Orbital ordering-induced unconventional magnetism

"SPICE-SPIN+X Seminar Series"
January 28th 2026, online

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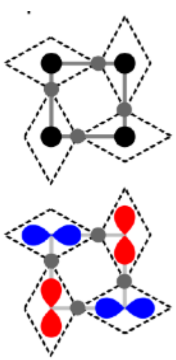
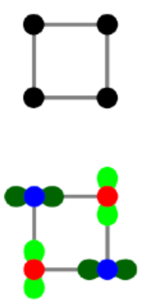
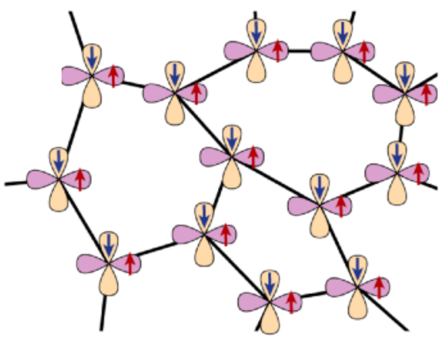
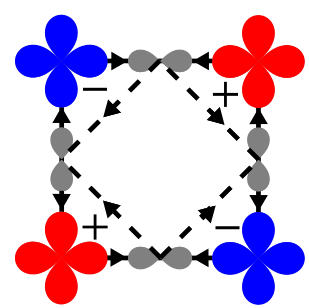
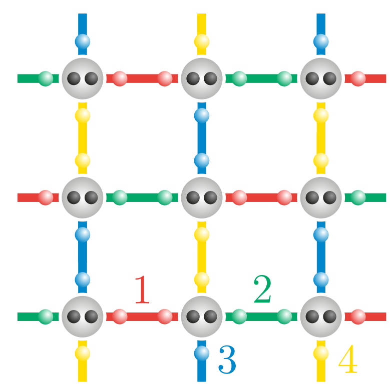
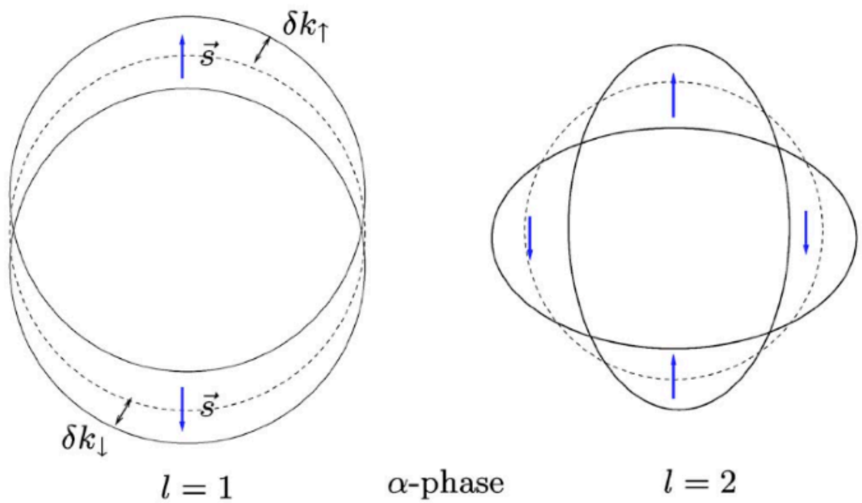
Technical University Munich
Imperial College London

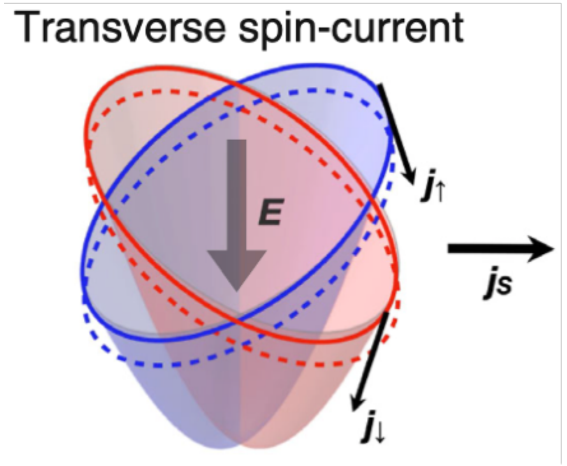






Intro: Altermagnetism

- **A form of d-wave magnetism and lattice generalisation**

  Pomeranchuk JETP (1958)
  Wu, Sun, Fradkin, Shou-Cheng Zhang PRB 2007
  Smejkal, Sinova, Jungwirth PRX 22

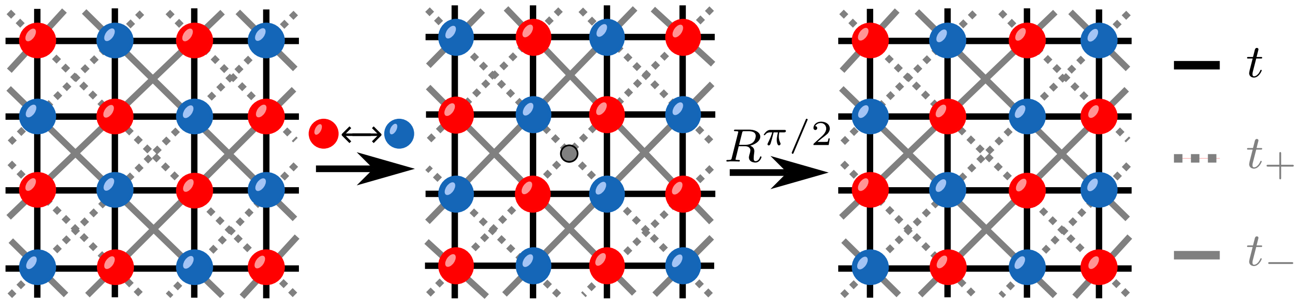




→ Interaction-induced as a spin-dependent Pomeranchuk instability

→ Unusual spin transport: **promising for spintronics and beyond…**

Intro: minimal Hubbard model

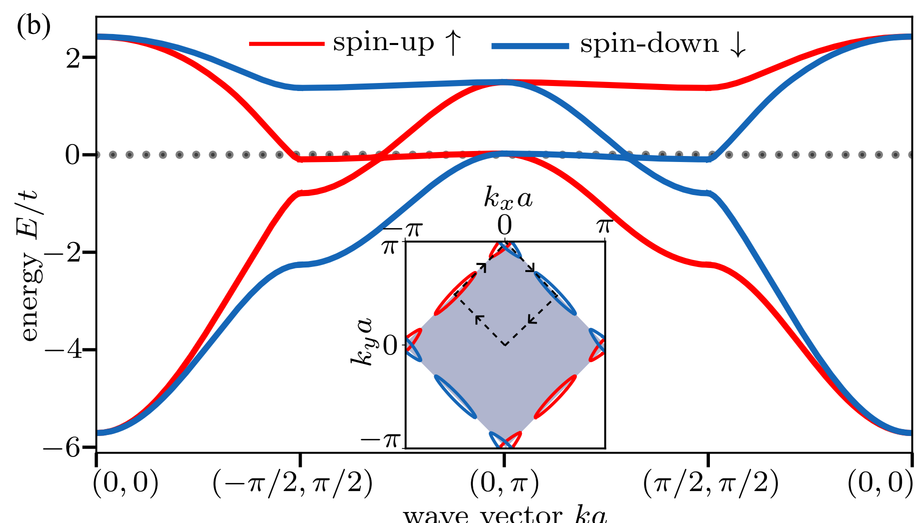


$$H = -t \sum_{\langle ij \rangle, s} c_{i,s}^{\dagger} c_{j,s} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow} - t' \sum_{\langle\langle ij \rangle\rangle, s} [1 + (-1)^i \delta] c_{i,s}^{\dagger} c_{j,s}$$

Imprint non-trivial rotation symmetry:

Anisotropic tight-binding Hamiltonian

- NNN hopping: $t'(1 + (-1)^i \delta)$
- Half filling: (π, π)-AFM



Das, Leeb, Knolle, Knap PRL 2024

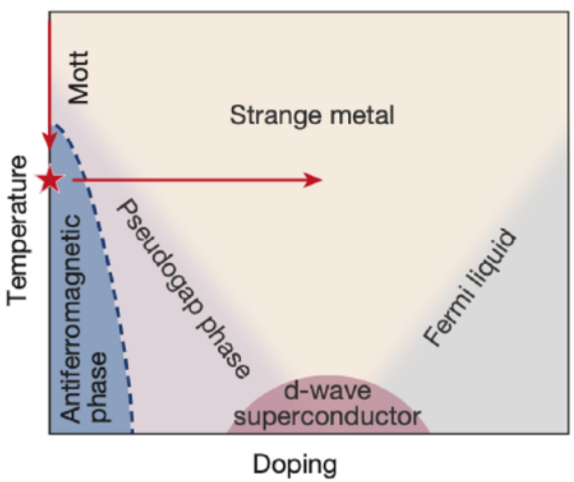
AM beyond magnetic materials

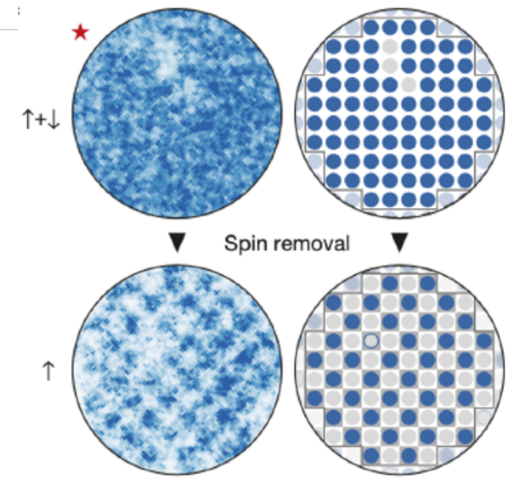
- **Cold atom Fermi-Hubbard quantum simulators**

A cold-atom Fermi–Hubbard antiferromagnet

Anton Mazurenko[1], Christie S. Chiu[1], Geoffrey Ji[1], Maxwell F. Parsons[1], Márton Kanász-Nagy[1], Richard Schmidt[1], Fabian Grusdt[1], Eugene Demler[1], Daniel Greif[1] & Markus Greiner[1]

Nature 2017



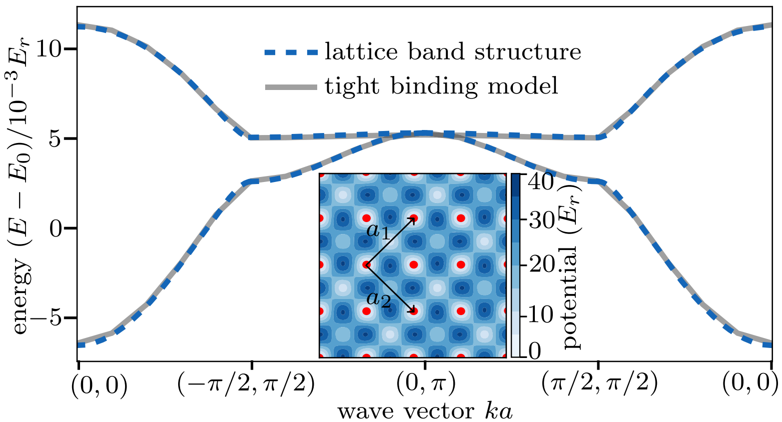


Greif, Uehlinger, Jotzu, Tarruell, Esslinger *Science* 2013

- **Direct real space detection of spin**
- **Longstanding challenge to reach low temperature (T/t~0.25)**

