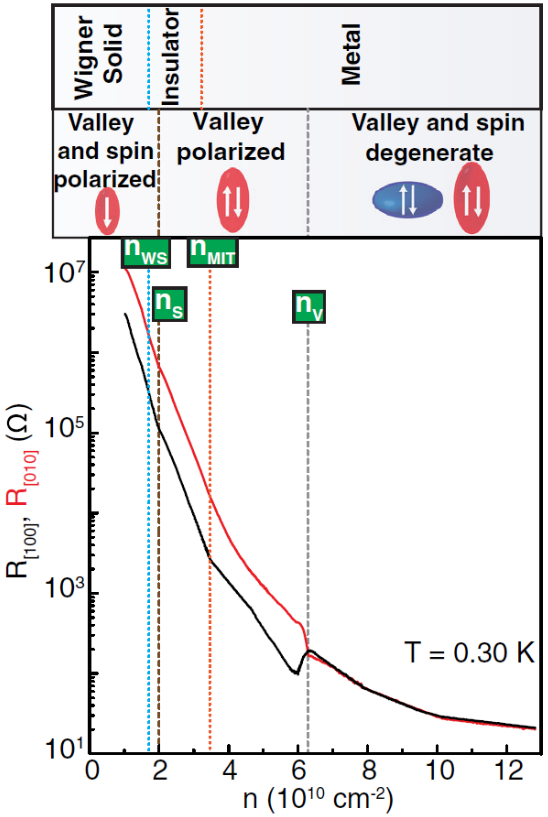


$$\epsilon_{\mathbf{k},\tau} = \frac{\eta^\tau k_x^2 + \eta^{-\tau} k_y^2}{2m} - \mu_0$$

