

# Superconductivity in Altermagnets

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# Outline



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## Introduction: Altermagnetism and Superconductivity

### Results:

- Finite-momentum and field-induced superconductivity
- Perfect superconducting diode effect
- Constraints on superconducting pairing
- Altermagnetism and superconductivity in  $\text{Sr}_2\text{RuO}_4$

# Acknowledgements



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Wallenberg  
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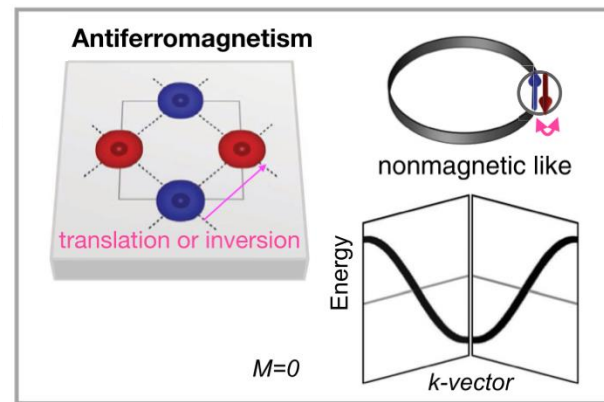
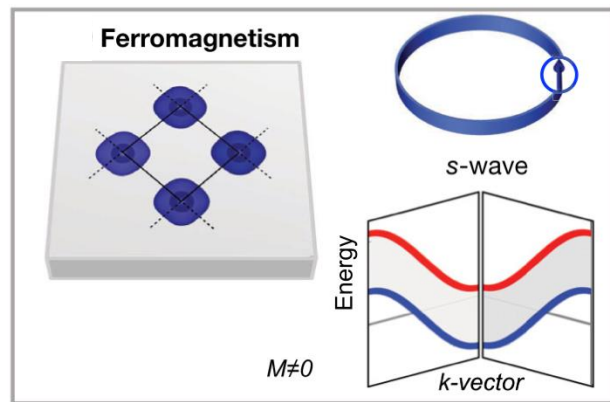
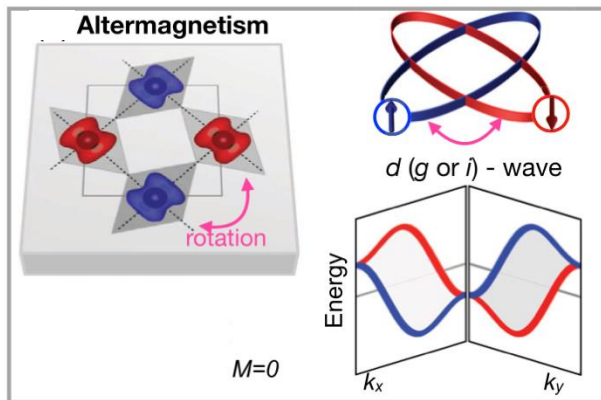


Wallenberg Initiative  
Materials Science  
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# Altermagnetism



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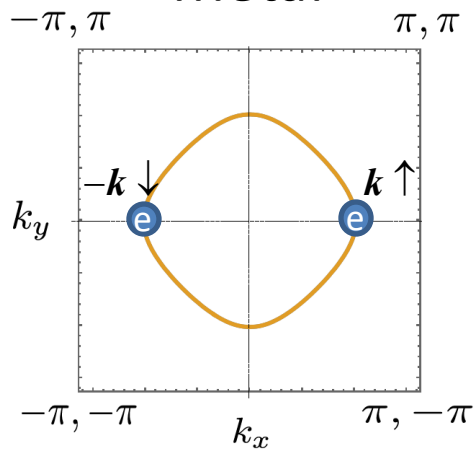


# Superconductivity in altermagnets

Intrinsic (or in heterostructures)

–  $\text{La}_2\text{CuO}_4$ ,  $\text{Sr}_2\text{RuO}_4$ , ...