AM in cold atoms

- **Optical lattice with** additional 45$^{\text{deg}}$ rotated laser



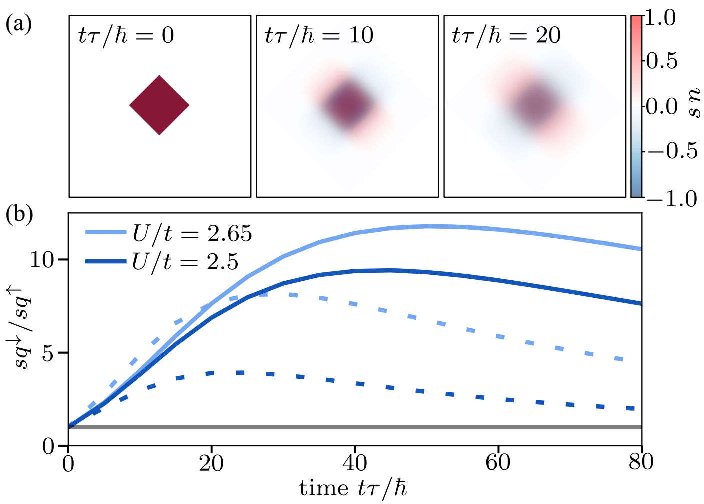
$$V_{\text{latt}} = E_r(V_{\text{sq}} + V_{d,1} + V_{d,2}),$$

$$V_{\text{sq}} = V_0[\sin^2(k_l x) + \sin^2(k_l y)],$$

$$V_{d,1} = V_1\left[\Delta_+ \sin^2[k_l(x+y)] + \Delta_- \sin^2\left[\frac{k_l}{2}(x+y)\right]\right]$$

$$V_{d,2} = V_1\left[\Delta_+ \sin^2[k_l(x-y)] + \Delta_- \cos^2\left[\frac{k_l}{2}(x-y)\right]\right]$$

- **Anisotropic diffusion - new probes of spin split transport**



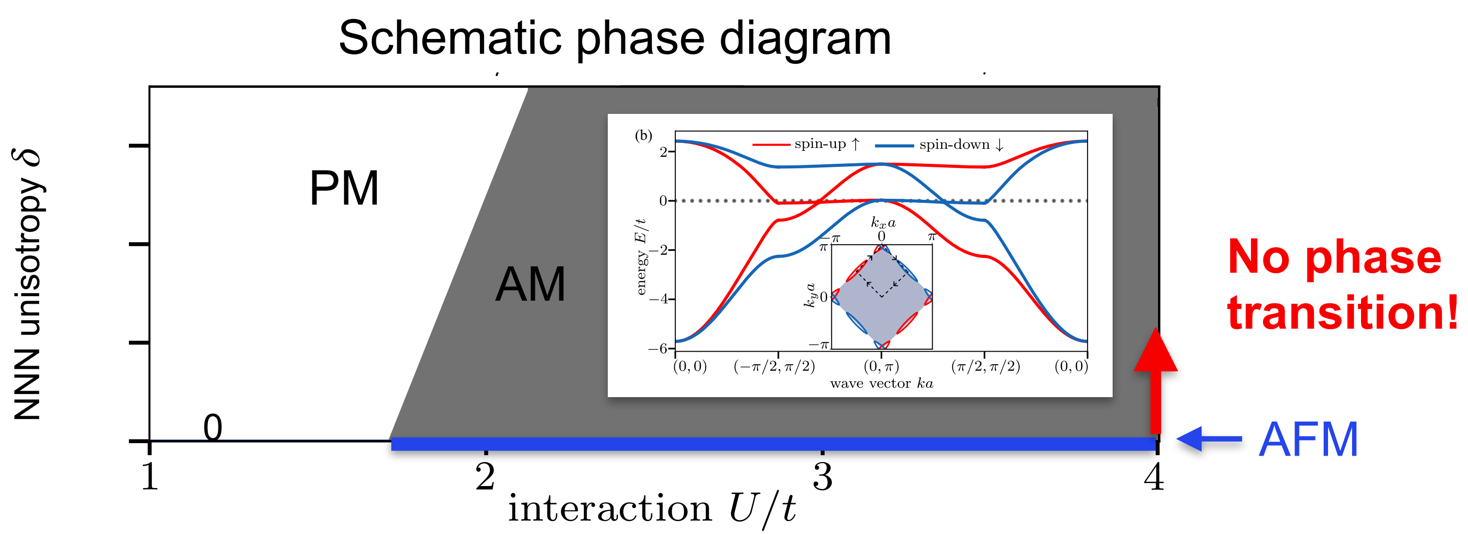
$$\frac{\partial n^s}{\partial \tau} = (D^s_{\tilde{x}\tilde{x}}\partial^2_{\tilde{x}} + D^s_{\tilde{y}\tilde{y}}\partial^2_{\tilde{y}})n^s$$

$$\sigma^s_{\alpha\beta} = \frac{n^s D^s_{\alpha\beta}}{T}$$

→ spin asymmetric diffusion

$$sq^s(\tau) = \frac{\int d^2\tilde{r}\,\tilde{x}^2\, n^s(\tilde{\boldsymbol{r}},t)}{\int d^2\tilde{r}\,\tilde{y}^2\, n^s(\tilde{\boldsymbol{r}},t)}$$

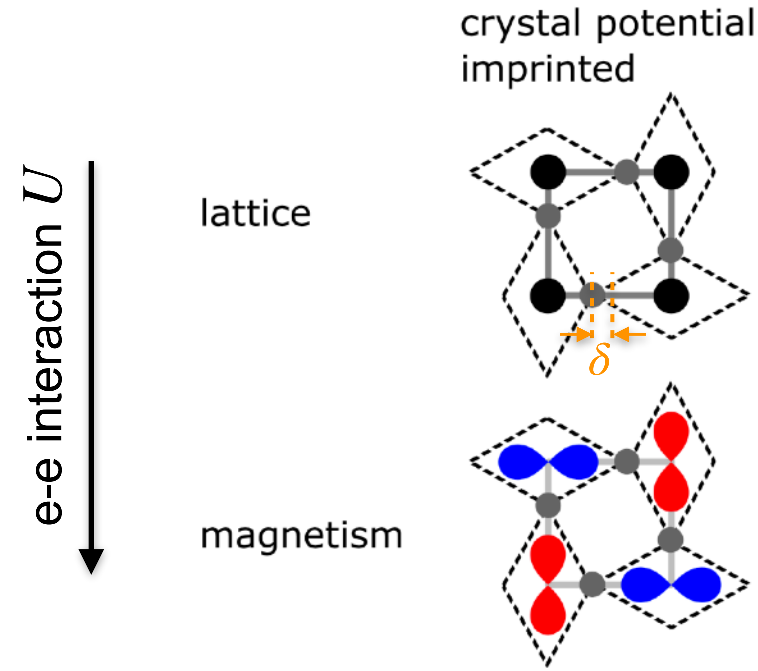
Interaction induced AM



Altermagnet with phase transition to AFM?

Spontaneous AM

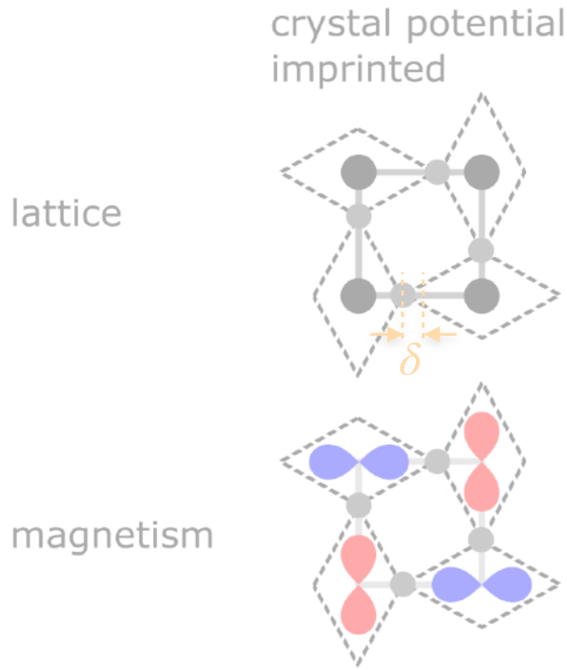
- **Normally anisotropic spin density from low crystal symmetry**

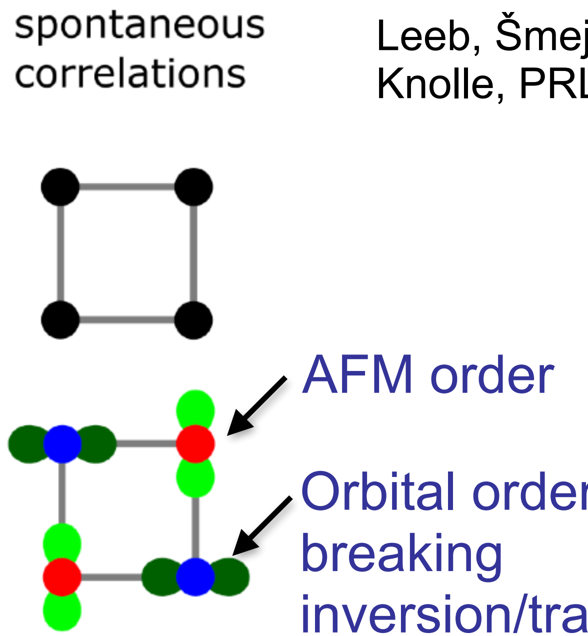


Question: Can interactions induce AM in a high symmetry crystal?

Spontaneous AM

- **Normally anisotropic spin density from low crystal symmetry**



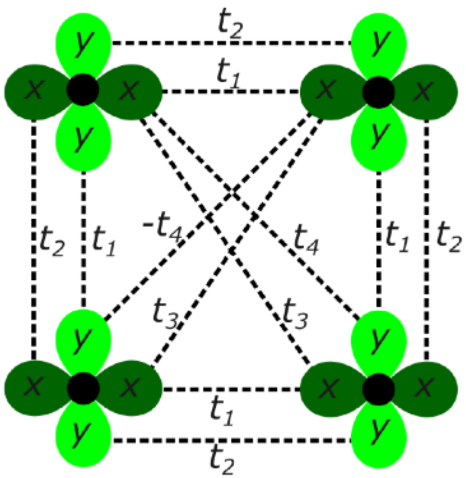


Leeb, Šmejkal, Mook, Knolle, PRL 2024

→ Yes, via a coexistence of Neel AFM with another order!

Orbital Ordering

- **Use a minimal two orbital model with d_{xz}, d_{yz}-orbitals**



$$H_0 = \sum_{\boldsymbol{k},s} \Psi^\dagger_{\boldsymbol{k}s} \begin{pmatrix} \varepsilon_x(\boldsymbol{k}) & \varepsilon_{xy}(\boldsymbol{k}) \\ \varepsilon_{xy}(\boldsymbol{k}) & \varepsilon_y(\boldsymbol{k}) \end{pmatrix} \Psi_{\boldsymbol{k}s}$$

Raghu, Qi, Liu, Scalapino, S.-C. Zhang PRB 2008

→ simple model for iron-based SC

→ Invariant under rotations $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ -x \end{pmatrix}$

- **Interactions inducing (π, π)-AFM and (π, π)-OO**

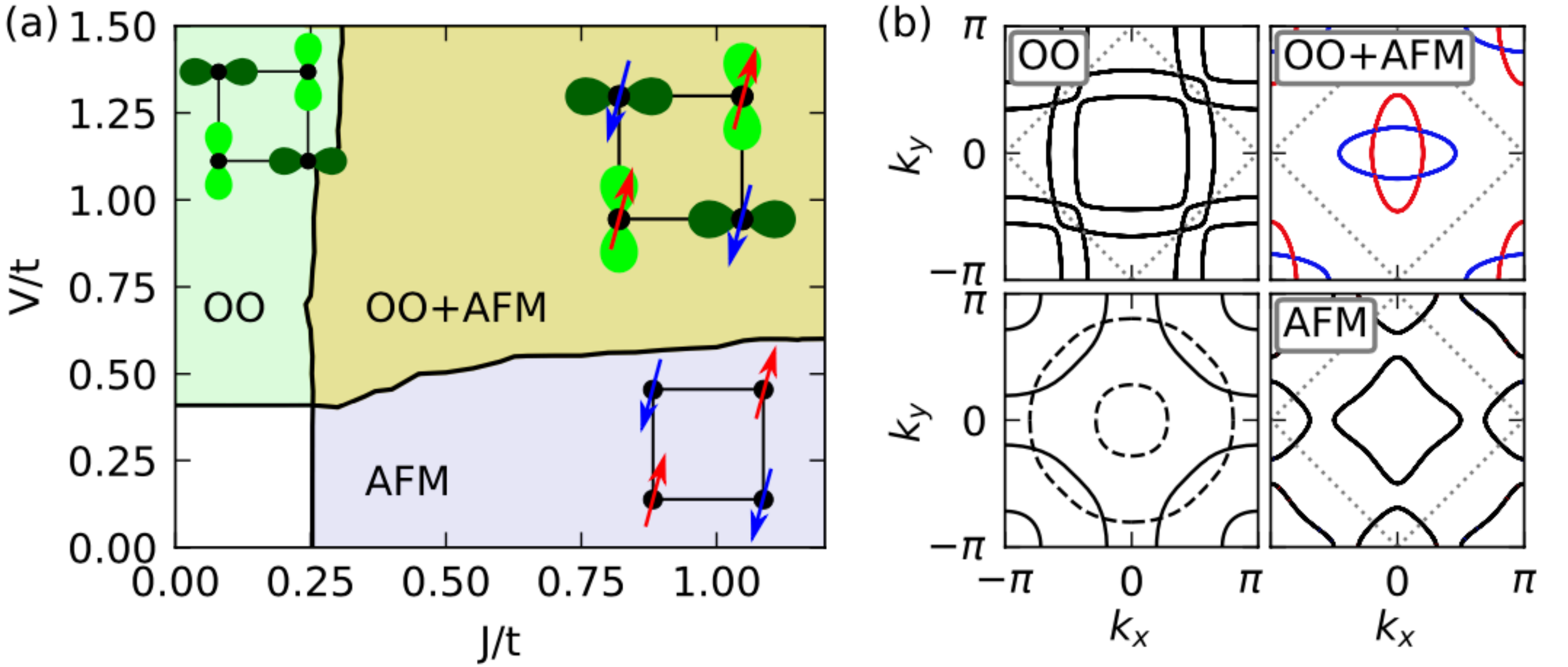
$$H_J = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \quad \text{and} \quad H_V = V \sum_{\langle ij \rangle} N^z_i N^z_j$$

$$S^z_i = c^\dagger_{i\uparrow} c_{i\uparrow} - c^\dagger_{i\downarrow} c_{i\downarrow} \qquad N^z_i = c^\dagger_{ix} c_{ix} - c^\dagger_{iy} c_{iy}$$