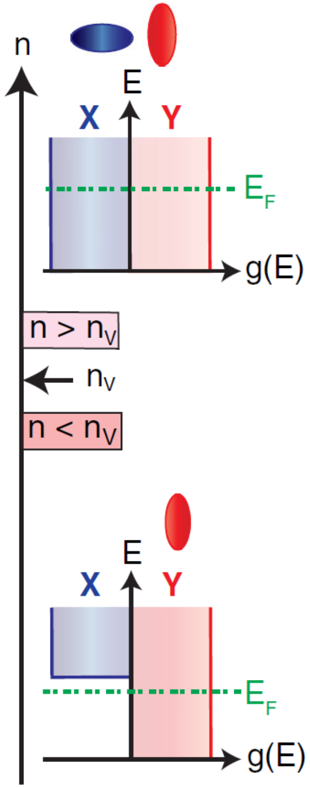
$\tau = +$ and $\tau = -$ valleys

Hossain et al, 2020, 2021, 2022
M. Shayegan's group

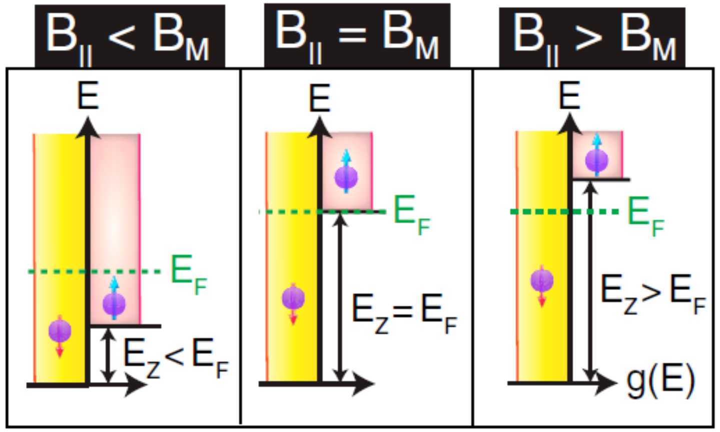
Valley/spin polarization

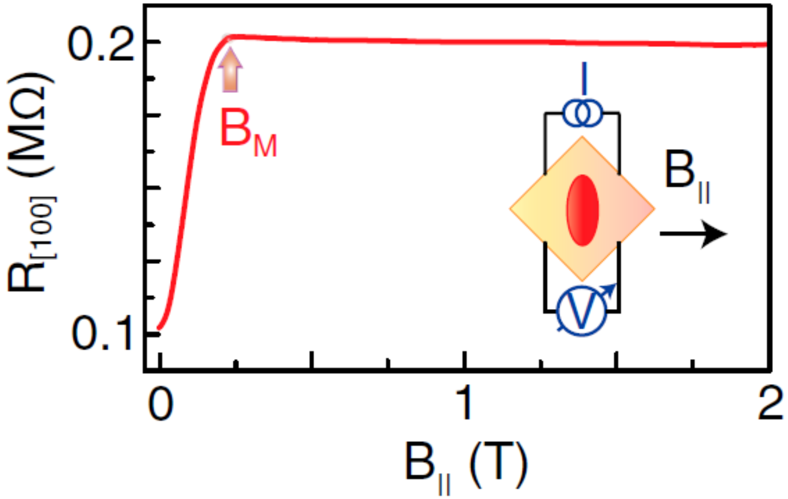


Valley polarization half-metal

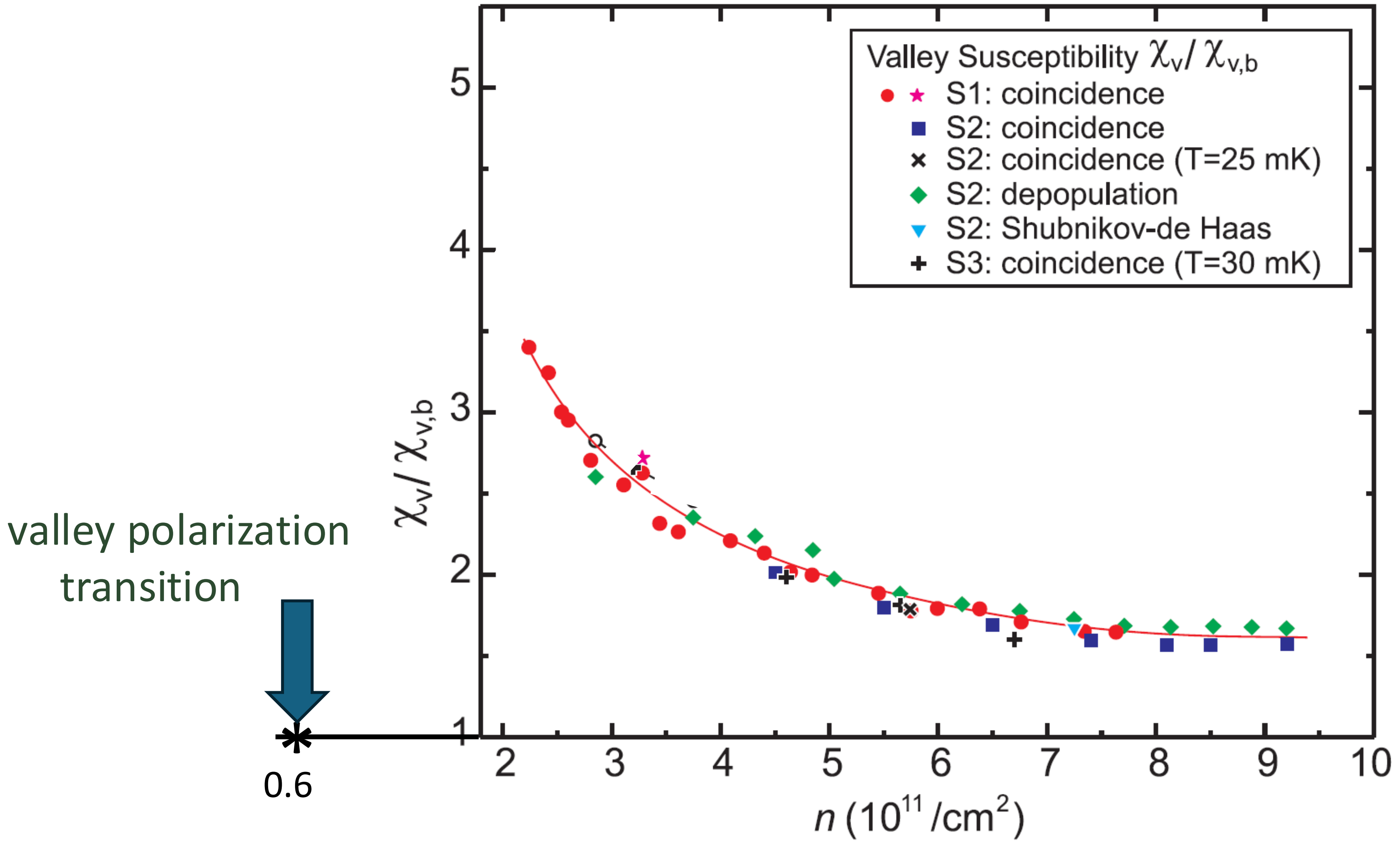


Valley and spin polarization quarter-metal





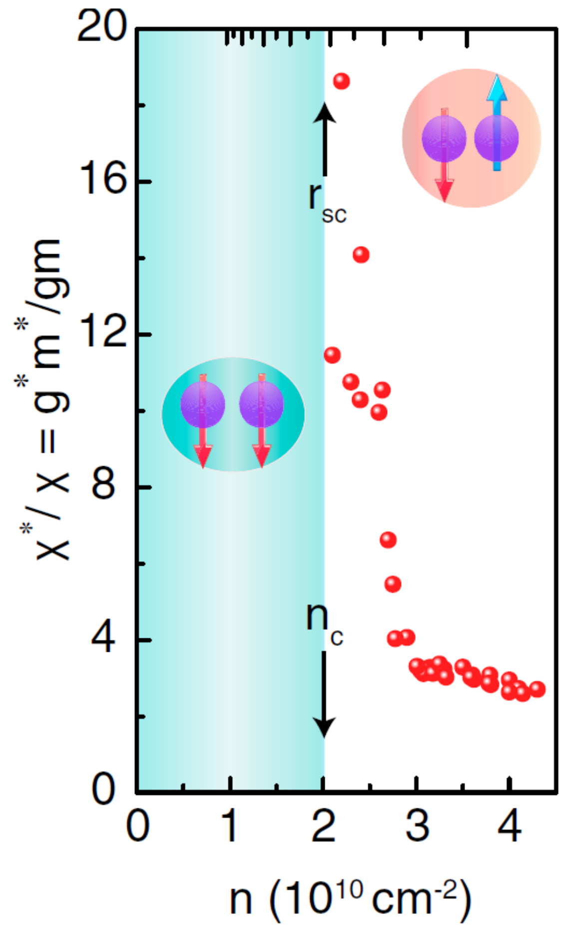
Valley susceptibility



Gunawan et al, 2006
(Shayegan group)

Spin susceptibility

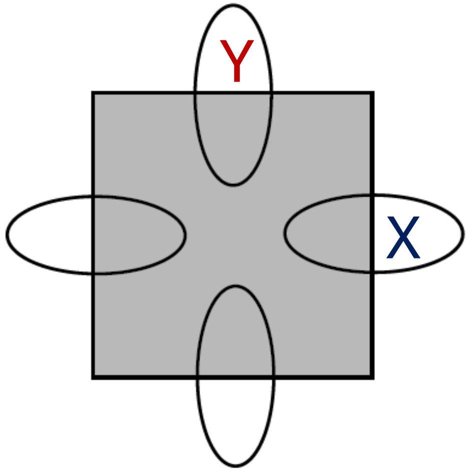
Magnetic susceptibility
diverges at the FM transition



Hossain et al, 2020 (Shayegan group)

Theory beyond Hartree-Fock

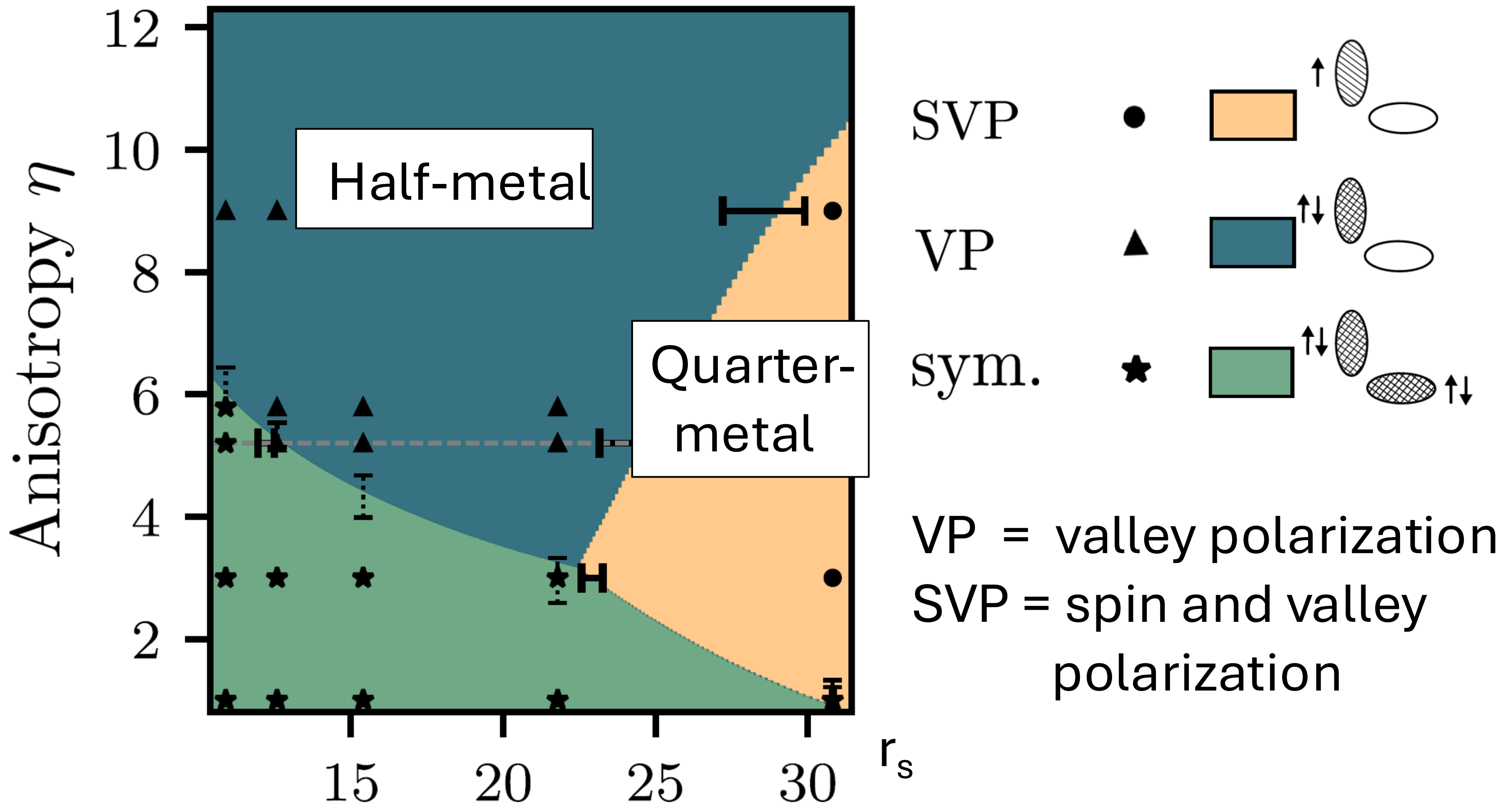
Variational QMC



$$\epsilon_X = \frac{k_x^2}{2m}\eta + \frac{k_y^2}{2m}\frac{1}{\eta}$$

$$\epsilon_Y = \frac{k_x^2}{2m}\frac{1}{\eta} + \frac{k_y^2}{2m}\eta$$

+ Coulomb or Hubbard



VP = valley polarization

SVP = spin and valley polarization

A. Valenti et al

Matches Hartree-Fock (Raines and A.C.)