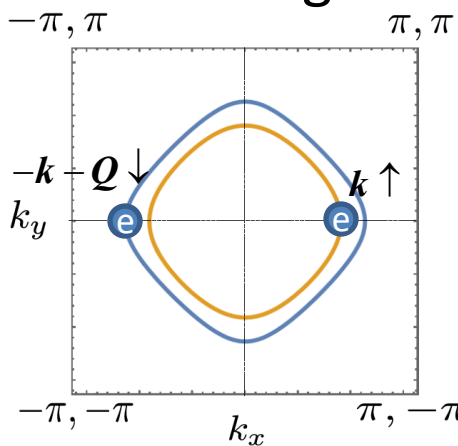
Metal



BCS pairing

(zero momentum pairing)

Ferromagnet



Finite momentum pairing

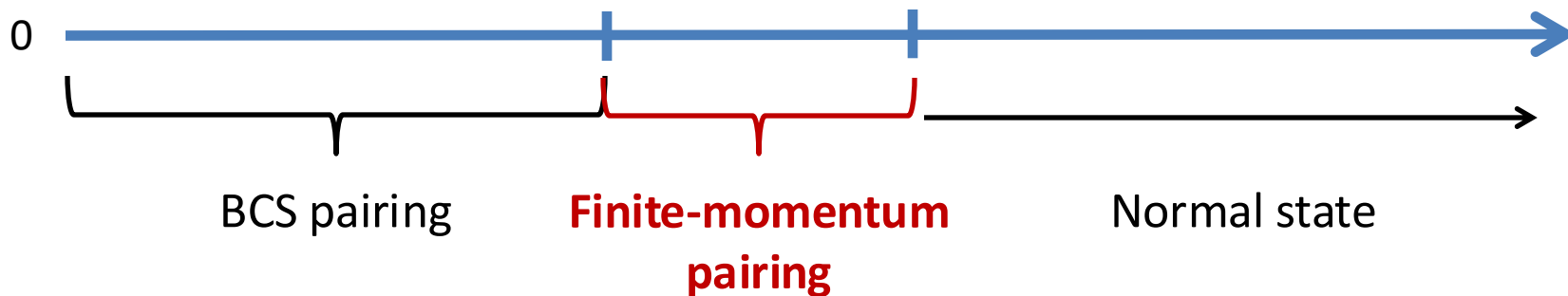
(finite center-of-mass Cooper pairs)

# Finite momentum pairing



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Increased spin-splitting (external magnetic field)



Fulde-Ferrel (FF):  $\Delta = |\Delta_0| e^{i\mathbf{Q} \cdot \mathbf{r}}$   
PR 135, A550 (1964)

Larkin-Ovchinnikov (LO):  $\Delta = \Delta_0 \cos(\mathbf{Q} \cdot \mathbf{r})$   
ZETF 47, 1136 (1964)

# Superconductivity in altermagnets

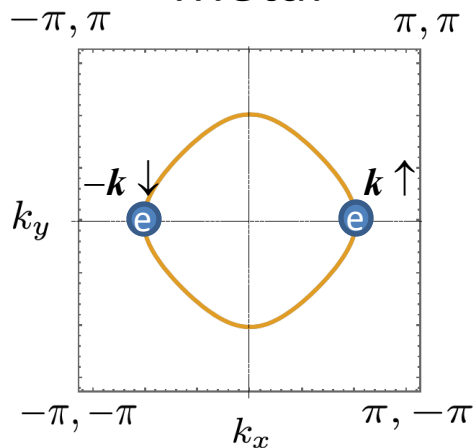


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Intrinsic (or in heterostructures)

–  $\text{La}_2\text{CuO}_4$ ,  $\text{Sr}_2\text{RuO}_4$ , ...