Orbital Ordering

- Self-consistent Hartree-Fock at fixed filling

Leeb, Šmejkal, Mook, Knolle, PRL 2024



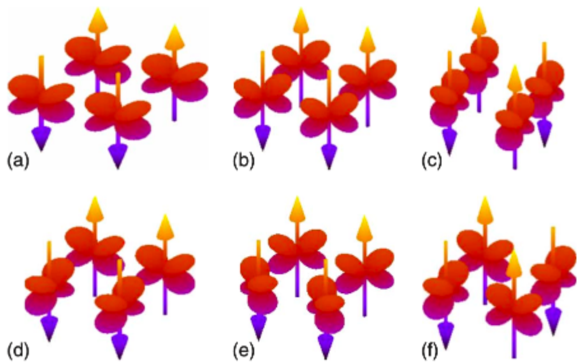
→ **Interaction induced AM from OO!**

Candidate Materials

- Considered for iron-based SC like FeSe

  Daghofer, Nicholson, Morea, Dagotto PRB 2010

  → **Violation of the Goodenough-Kanamori rules**



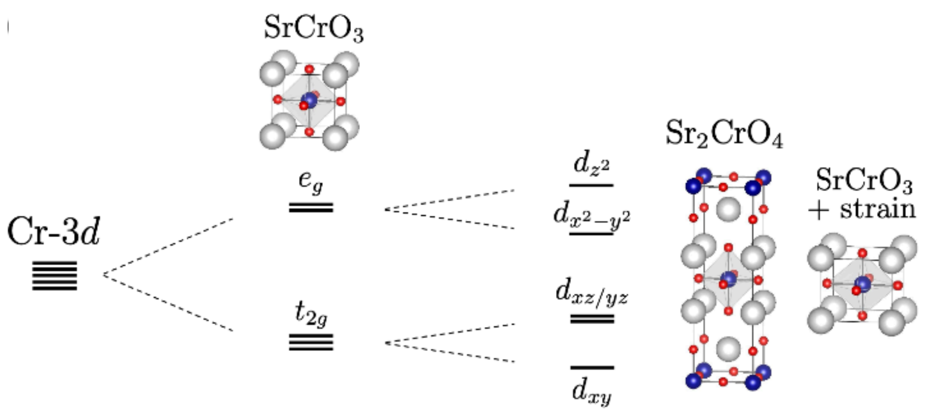
- Cubic Vanadates: $LaVO_3$

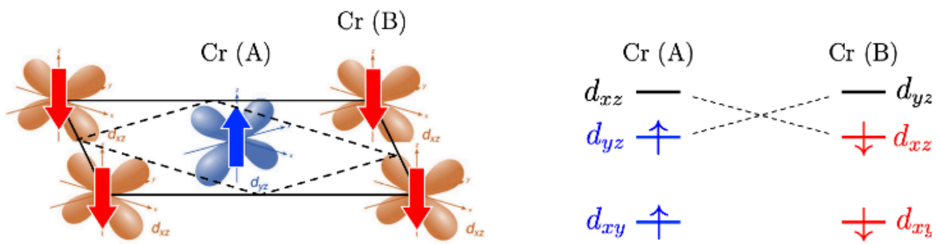
  Khaliullin, Horsch, Oles PRL 2001;
  Daghofer, Wohlfeld, v.d. Brink
  arXiv:2506:03261

- Ruddlesden-Popper Chromates $Sr_{n+1}Cr_nO_{3n+1}$

  Meier, Carta, Ederer, Cano
  arXiv:2502.01515

  → **compensated Anti-AM**

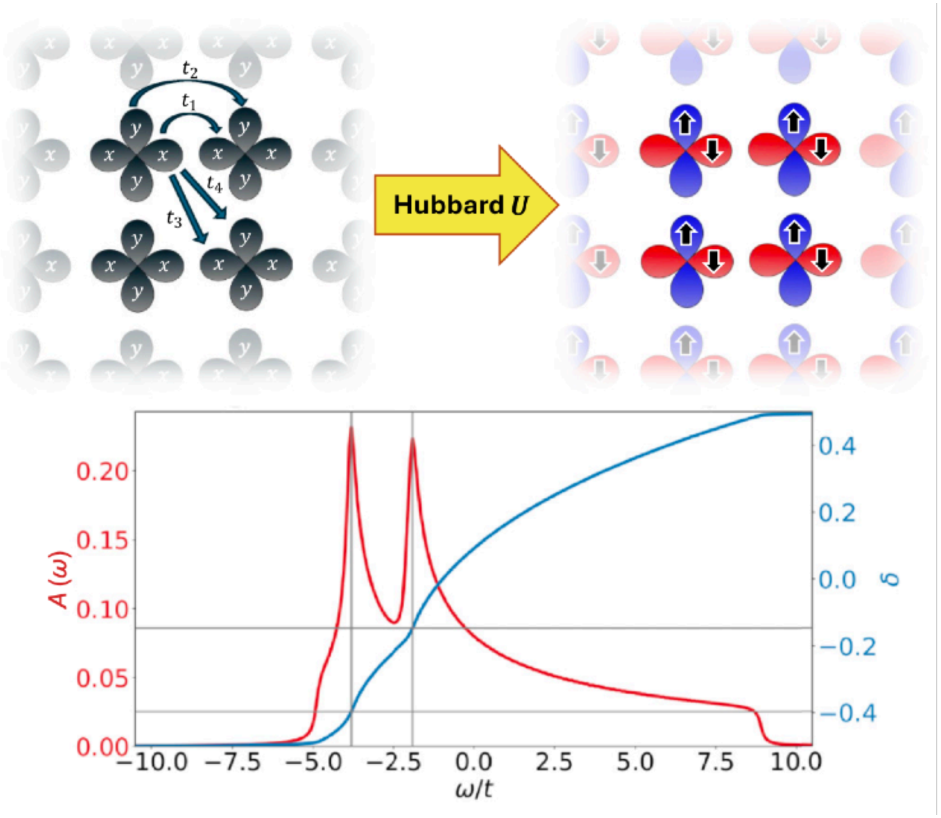




Q=(0,0) orbital AM

- Even simpler: no spatial symmetry breaking required

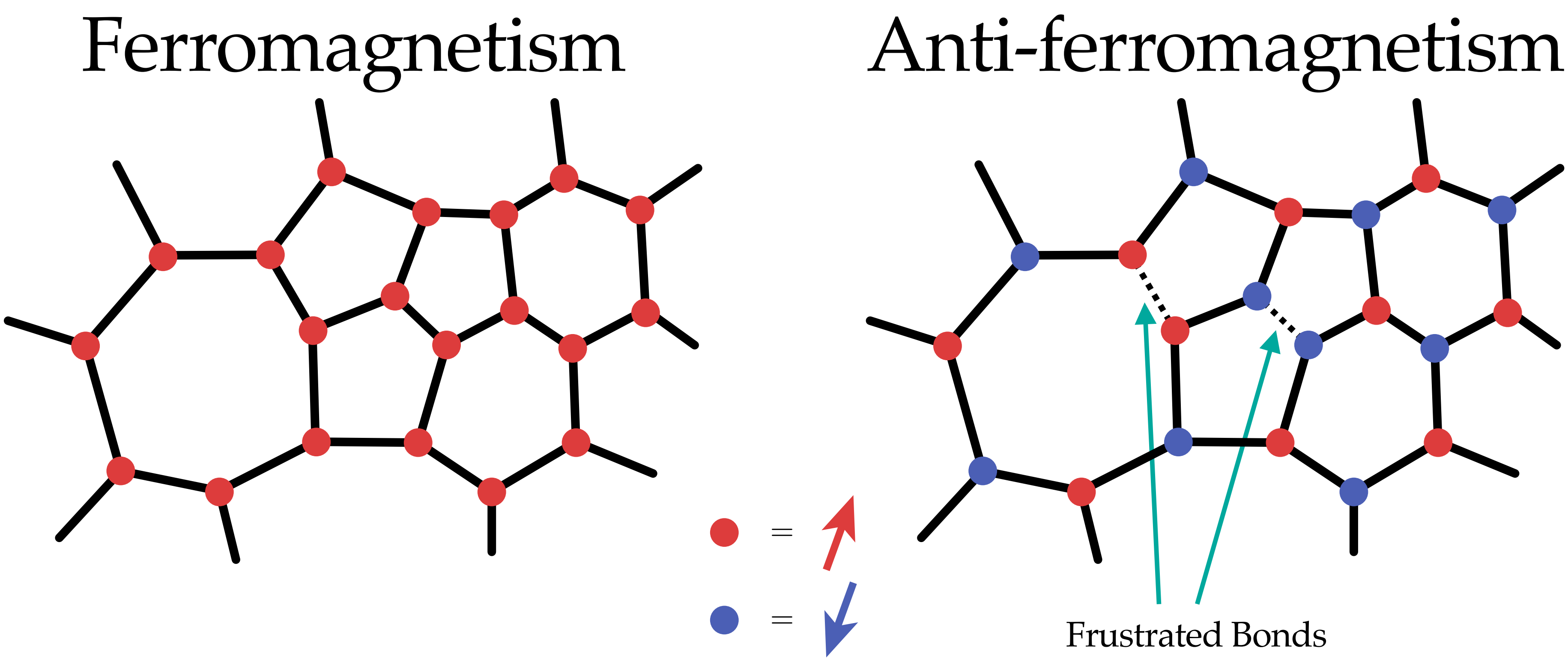
Guili, Zaera, Capone Phys. Rev. B 111, L020401 (2025)



→ **hard to stabilise in an itinerant model**

Orbital AM without crystal symmetry

- Do we need crystal symmetries at all?
  Can we get spontaneous AM in **amorphous magnets**?