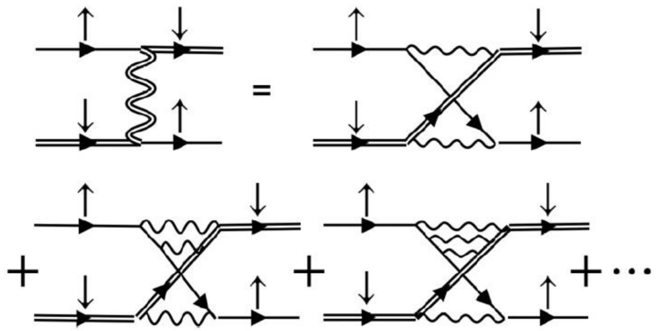
Superconductivity

Kohn-Luttinger type analysis shows that there is an attractive interaction in at least one pairing channel near each isospin order, either with $q=0$ (spin/valley ferromagnetism), or with a finite $Q = K\text{-}K'$ (IVC/spin IVC)

Guinea et al, Cea et al, Dong et al, Kozii et al, Chatterjee et al, You and Vishwanath, Berg et al, MacDonald et al, Gonzales et al, Classen et al Thomale et al, Polini et al

I will only consider superconductivity near a Q=0 magnetic order, due to an exchange of ferromagnetic fluctuations

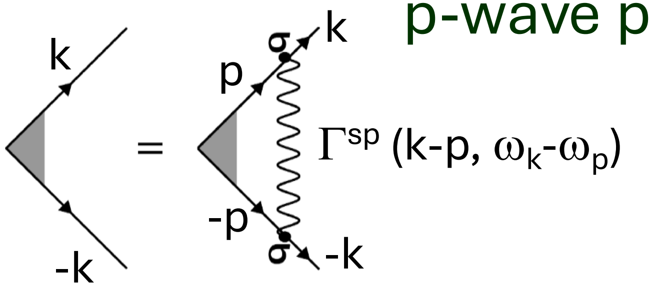
This is not orthogonal to Kohn-Luttinger scenario: an exchange of magnetic fluctuations is an extension of this scenario to higher orders



Let’s start with one-valley system (a single band)

I. Paramagnetic phase (c<1)

p-wave pairing mediated by ferromagnetic spin fluctuations



$$\Gamma^{sp}(q,\Omega_m) = -\frac{U}{2(1-U\Pi(q,\Omega_m))}$$

$q = k - p$, $\Omega_m = \omega_k - \omega_p$

Fay, Appel, Berk, Schrieffer; Monthoux, Lonzarich; Millis, Z. Wang, Bedell; Klein, Maslov, Abanov, A.C.; Berg, Fernandes…

For a parabolic dispersion

$$\Pi(q,\Omega_m) = \nu\left(1 - \frac{2k_F}{q}\,\mathrm{Re}\sqrt{\left(\frac{q}{2k_F}+\frac{i\Omega_m}{v_F q}\right)^2 - 1}\right)$$

Small Ω, finite q

$$\Gamma^{sp}(q,\Omega_m) = -\frac{U}{2\left(1-c+c\frac{|\Omega_m|}{v_F q\sqrt{1-\left(\frac{q}{2k_F}\right)^2}}\right)}$$

For a non-parabolic dispersion, p-wave Tc is non-zero, but numerically quite small

No odd-momentum components, no superconductivity

II. Ferromagnetic phase (c>1)

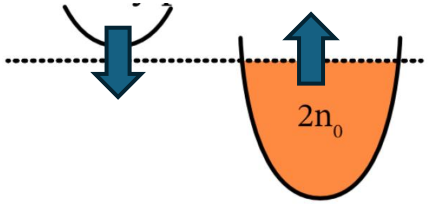
A fully saturated ferromagnetic order:

- No longitudinal excitations
- Goldstone transverse excitations are present

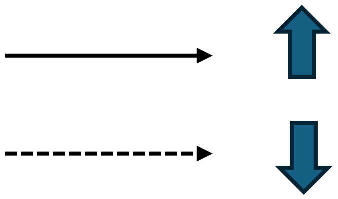
Pairing interaction between fermions must be mediated by a Goldstone boson

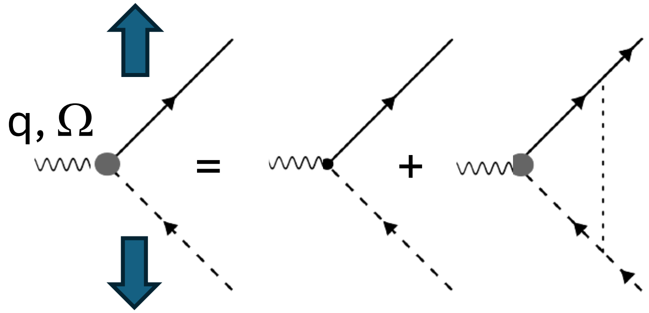
Half-metal

$c = U\,N_F > 1$



Magnon propagators

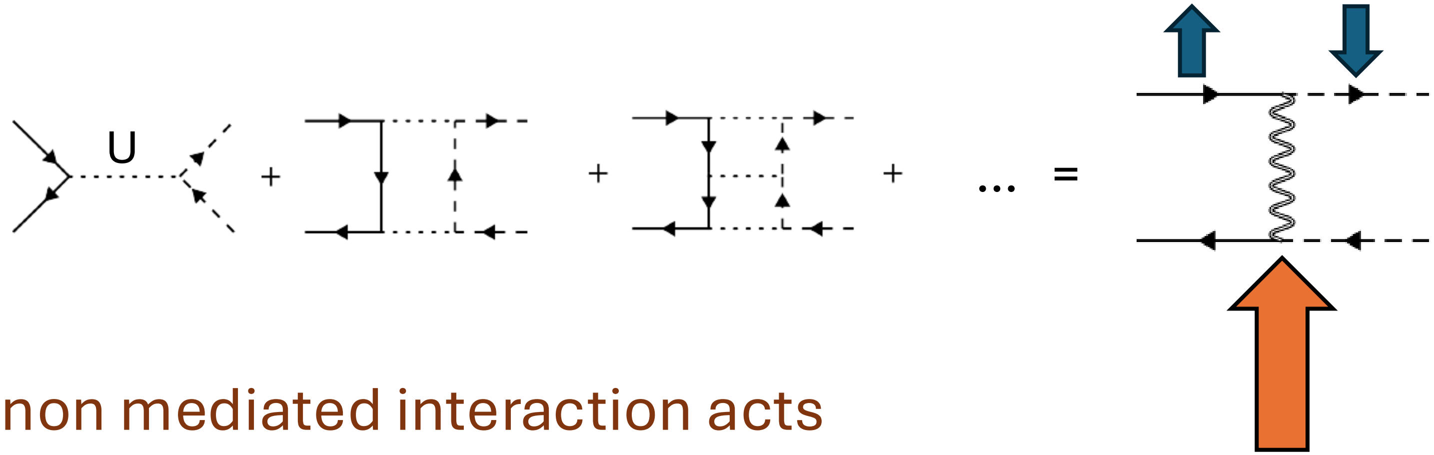




$$\chi_{\uparrow\downarrow}(q,\Omega_m) = \frac{2\mu_0 c}{i\Omega_m + \frac{q^2}{2m}\frac{c-1}{c}}$$

$$\chi_{\downarrow\uparrow}(q,\Omega_m) = \frac{2\mu_0 c}{-i\Omega_m + \frac{q^2}{2m}\frac{c-1}{c}}.$$

We can also derive magnon-mediated interaction between fermions

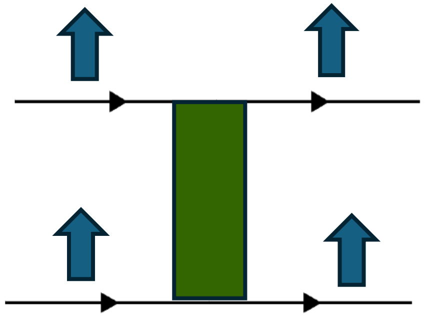


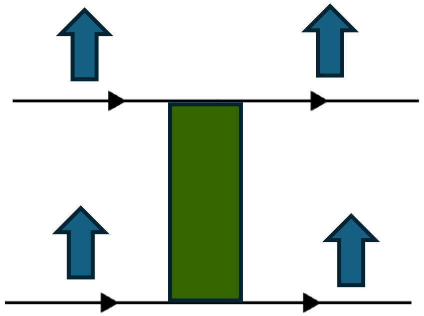
Single- magnon mediated interaction acts between spin-up and spin-down fermions

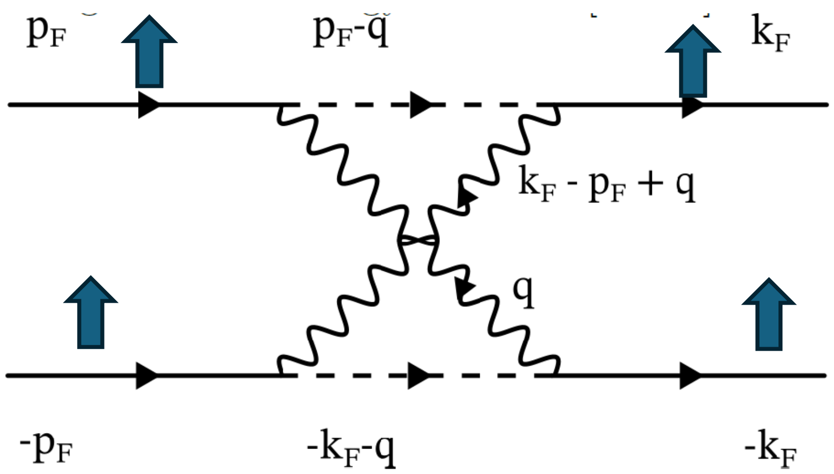
$$\Gamma_1(q, \Omega_m) = U\chi_{\uparrow\downarrow}(q, \Omega_m) = U\frac{2\mu_0 c}{i\Omega_m + \frac{q^2}{2m}\frac{c-1}{c}}$$