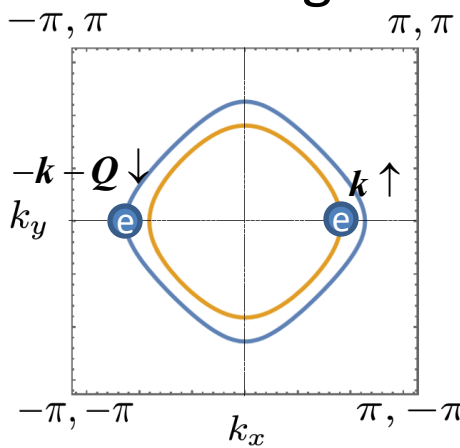
Metal



BCS pairing

(zero momentum pairing)

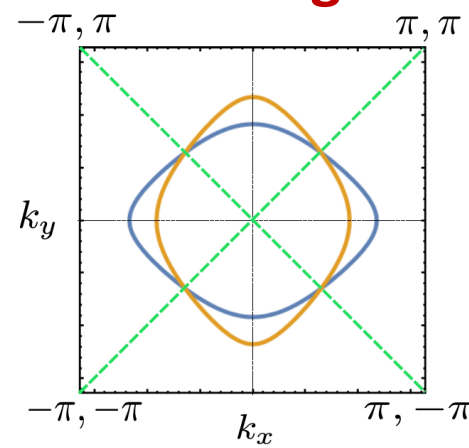
Ferromagnet



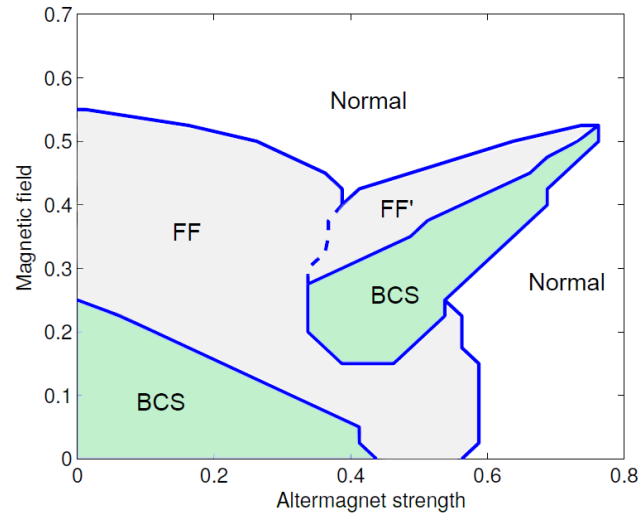
Finite momentum pairing

(finite center-of-mass Cooper pairs)

Altermagnet



Pairing?





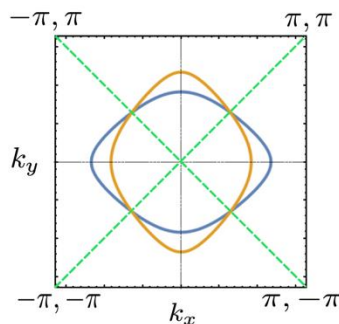
# Modeling SC in altermagnets



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$$H = \sum_{k,\sigma} \underbrace{\left( \xi_k - \sigma \frac{t_{\text{am}}}{2} (\cos(k_x) - \cos(k_y)) \right)}_{\text{Altermagnet band structure}} + \underbrace{\sigma B}_{\text{External magnetic field}} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k',q} V_{k,k'} c_{k+q\uparrow}^\dagger c_{-k+q\downarrow}^\dagger c_{-k'+q\downarrow} c_{k'+q\uparrow}$$

Altermagnet band structure      External magnetic field      Superconducting pairing



**d-wave spin-split  
Fermi surfaces (FS)**

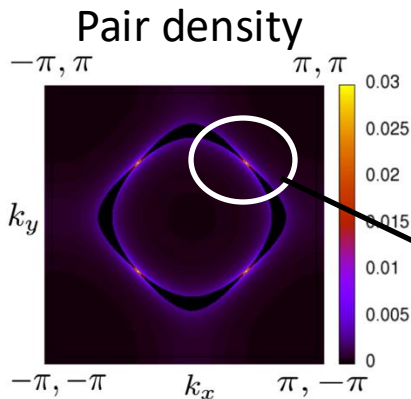
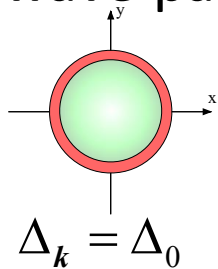
Solve self-consistently for  $\Delta_k^Q = \sum_{k'} V_{k,k'} \langle c_{k'+Q/2\uparrow}^\dagger c_{-k'+Q/2\downarrow}^\dagger \rangle$