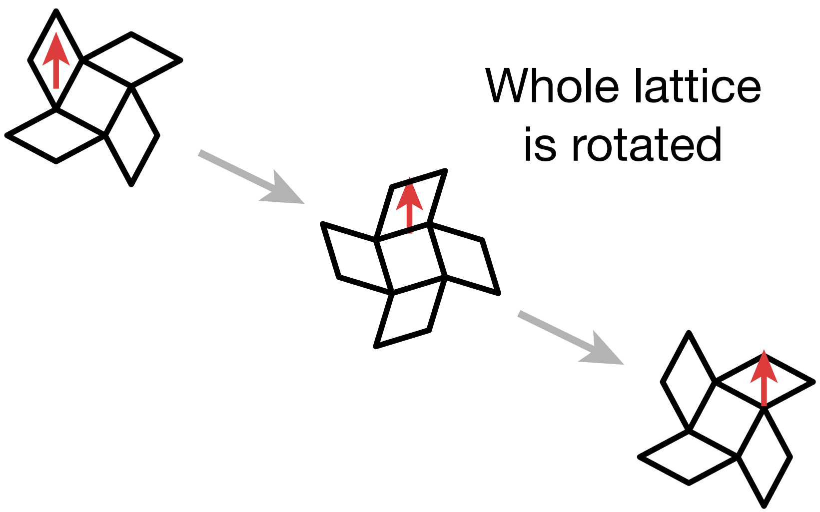


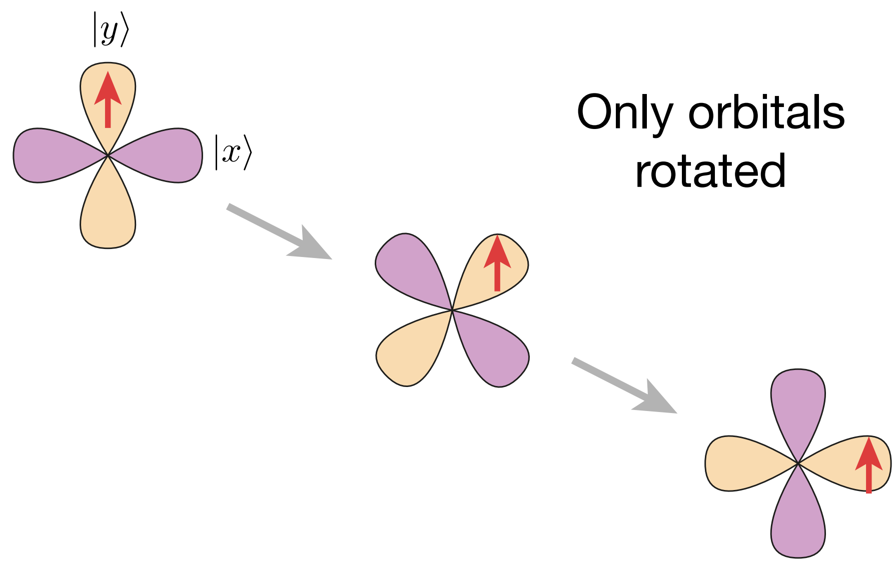
For amorphous QSL: Cassella, d'Ornellas, Hodson, Natori, Knolle Nature Comm. 14 (2023)

Rotation in orbital space

Real space coordinate transform: $\mathbf{x} \to R(\theta)\mathbf{x}$

On-site orbital transform: $\begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix} \to \mathcal{R}(\theta) \begin{pmatrix} |x\rangle \\ |y\rangle \end{pmatrix}$





- **Questions:**

  Can we build a toy model with an AM ground state on any geometry?

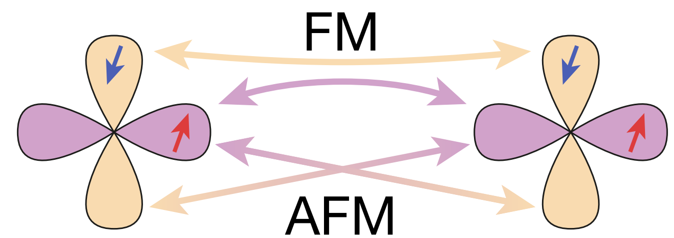
SSB in spin and orbital sector

- **Kugel-Khomskii t-J model**

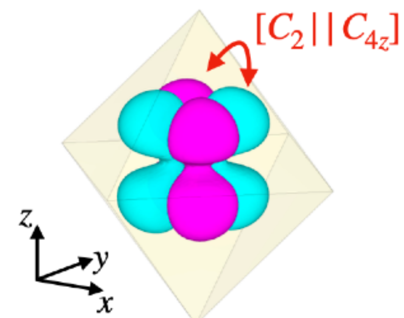
Two orbitals + two spin states per site

$$H = H_{Kin} + H_{Int}$$

$$H_{Int} = -J \sum_{\langle jk \rangle} (\boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k)(\boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k) - n_j n_k$$



Kugel, Khomskii, Sov. Phys. Usp. 1983



- **Similar to:** (nothing but a d-wave spin density)

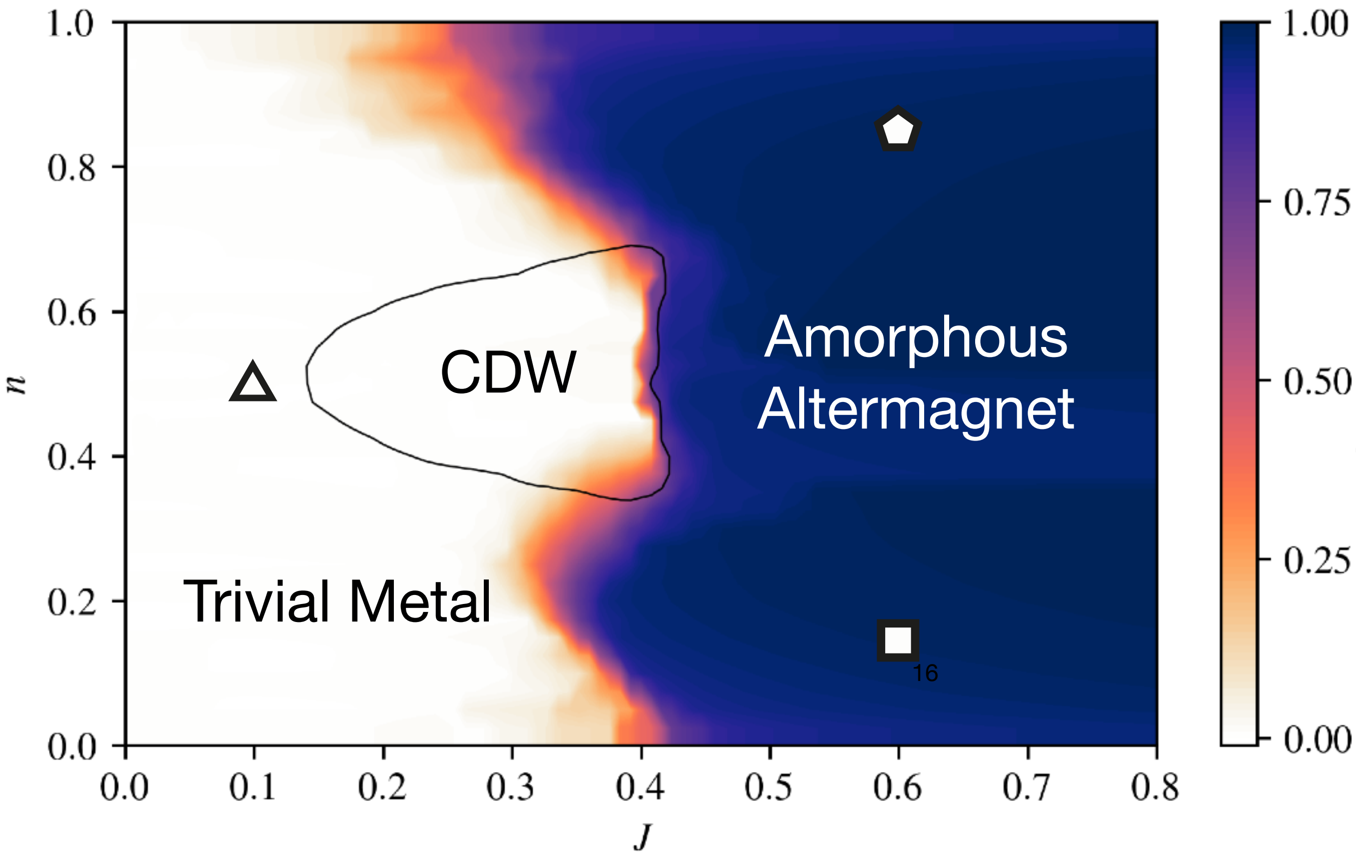
Atomic altermagnetism

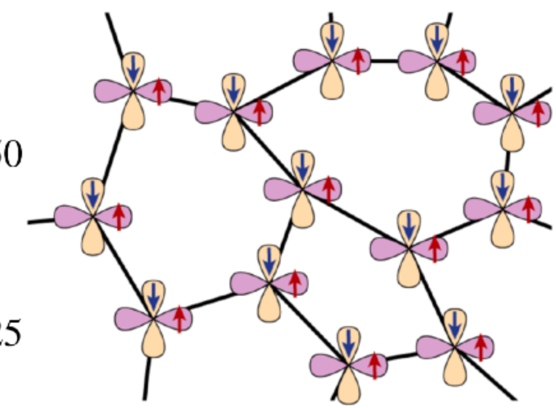
Rodrigo Jaeschke-Ubiergo,[1] Venkata-Krishna Bharadwaj,[1] Warlley Campos,[2] Ricardo Zarzuela,[1] Nikolaos Biniskos,[3] Rafael M. Fernandes,[4,5] Tomas Jungwirth,[6,7] Jairo Sinova,[1] and Libor Šmejkal[2,8,1,6]

Phase diagram amorphous AM

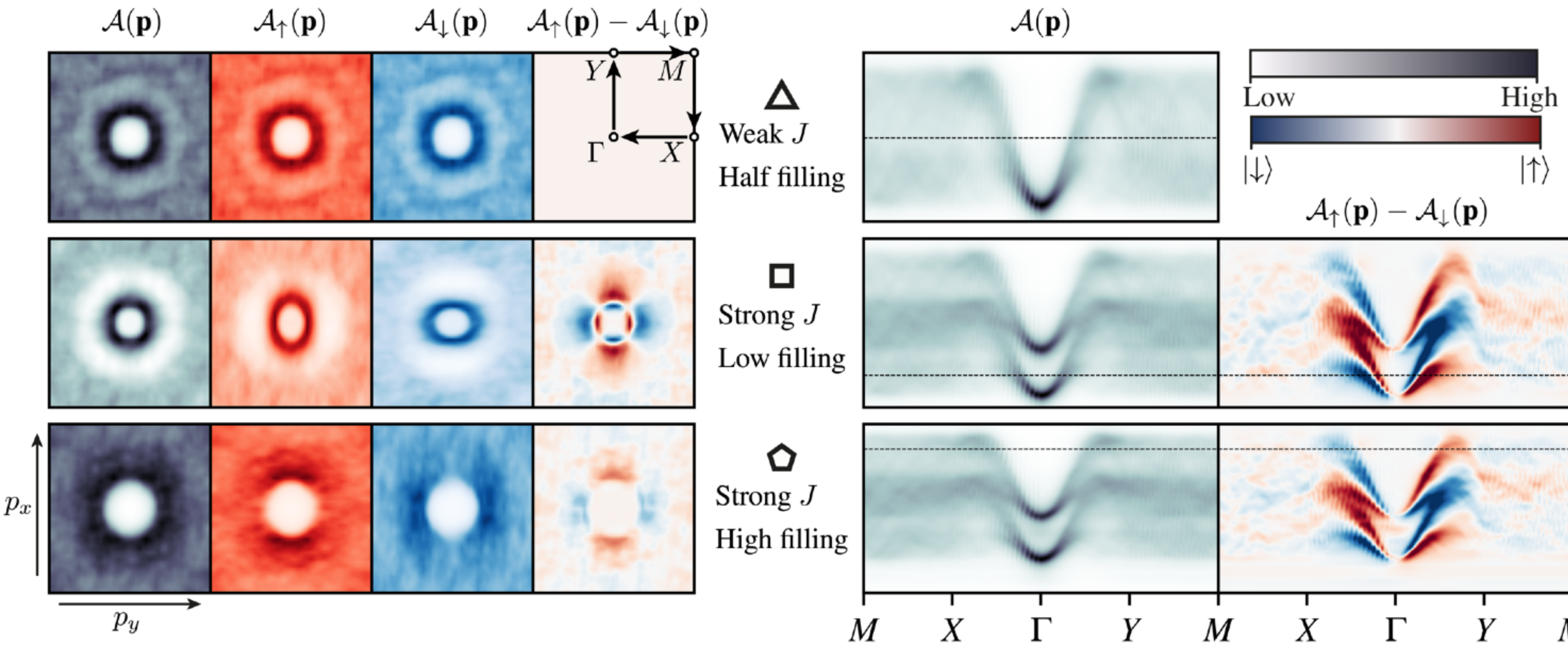
- **Decouple in Hartree-Fock**

$$m_j = \sigma_j^z \tau_j^z$$
$$= n_{\uparrow x} - n_{\uparrow y} - n_{\downarrow x} + n_{\downarrow y}$$