Z. Dong, J. Alicea, E. Lantagne-Hurtubise

For superconductivity, we need a different beast: pairing interaction between low-energy fermions. They all have spin-up

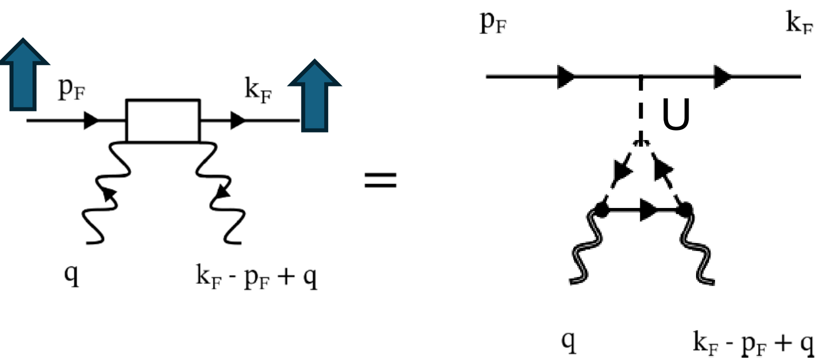


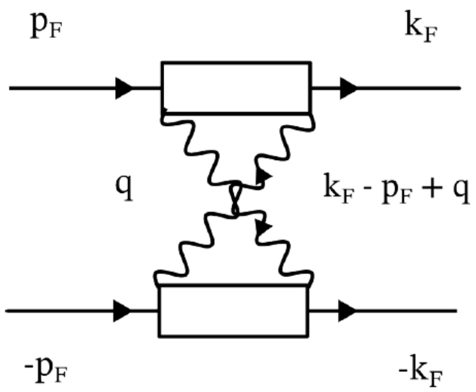


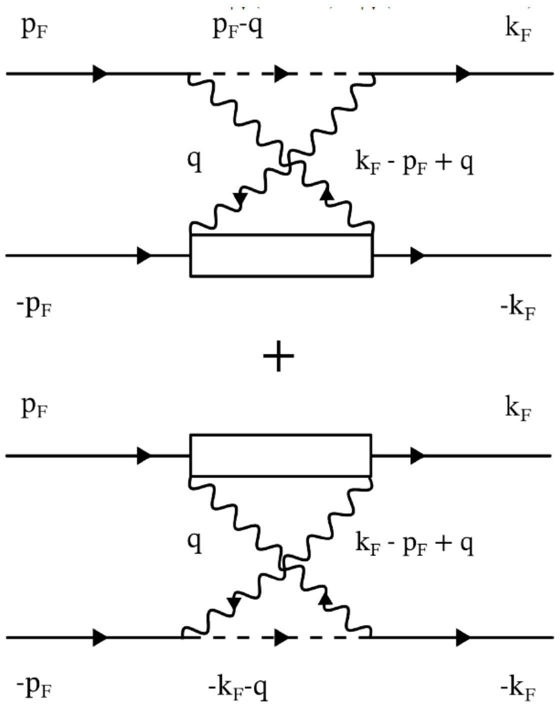


An effective two-magnon scattering process: a convolution of the two Goldstone modes

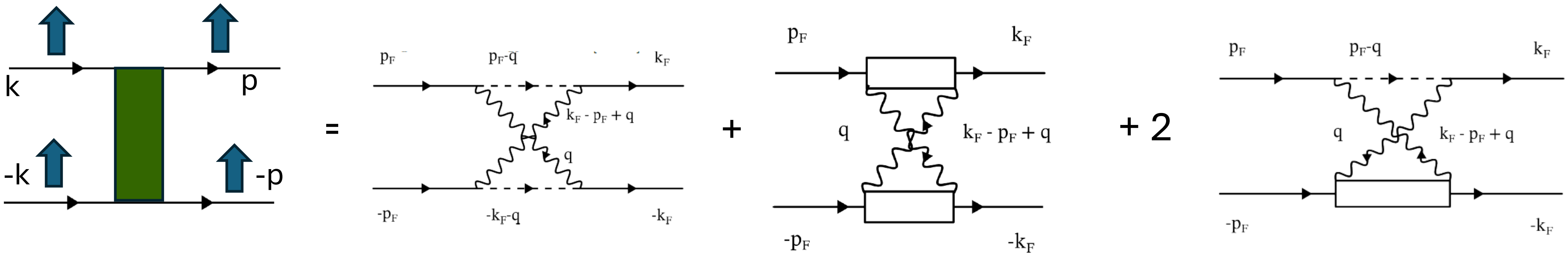
There also exists a direct two-magnon scattering







The full magnon-mediated pairing interaction is the sum of two single-magnon and one two-magnon scatterings



$$\Gamma_{2,\mathrm{tot}}(\boldsymbol{\delta}\mathbf{k}) = -U^2 \int \frac{d^2 q}{4\pi^2} \frac{d\Omega_m}{2\pi} S\left[\mathbf{q}, \Omega_m, \boldsymbol{\delta}\mathbf{k}\right] \times \chi_{\uparrow\downarrow}(q, \Omega_m)\chi_{\uparrow\downarrow}(\mathbf{q} + \boldsymbol{\delta}\mathbf{k}, \Omega_m)$$

δk = k-p

S[0,0,0] =0 Adler principle is satisfied. Interaction with low-energy fermions does not destroy a Goldstone boson

H. Watanabe and A. Vishwanath

However, frequency integral vanishes

For spin SU(2)-invariant system, there is no magnon-mediated superconductivity

We added a small easy-plane magnetic anisotropy, $\Omega_0 << E_F$

$$\chi_{\uparrow\downarrow}(q,\Omega_m) = \frac{2\mu_0 c}{i\Omega_m + \frac{q^2}{2m}\frac{c-1}{c}} \qquad \chi_{\uparrow\downarrow}(q,\Omega_m) = \frac{2\mu_0 c}{B(i\Omega_m) + \frac{q^2}{2m}\frac{c-1}{c}}$$

$$B(i\Omega_m) = \begin{cases} i\,\Omega_m, & \Omega_m > \Omega_0 \\ (\Omega_m)^2/\Omega_0, & \Omega_m < \Omega_0 \end{cases}$$