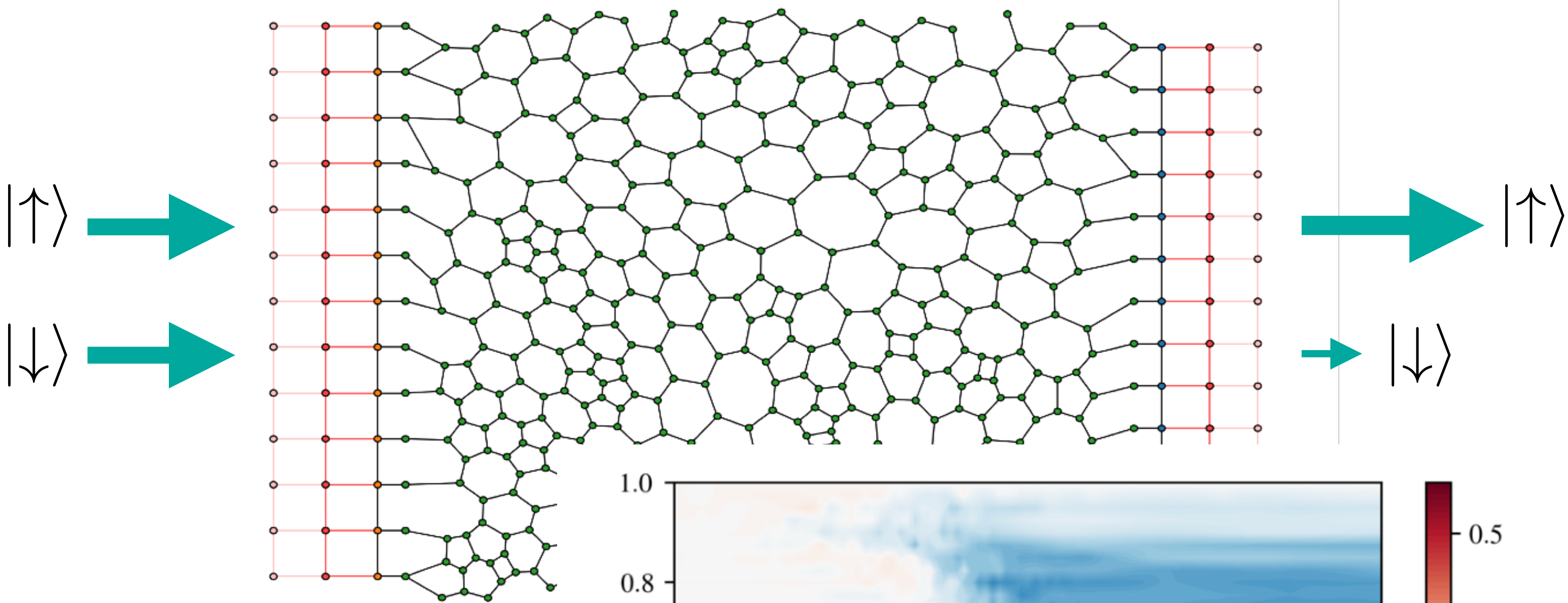


Spectral function is polarised

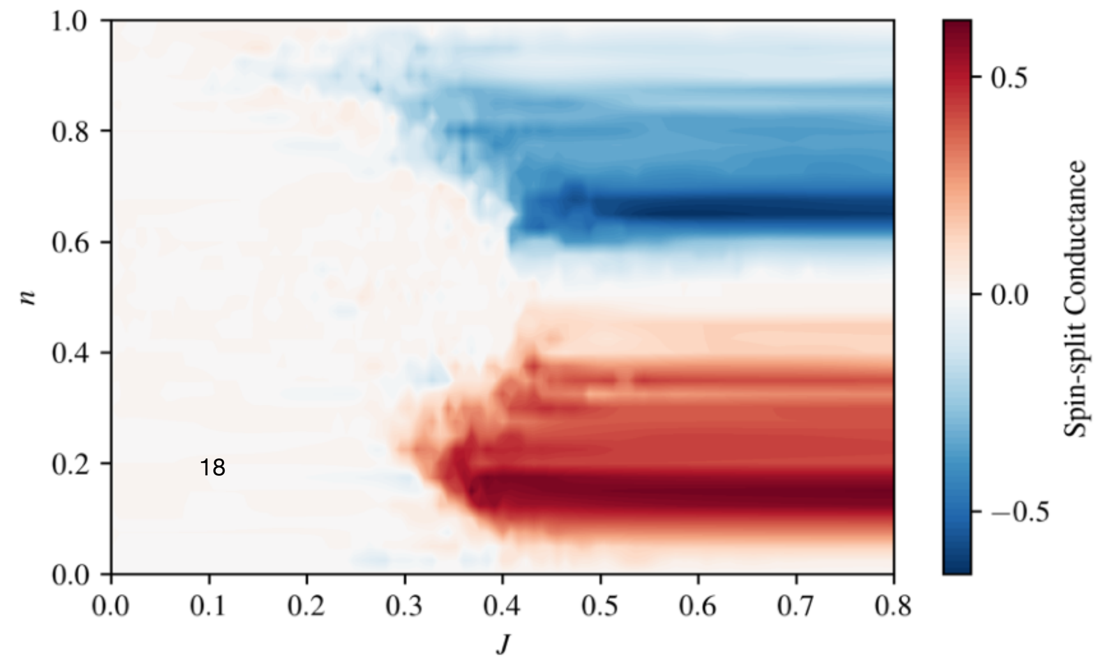


d'Ornellas, Leeb, Grushin, Knolle PRB 113 024426

Spin-Split Conductance

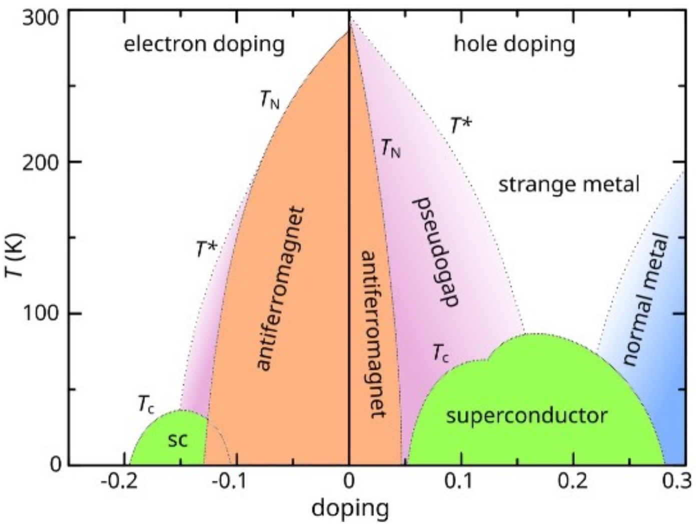


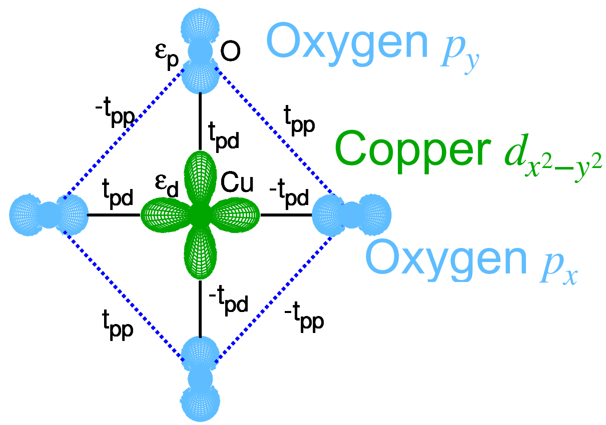
Left lead

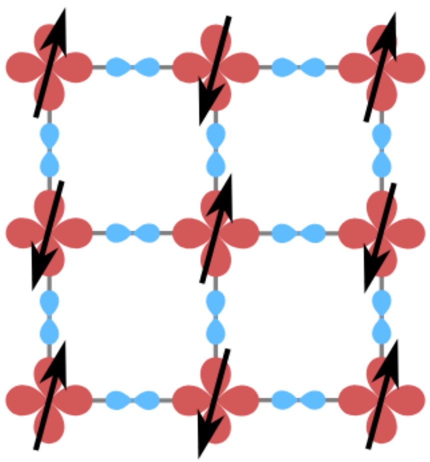


What about Cuprates?

- **Complex phase diagram with a (π, π)-AFM and *d*-wave SC**







- **3 band Emery model = Lieb lattice model**

$$H = \sum_{\langle ij \rangle, \alpha, \beta, s} t_{ij\alpha,\beta} c^\dagger_{i\alpha s} c_{j\beta s} + U_p \sum_i \left(n_{i,p_x,\uparrow} n_{i,p_x,\downarrow} + n_{i,p_y,\uparrow} n_{i,p_y,\downarrow} \right) + U_d \sum_i n_{i,d,\uparrow} n_{i,d,\downarrow}$$

$\rightarrow$ Hubbard interaction O $\rightarrow$ Hubbard interaction Cu

Intra-unit cell magnetism

PRL 96, 197001 (2006) PHYSICAL REVIEW LETTERS week ending 19 MAY 2006

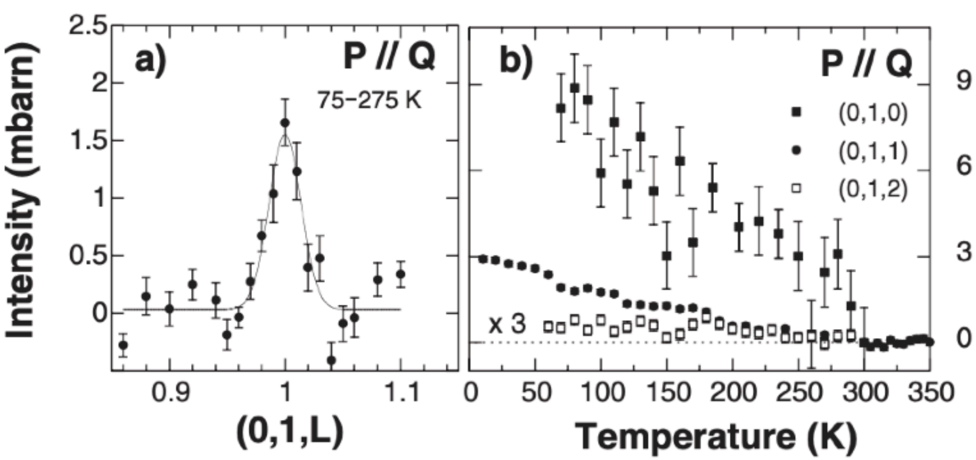
Magnetic Order in the Pseudogap Phase of High-T_C Superconductors

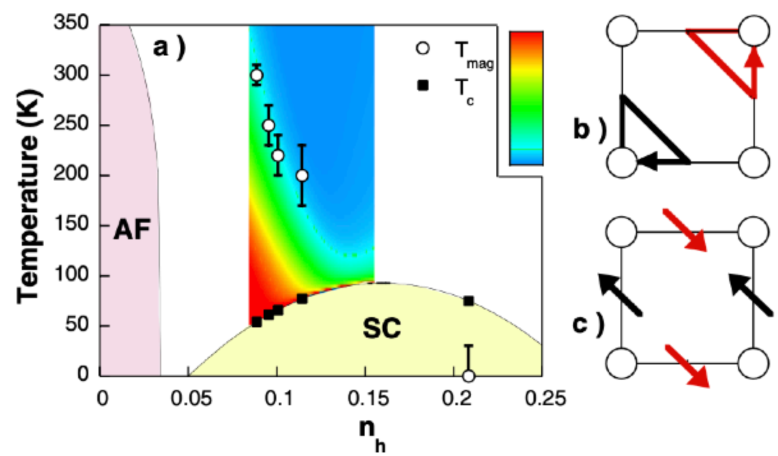
B. Fauqué,[1] Y. Sidis,[1] V. Hinkov,[2] S. Pailhès,[1,3] C. T. Lin,[2] X. Chaud,[4] and P. Bourges[1,*]

nature Vol 455 | 18 September 2008 | doi:10.1038/nature07251

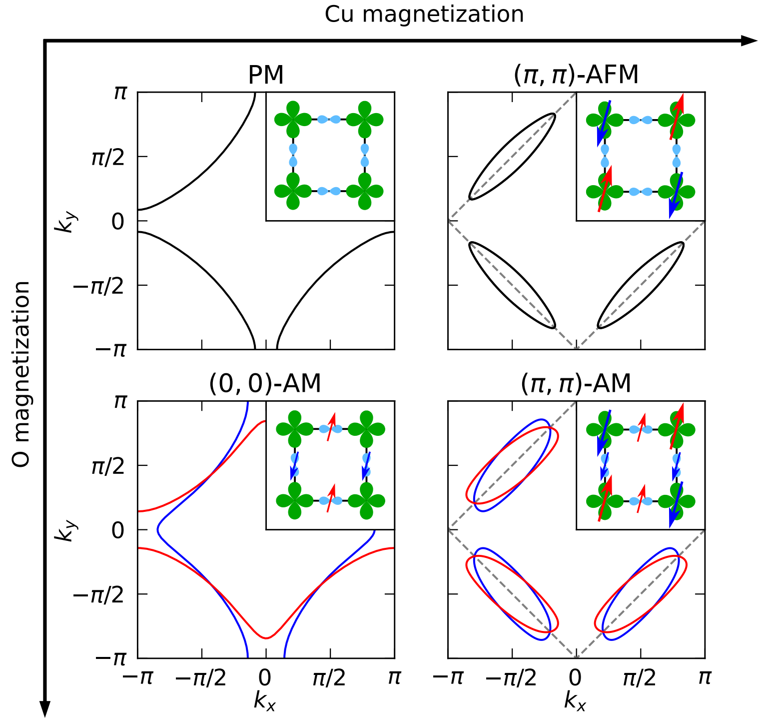
Unusual magnetic order in the pseudogap region of the superconductor $HgBa_2CuO_{4+\delta}$

Y. Li[1], V. Balédent[2], N. Barišić[3,4], Y. Cho[3,5], B. Fauqué[2], Y. Sidis[2], G. Yu[1], X. Zhao[3,6], P. Bourges[2] & M. Greven[3,7]





Cuprates with spontaneous oxygen magnetization?

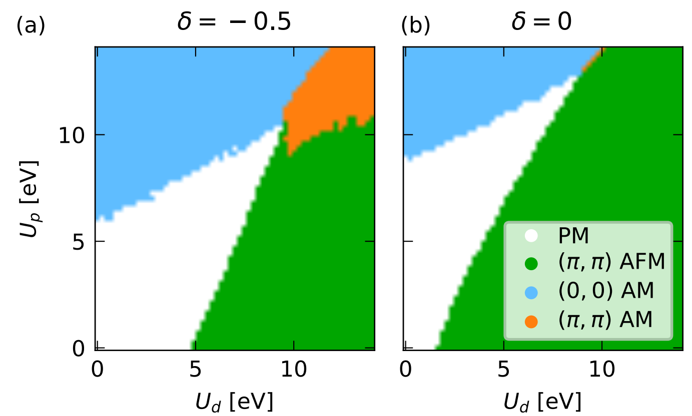


Li, Leeb, Wohlfeld, Valentí, Knolle PRB 2025

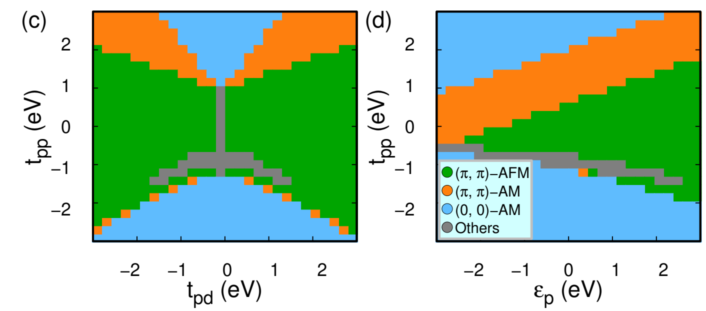
→ (0,0)-AM has been discussed as **nematic-spin-nematic order**

Fischer, Kim *PRB* 2011

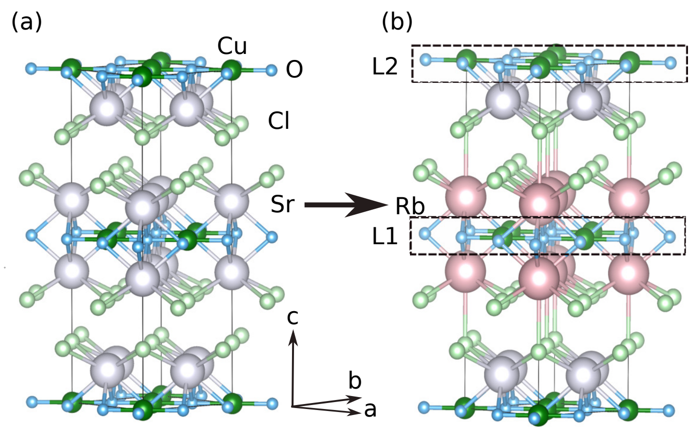
Phase diagram

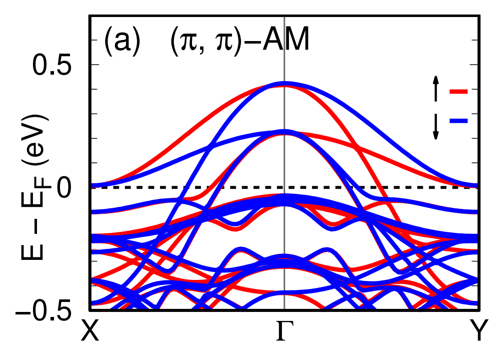


- **MFT and 4-site cluster ED**
  → Cell perturbation theory:
  → Superexchange over Cu



- **DFT+U**
  → $SrRbCuO_2Cl_2$

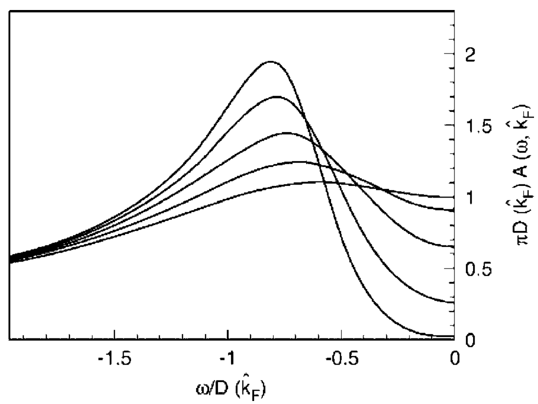


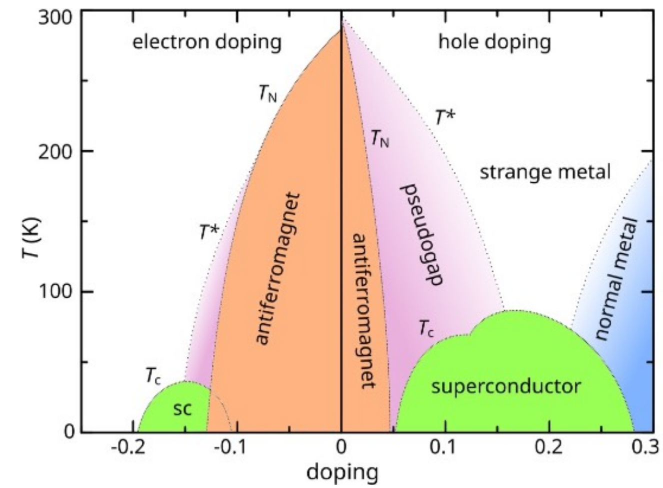


Li, Leeb, Wohlfeld, Valentí, Knolle PRB 2025

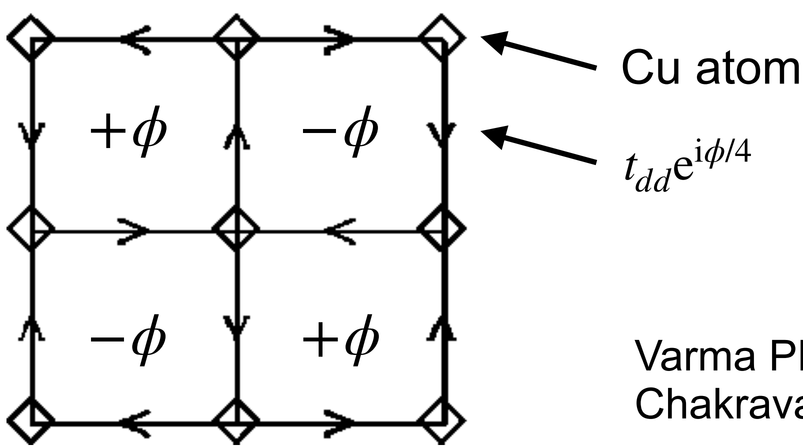
What about AM and loop currents?