Pairing interaction is non-zero and attractive in p-wave channel

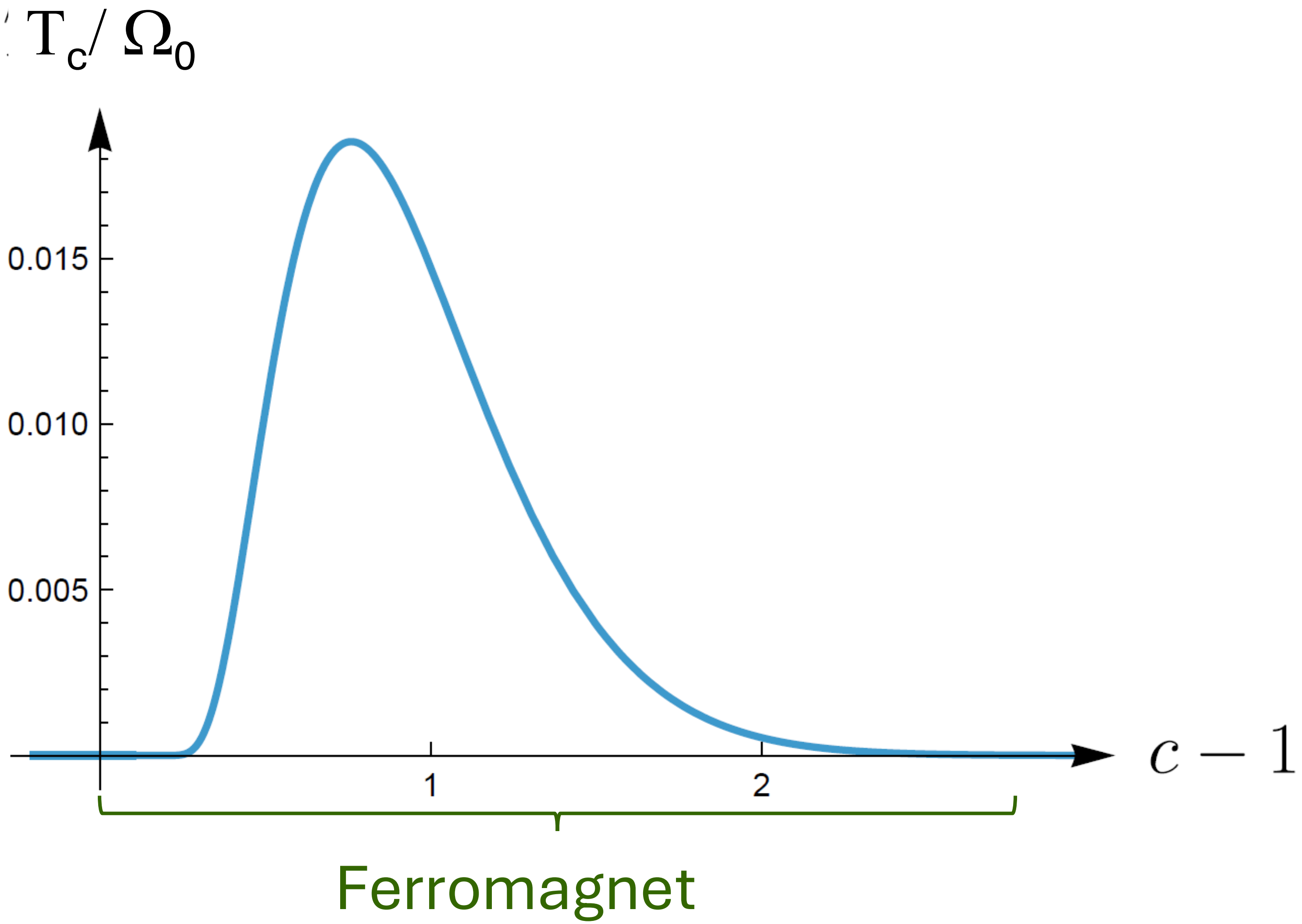
$Tc \sim \Omega_0\, e^{-1/\lambda p}$

$$\lambda_p = \frac{c}{c-1} F\left((c-1)\frac{E_F}{4c\Omega_0}\right), \qquad F(z) \approx 0.01/z^{3/2}$$

Deep inside FM phase, p-wave coupling is small in $(\Omega_0/E_F)^{3/2}$
In practice, this means no superconductivity

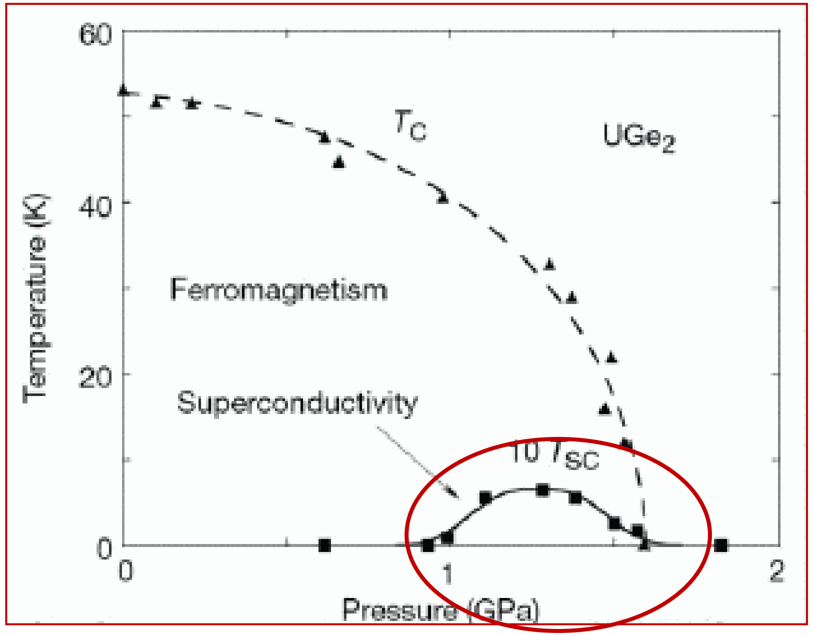
However, near the onset of FM, $(c-1<<1)$, λ_p reaches 1 at $c-1 \approx 0.38(\Omega_0/E_F)^{3/5}$

Key message: superconducting Tc is induced by a magnetic anisotropy and is the largest inside the ferromagnetic phase near its onset



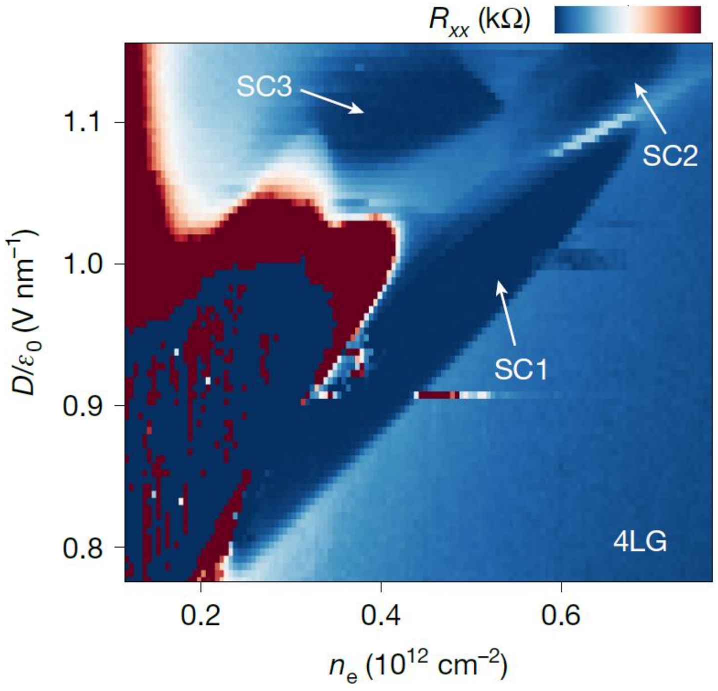
Experiment (one valley)