- **What is the origin of the pseudogap?**





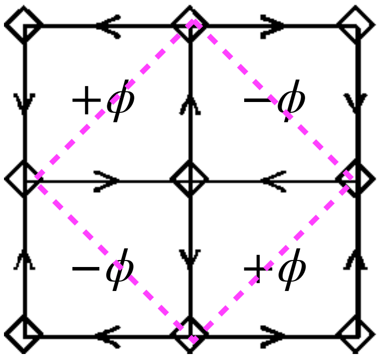
- **Orbital current states?**

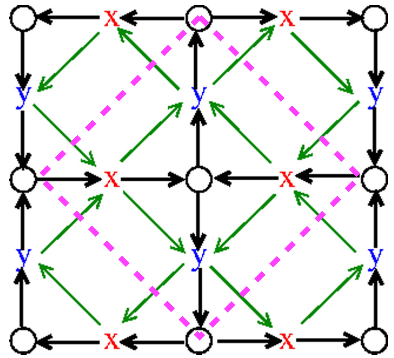


Varma PRL 1999
Chakravarty, Laughlin, Morr, Nayak PRB 2001

Zoo of orbital current patterns

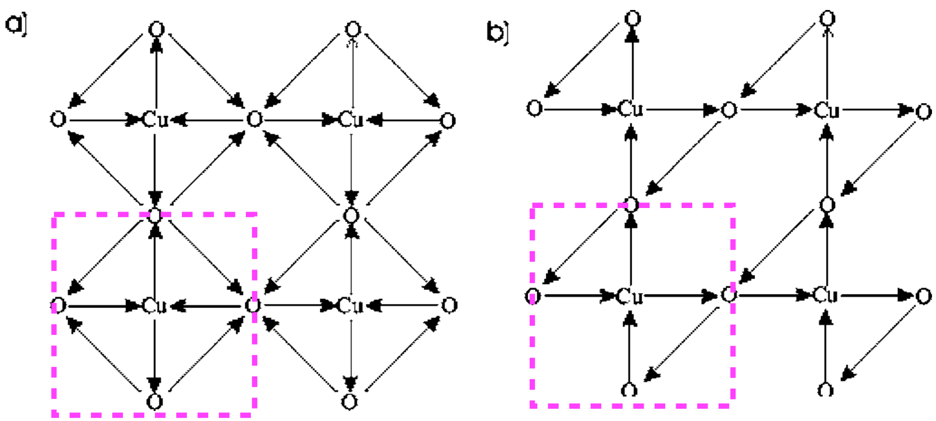
- Staggered orbital currents





…

- Intra unit cell orbital currents

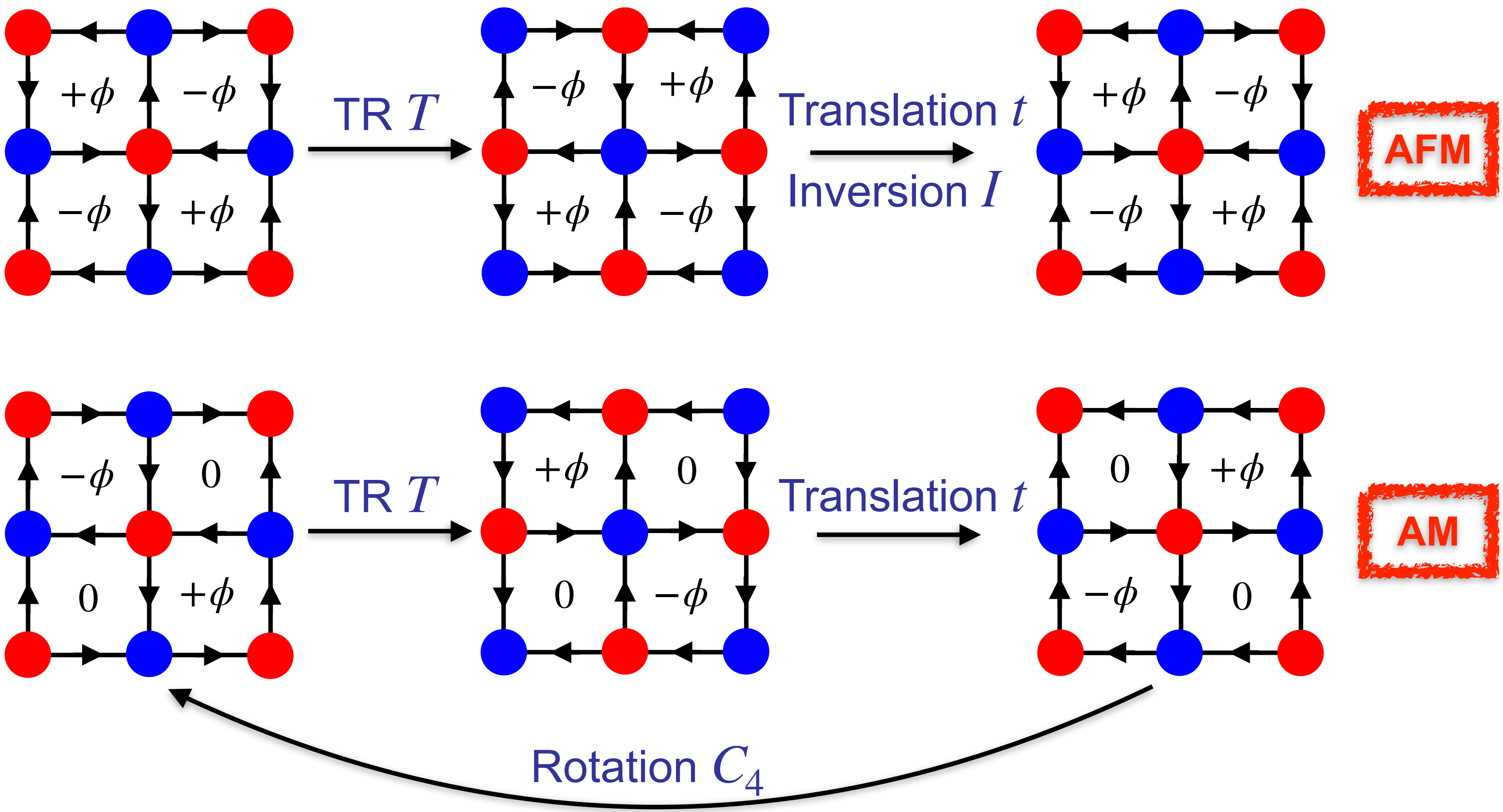


…

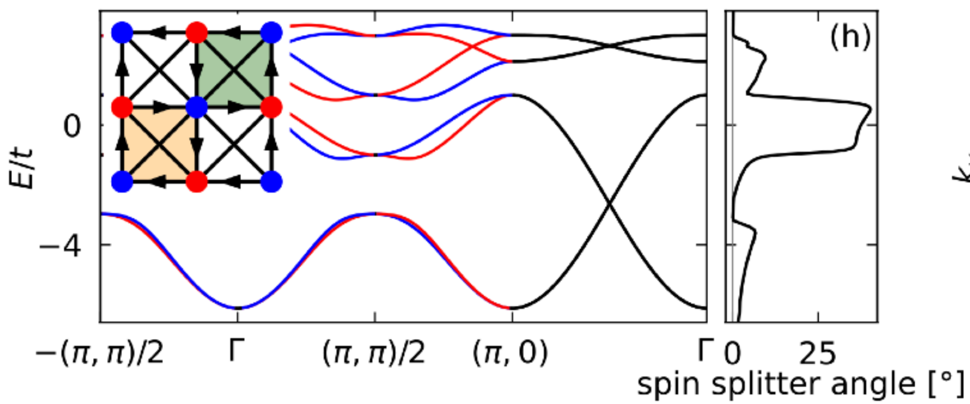
Chakravarty, Laughlin, Morr, Nayak PRB 2001
Varma PRB 2006
Bulut, Kampf PRB 2015
Scheurer, Sachdev PRB 2016

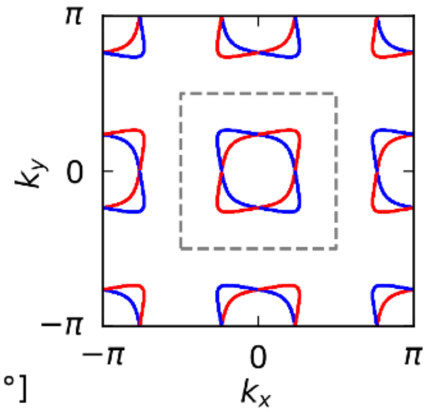
Can orbital currents lead to AM?

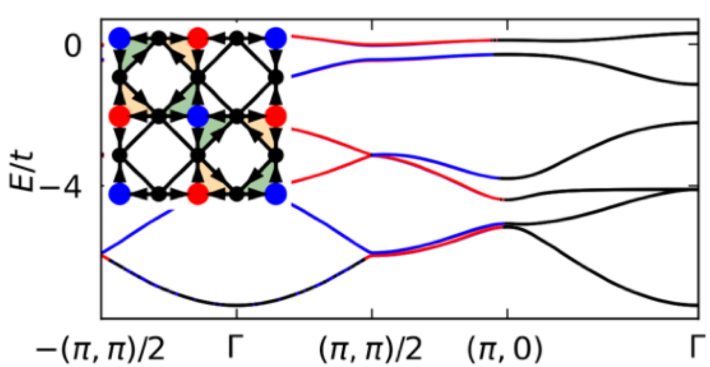
Yes, but **Orbital currents are odd under TR!**

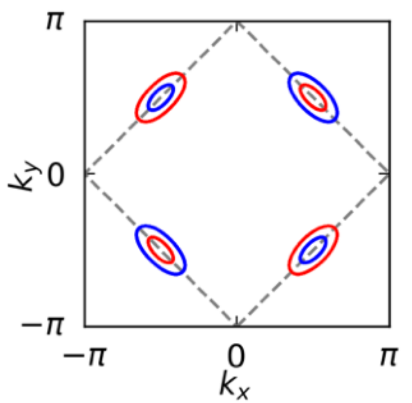


Spin splitting not from SO but magnetic order









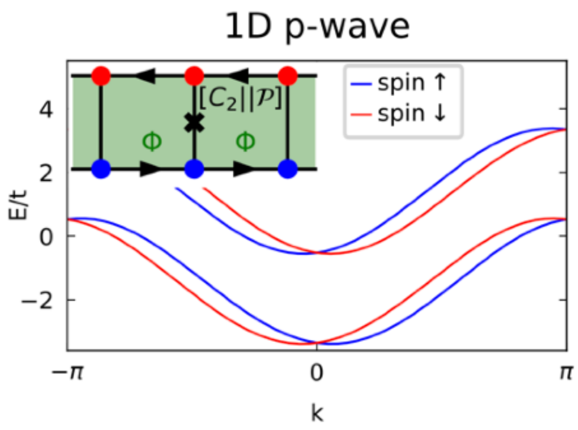
→ **compare to Kagome LC**

Leeb, Knolle arXiv:2601.07418

Chakraborty, Yang, Birol, Fernandes, arXiv:2509.26596

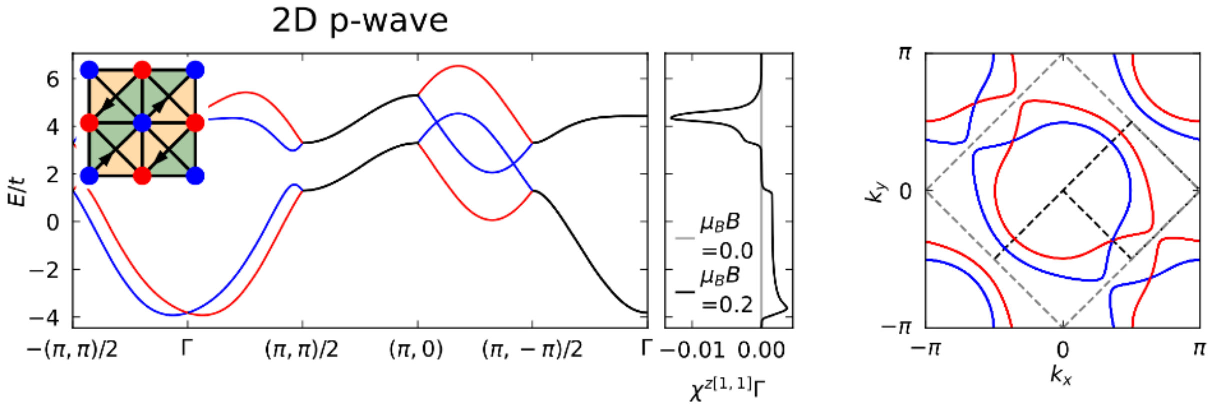
Collinear Odd-Wave Magnetism

- Spin-space group classification rules out collinear p-wave magnets
  → **but with LC the lattice transforms non-trivially under TRS!**



$$H_{1D} = \sum_{j,s=\pm} sm(c^\dagger_{j,B}c_{j,B} - c^\dagger_{j,A}c_{j,A}) - tc^\dagger_{j,B}c_{j,A}$$
$$- t\left(\mathrm{e}^{-\mathrm{i}\Phi/2}c^\dagger_{j+1,B}c_{j,B} - \mathrm{e}^{\mathrm{i}\Phi/2}c^\dagger_{j+1,A}c_{j,A}\right)$$

$$\epsilon_{\pm,s}(k) = -2t\cos k\cos\frac{\Phi}{2} \pm \sqrt{t^2 + (2t\sin k\sin(\Phi/2) + sm)^2}$$



Leeb, Knolle arXiv:2601.07418

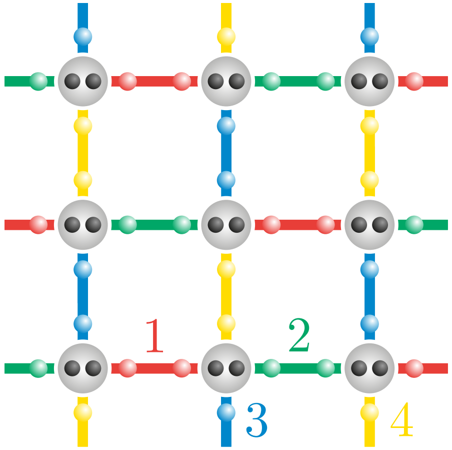
Fractionalized AM

- **Altermagnetic symmetries with topological order?**

  Sobral, Mandal, Scheurer PRR 2025
  Vijayvargia, Day-Roberts, Botana, Erten PRL 2025

- **Consider KK model on square lattice**

$$H = J \sum_{\langle ij \rangle_\mu} \left(\mathsf{s}_i^x \mathsf{s}_j^x + \mathsf{s}_i^y \mathsf{s}_j^y \right) \mathsf{t}_i^\mu \mathsf{t}_j^\mu \quad \leftarrow \quad \mathsf{t}^\mu = \begin{pmatrix} \mathsf{t}^x \\ \mathsf{t}^y \\ \mathsf{t}^z \\ \mathbb{1} \end{pmatrix}$$



→ exactly soluble like Kitaev honeycomb model

$$\mathsf{s}^z = -\mathrm{i} c^x c^y \qquad \mathsf{s}^x = -\mathrm{i} c^y b^4 \qquad \mathsf{t}^\alpha = -\mathrm{i}\epsilon^{\alpha\beta\gamma} b^\beta b^\gamma / 2$$

↖ Majorana fermions

Fractionalized AM

- **Parton mapping**

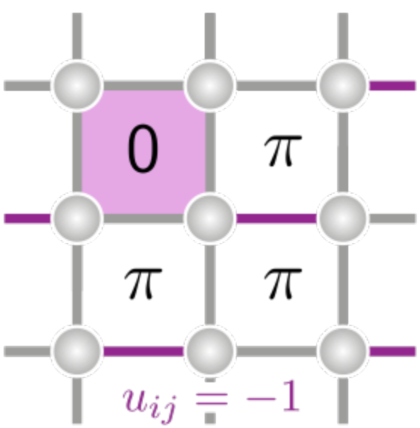
$$H = 2J \sum_{\langle ij \rangle} u_{ij} \left(f_i^\dagger f_j + \text{h.c.} \right)$$