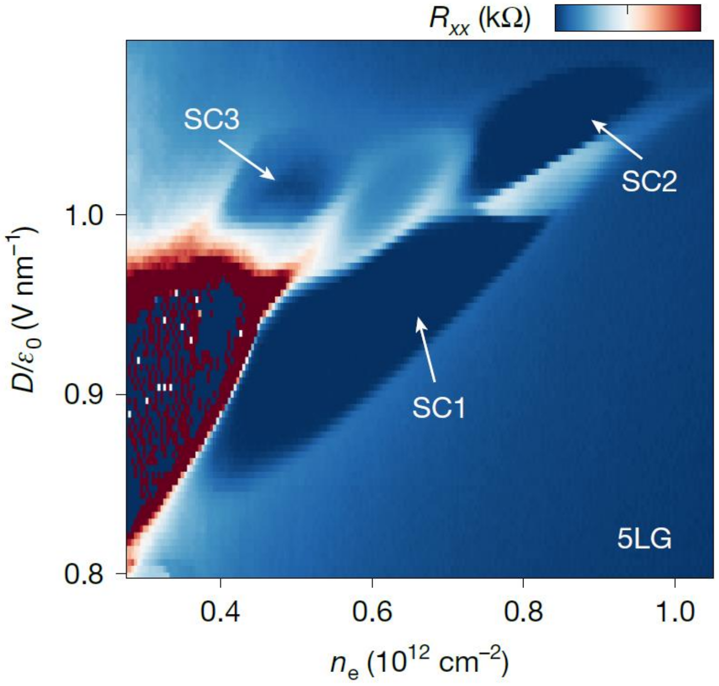
A heavy fermion superconductor



S.S. Saxena et al

A quarter-metal is also a single valley ferromagnet



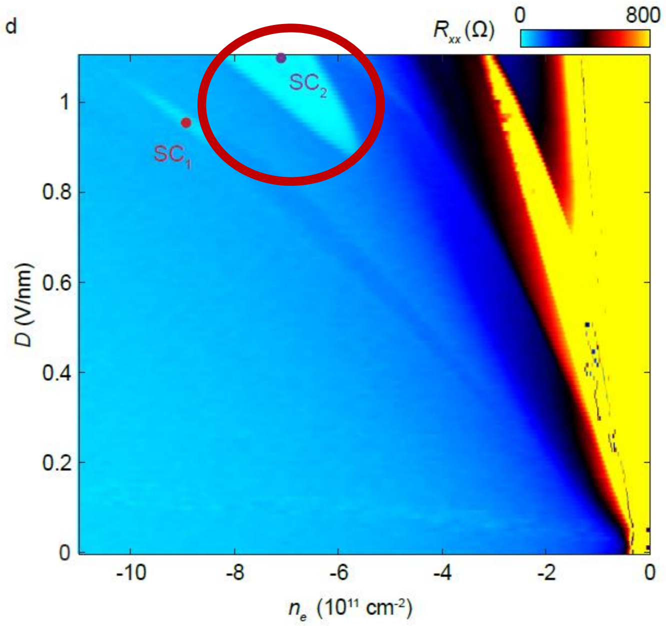


T. Han, L. Ju et al

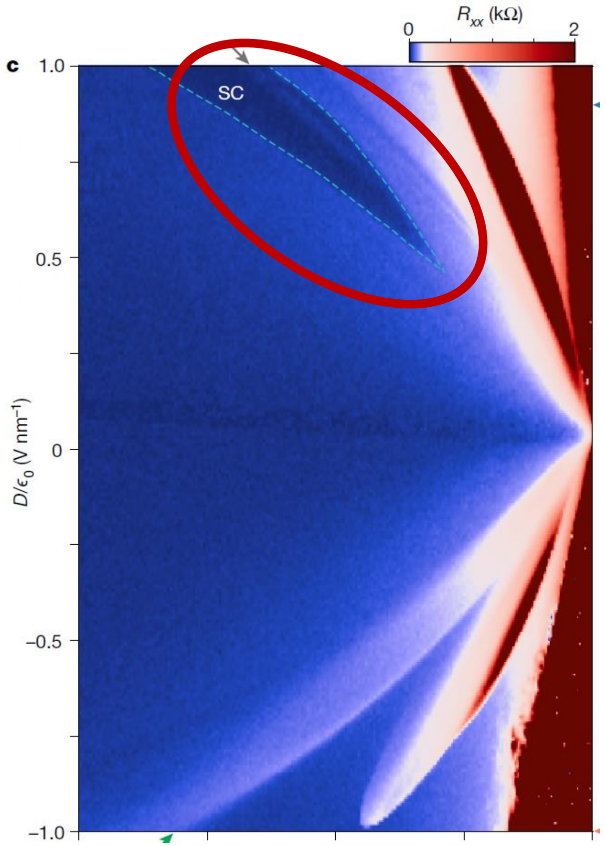
Valley order: T-reversal symmetry breaking, p+ip SC

A. Kapitulnik

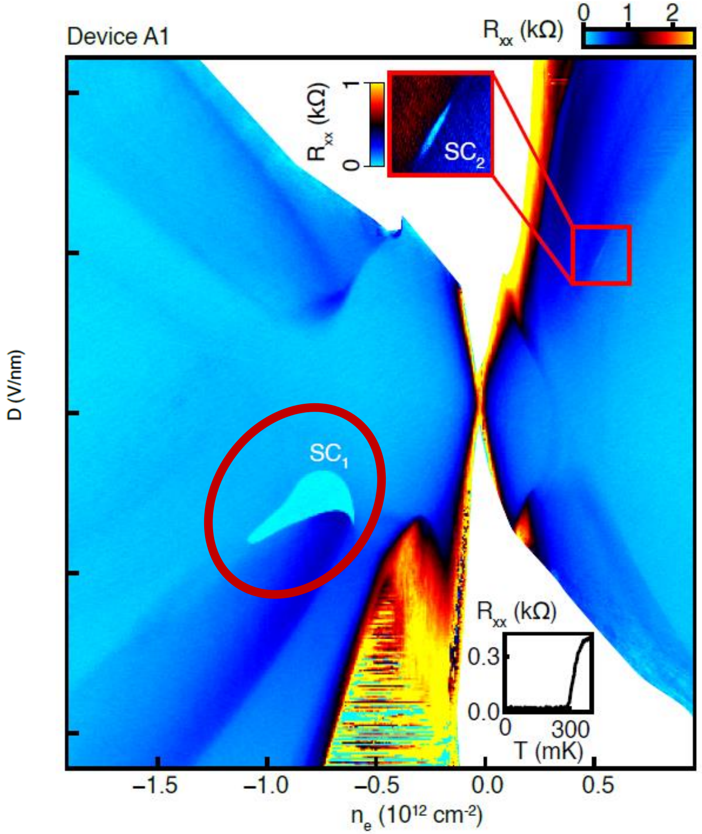
Two valley ferromagnetic half-metal



Y. Zhang et al



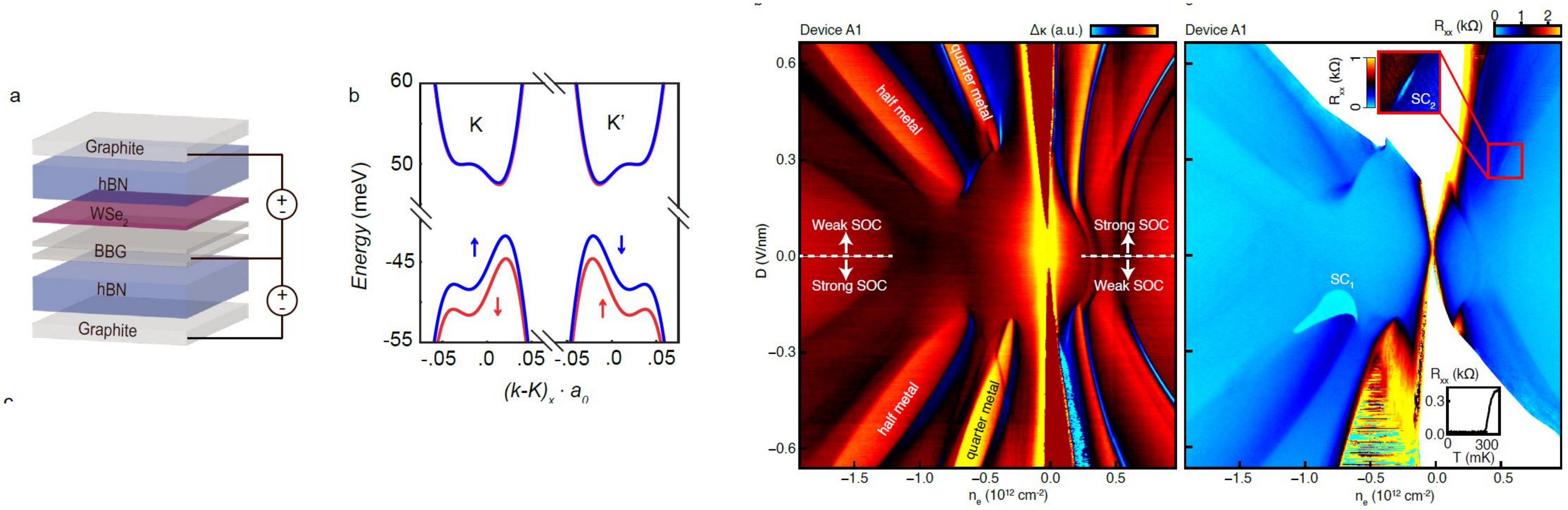
Holleis et al



Patterson et al

Systems that display SC are placed near WSe_2, which induces Ising spin-orbit coupling (an effective magnetic field of different sign in the two valleys)

Levitov et al, Nadj-Perge et al



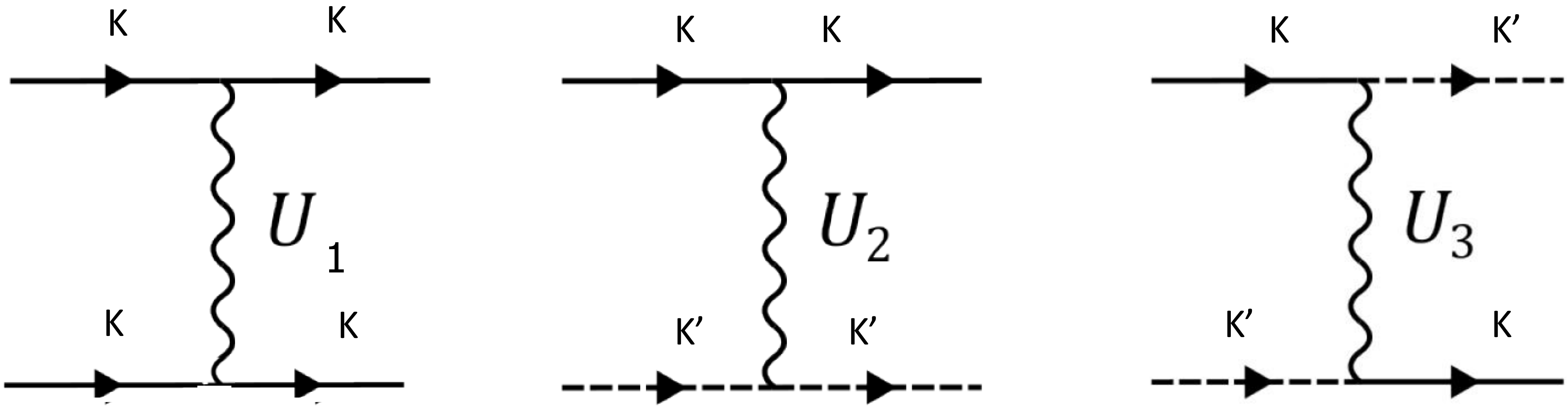
- Superconductivity only in the regions where SOC is strong
- Tc increases when SOC increases

Superconductivity may be induced by SOC

Patterson et al

The same model of electrons with a parabolic dispersion, but with two valleys + Ising SOC

Three interactions instead of one



intra-valley density-density interaction

inter-valley density-density interaction

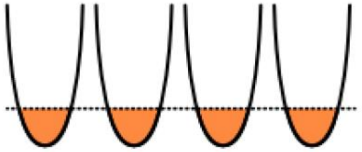
inter-valley scattering

$U_1, U_2 \gg U_3$

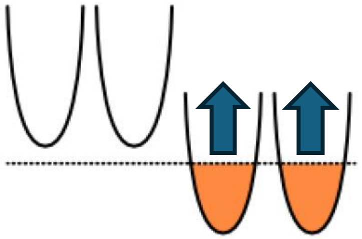
Ferromagnetism in presence of Ising SOC:

$c = (U_1 + U_3)\, N_F$ FM order for $c>1$

Fermi liquid

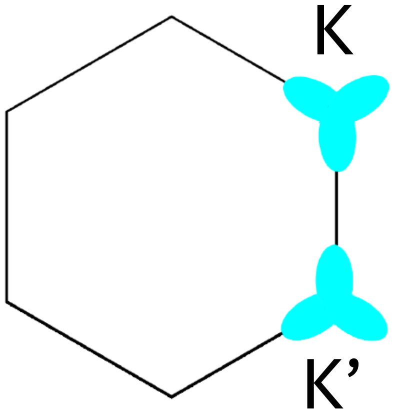


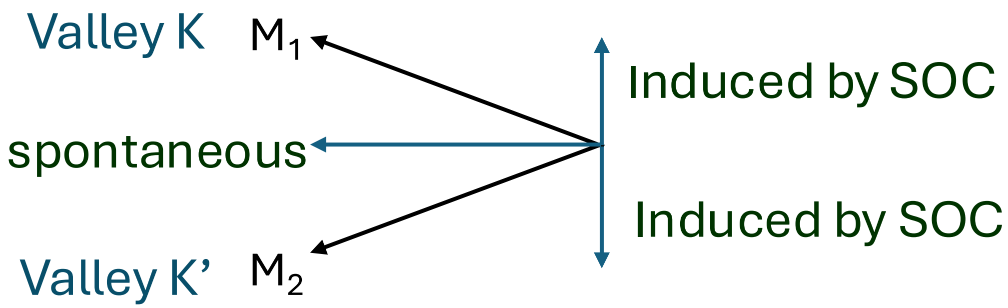
Ferromagnet



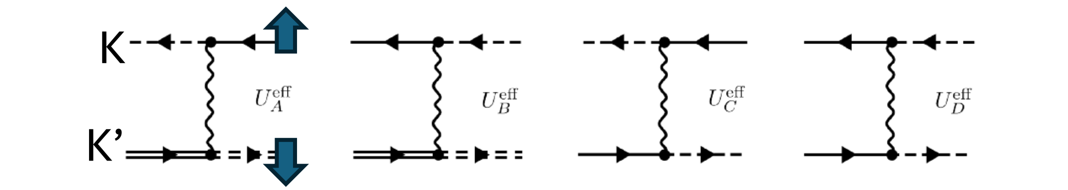
Still, full magnetization (half-metal)

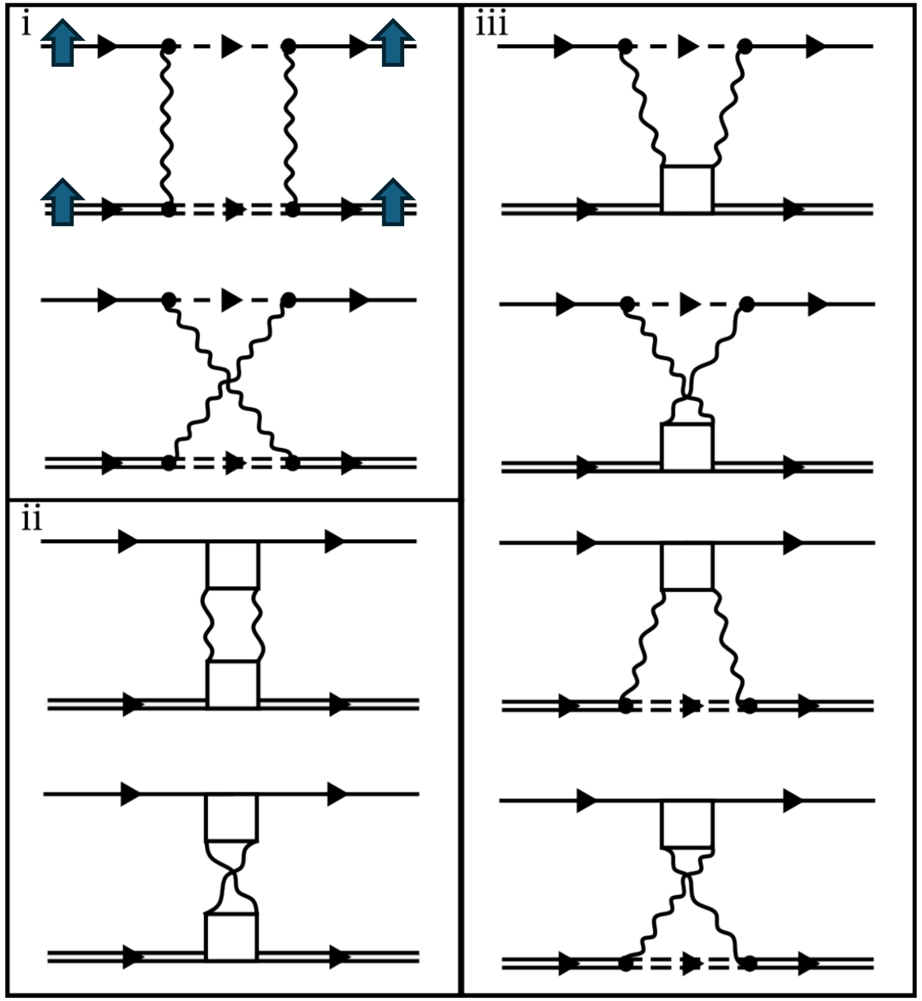
In the presence of SOC, an ordered state is a canted ferromagnet:





The same analysis as before, but for 2 valleys + SOC
(a bit more cumbersome)





Theory kitchen

In the presence of SOC, the order parameter manifold is O(2)

Spin-wave spectrum:

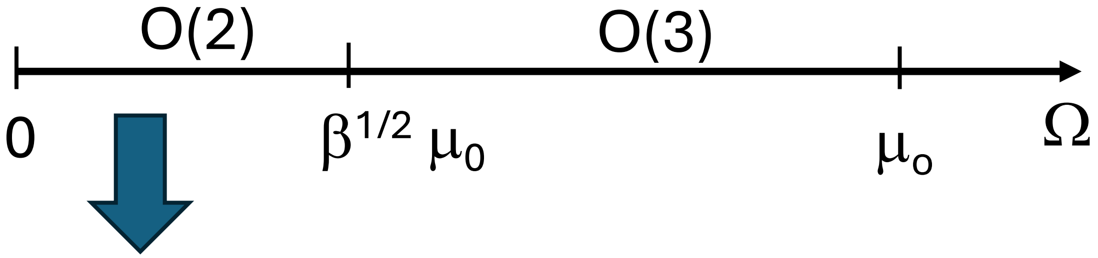
$$\Omega^2 = 2\beta\mu_0(c-1)\frac{q^2}{2m} + \left(\frac{c-1}{c}\right)^2 \frac{q^4}{4m^2}$$