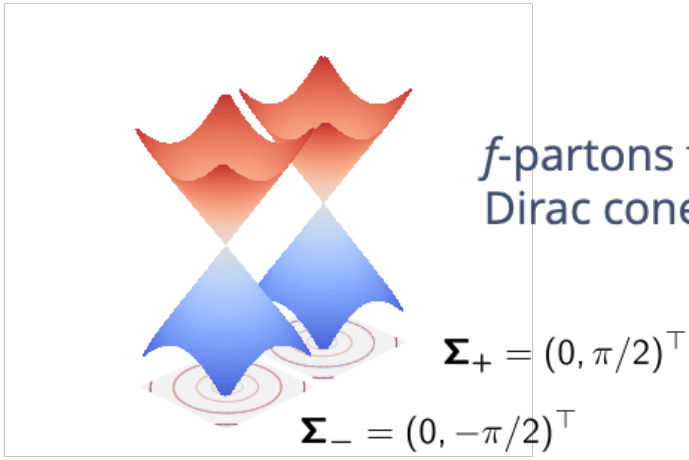
complex (spinless) fermion

Z_2 gauge field (on links)

- **Spins and orbitals fractionalize into fluxes and visons**

gapped vison





f-partons form Dirac cones

Add interactions

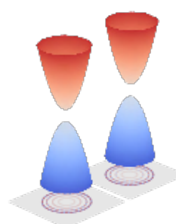
$$H = J\sum_{\langle ij\rangle_\mu}\left(s_i^x s_j^x + s_i^y s_j^y\right) t_i^\mu t_j^\mu \longrightarrow H = 2J\sum_{\langle ij\rangle} u_{ij}\left(f_i^\dagger f_j + \text{h.c.}\right)$$

Parton construction

$$+g\sum_{\langle ij\rangle} s_i^z s_j^z \longrightarrow +4g\sum_{\langle ij\rangle}\left(f_i^\dagger f_i - \frac{1}{2}\right)\left(f_j^\dagger f_j - \frac{1}{2}\right)$$

Gauge field remains static!

Dirac semimetal of *f*-fermions

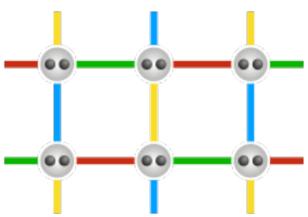


Charge Density Wave with $\langle f_i^\dagger f_i\rangle \propto (-1)^i$

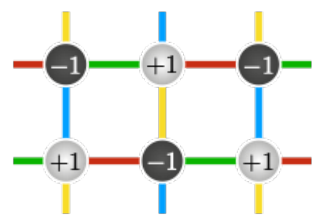
g/J

$(g/J)_c = 0.64$

(Z_2 Gross-Neveu universality)



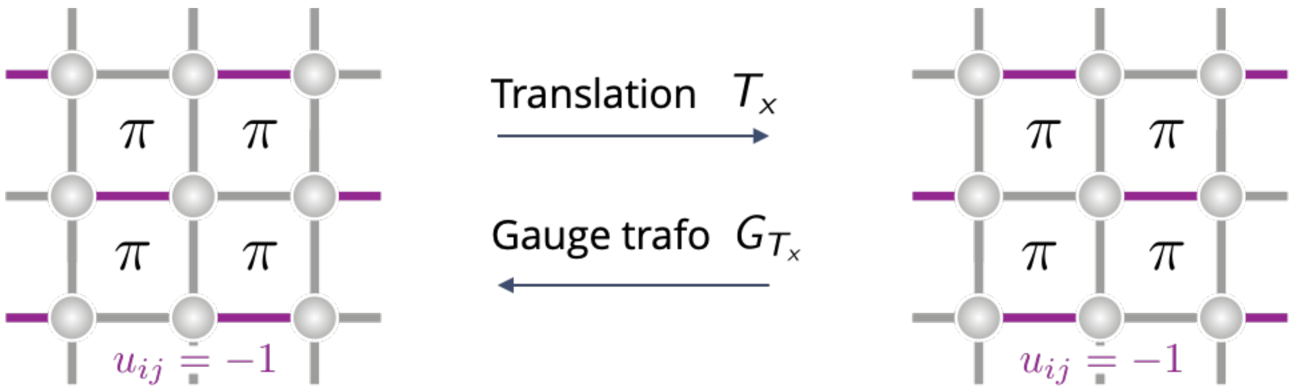
Gapless quantum spin liquid



Ising antiferromagnet with Z_2 topological order (AF*)

Projective Symmetry Group I

- **For symmetry of parton bands consider fixed gauge**



Symmetry is restored *with* gauge transformation $G_{T_x} : u_{ij} \to s_i u_{ij} s_j$

⇒ "Projective Symmetry Group": symmetries act *projectively* in parton theory

$$(G_U U)^{-1} H (G_U U) = H$$

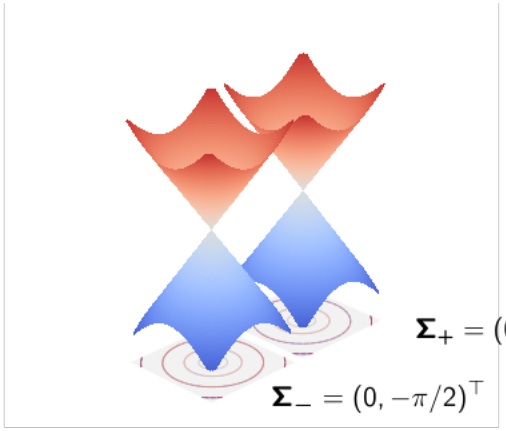
X.G. Wen PRB 1990

- **Spin and TRS implemented projectively**

$$\hat{S} : \mathbf{s}^z \to -\mathbf{s}^z \longrightarrow f_i \to (-1)^i f_i^\dagger$$

Projective Symmetry Group II

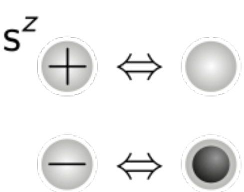
- **Bands transform projectively**



Translations

swaps valleys

$$\psi(\mathbf{q}) = \begin{pmatrix} \psi_{+,A} \\ \psi_{+,B} \\ \psi_{-,A} \\ \psi_{-,B} \end{pmatrix} \xrightarrow{G_{T_x} T_x} [\tau^y \otimes U_{T_x}(\mathbf{q})]\, \psi(\mathbf{q})$$



$$s_i^z = 1 - 2 f_i^\dagger f_i$$

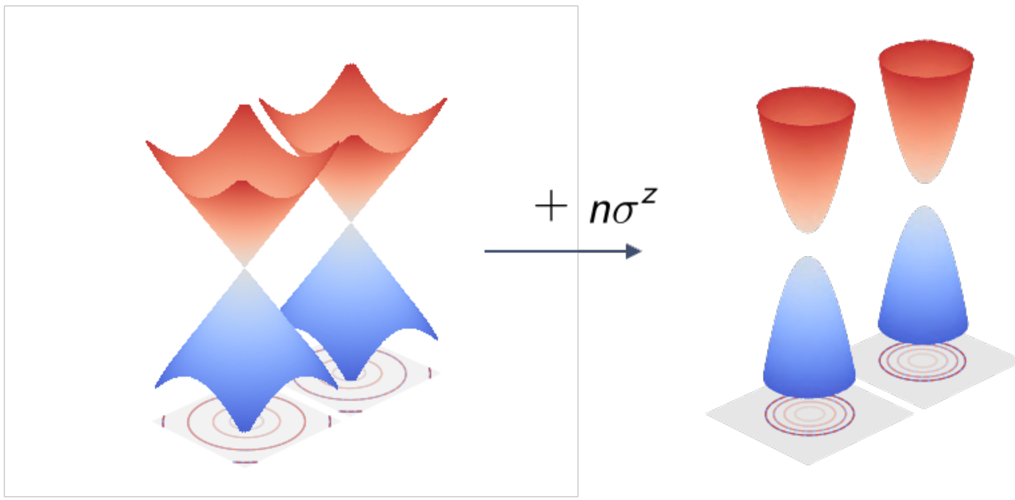
Time-reversal

Projective implementation: particle-hole transformation!

$$\psi(\mathbf{q}) \xrightarrow{G_{\mathcal{T}} \mathcal{T}} \mathbb{1} \otimes \sigma^z \psi^\dagger(\mathbf{q})$$

⇒ Time-reversal symmetry implies $\varepsilon_{\mathcal{T}}^{+}(\mathbf{q}) = -\varepsilon_{\mathcal{T}}^{-}(\mathbf{q})$

Projective Symmetry Group III



Quantum spin liquid
Z_2 gauge theory with Dirac fermions

Fractionalized antiferromagnet (AF*)
gapped fermions $\langle s_i^z \rangle \sim (-1)^i n$,
and deconfined Z_2 gauge fields

→ spin degeneracy is now a particle-hole +valley swap symmetry!

Constructing the AM*

σ^α : sublattice index
τ^α : valley index

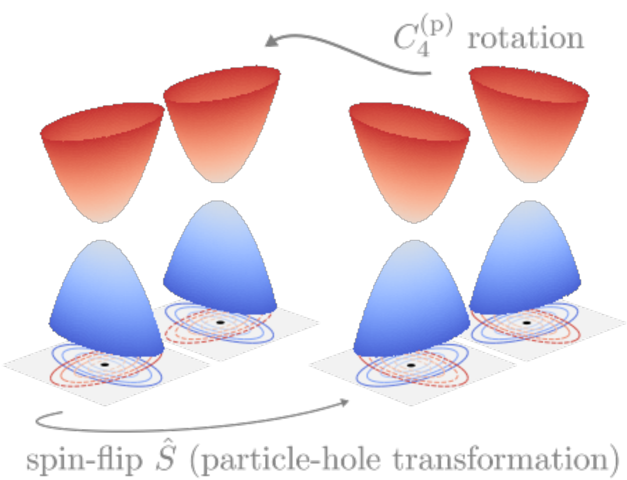
Néel order parameter *n:*
odd under C_4 , $R_y(\pi)$, $\mathcal{T}$ and $T_{x,y}$

$$H^{\text{eff}}(q_x, q_y) = H_0(q_x, q_y) \quad + \quad n\sigma^z \quad + \quad n\kappa(q_x^2 - q_y^2)\,\mathbb{1}$$

bare parton dispersion
(Dirac cones)

Staggered lattice distortion κ:
odd only under $T_{x,y}$

$\Rightarrow n\kappa \neq 0$ breaks $T_x R_y(\pi)$
but preserves $C_4 R_y(\pi)$!



d-wave **particle-hole asymmetry** of parton bands:

$$\varepsilon_\tau^\pm(\mathbf{q}) = \pm|n| \pm \frac{|\mathbf{q}|^2}{2m_{\text{eff}}} - \kappa n(q_x^2 - q_y^2)$$

(encodes momentum-dependent spin splitting)

Neehus, Rosch, Knolle, Seifert,
Phys. Rev. Lett. 135, 256504 (2025)

Thank you!

Valentin Leeb, Šmejkal, Mook, Knolle, PRL 2024

Das, Leeb, Knolle, Knap PRL 2024

d'Ornellas, Leeb, Grushin, Knolle arXiv:2504.08597

Li, Leeb, Wohlfeld, Valentí, Knolle PRB 2025

Avedis Neehus, Rosch, Knolle, **Urban Seifert**, PRL 2025

Valentin Leeb, Knolle arXiv:2601.07418

Thanks to all collaborators

$$H = \sum_{\langle jk \rangle, s} \Psi^{\dagger}_{j,s} T(\theta_{jk}) \Psi_{k,s} - J \sum_{\langle jk \rangle} (m_j m_k - n_j n_k)$$

Use Hartree Fock:

$$m_j \rightarrow \langle m_j \rangle$$

$$n_j \rightarrow \langle n_j \rangle$$

$$H_{Int} \rightarrow J \sum_{\langle jk \rangle} (\langle m_j \rangle m_k - \langle n_j \rangle n_k)$$