Dispersion is linear in q at the smallest q, becomes quadratic in q at larger q

$$\beta = 2\frac{U_3}{(U_1+U_3)}\left(\frac{\lambda}{2U_3 N_F \mu_0}\right)^2$$

λ is SOC

β is a small parameter in the theory



$\mu_o = E_F$ in a paramagnet

There exists an attractive pairing interaction in valley-odd, spatially-even channel, which is confined to the O(2) regime, induced by SOC

Formulas:

An effective pairing interaction at zero momentum transfer is

$$\Gamma^{sc}_{2mag}(0) = \int \frac{d^2q d\Omega_m}{(2\pi)^3} A(q,\Omega_m)\chi^2_\perp(q,\Omega_m)$$

$$\chi_\perp(q,\Omega) = \frac{4\mu_0^2 c^2}{\beta\mu_0(c-1)q^2/m + \Omega_m^2}$$

$$\beta = 2\frac{U_3}{(U_1+U_3)}\left(\frac{\lambda}{2U_3 N_F \mu_0}\right)^2$$

$$A(q,\Omega_m) = -\frac{U_3^2}{(2\mu_0^2c^2)^3}\left(\frac{\lambda}{2U_3N_F\mu_0}\right)^4 \bar{A}(q,\Omega_m)$$

$$\bar{A}(q,\Omega_m) = \Omega_m^4 + \frac{q^2\mu_0}{m}\Omega_m^2(c+1) + \left(\frac{q^2\mu_0}{2m}\right)^2 (c-1)^2$$

Adler principle at work

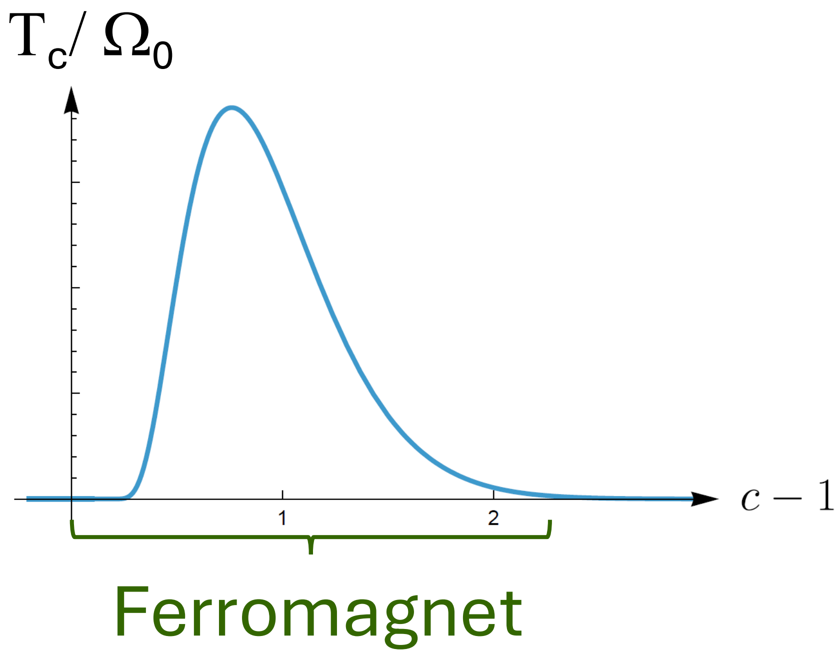
$$\lambda^{sc}_{2mag} = a\frac{c^4}{(c-1)^{3/2}}\beta^{5/2}$$ a = O(1)

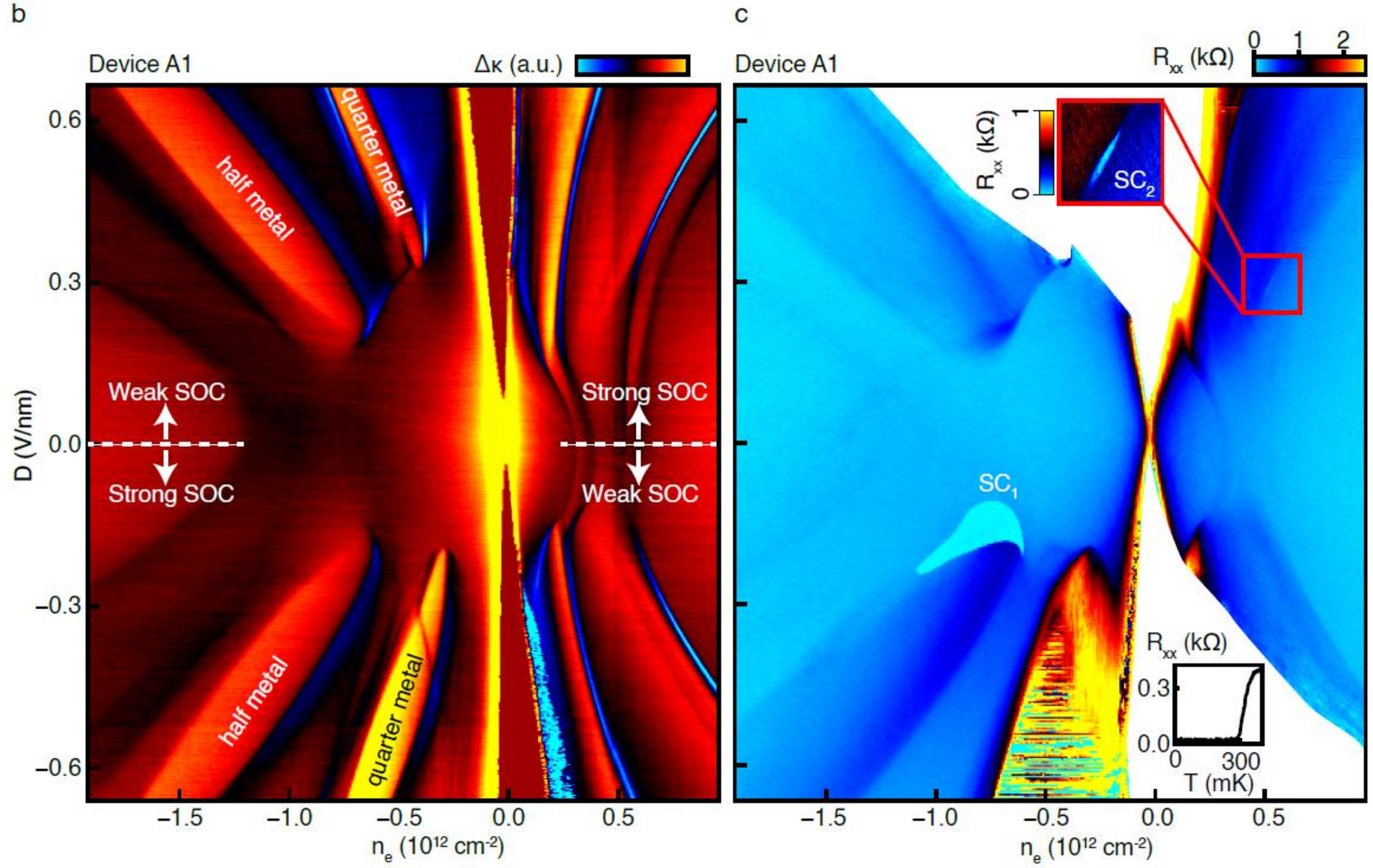
Exactly the same story as in the one-valley case:
the smallness of β is compensated by the smallness of c-1

Two-valley system

Key message: superconducting Tc is induced by Ising SOC and is the largest inside the ferromagnetic phase near its onset

SC$_1$ with the highest Tc ~ 300mK is near the onset of a canted FM





Conclusions:

- A highly unconventional Stoner transition to isospin ferromagnetism

  1^{st} order transition with divergent fluctuations

- A very weak superconductivity on the paramagnetic side

- Superconductivity in the ordered state is mediated by two-magnon exchange

  It does not develop in an SU(2) invariant system

  It is induced by a magnetic anisotropy (a single valley system) and Ising SOC (a two-valley system)

Near the onset of FM, the pairing coupling constant becomes O(1) even when anisotropy/SOC are weak

THANK YOU