Minimize ground state energy  $E(Q)$  to find optimal  $Q^*$

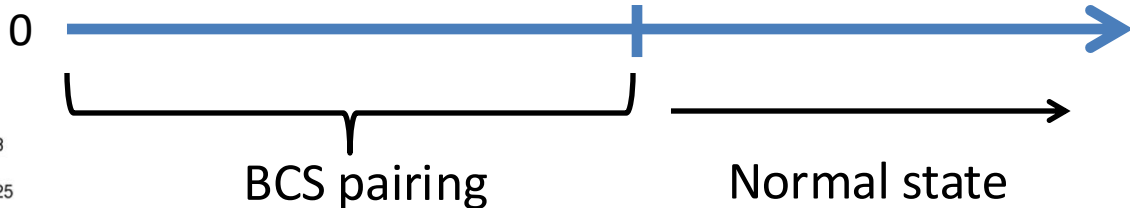
# Conventional s-wave pairing

$$H = \sum_{k,\sigma} (\xi_k - \sigma \frac{t_{am}}{2} (\cos(k_x) - \cos(k_y))) c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k',q} V_{k,k'} c_{k+q\uparrow}^\dagger c_{-k+q\downarrow}^\dagger c_{-k'+q\downarrow} c_{k'+q\uparrow}$$

s-wave pairing ( $V$  constant, on-site attraction)



Increased spin-splitting ( $t_{am}$ )

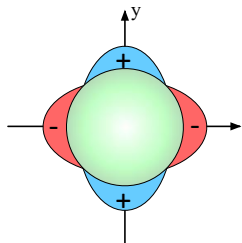


Sufficient condensation of  
BCS pairs at AM nodes  
(s-wave not likely in AMs)

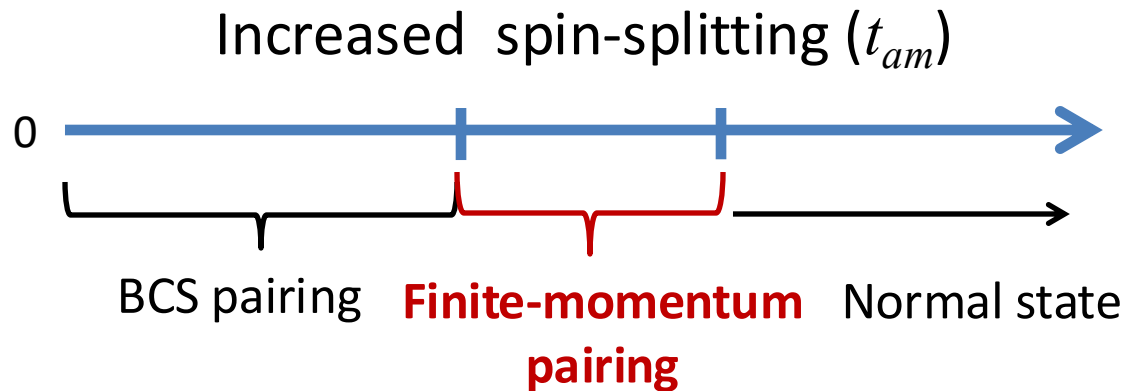
# Nodal $d$ -wave pairing

$$H = \sum_{k,\sigma} \left( \xi_k - \sigma \frac{t_{\text{am}}}{2} (\cos(k_x) - \cos(k_y)) \right) c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k,k',q} V_{k,k'} c_{k+q\uparrow}^\dagger c_{-k+q\downarrow}^\dagger c_{-k'+q\downarrow} c_{k'+q\uparrow}$$

$d$ -wave pairing (NN attraction)



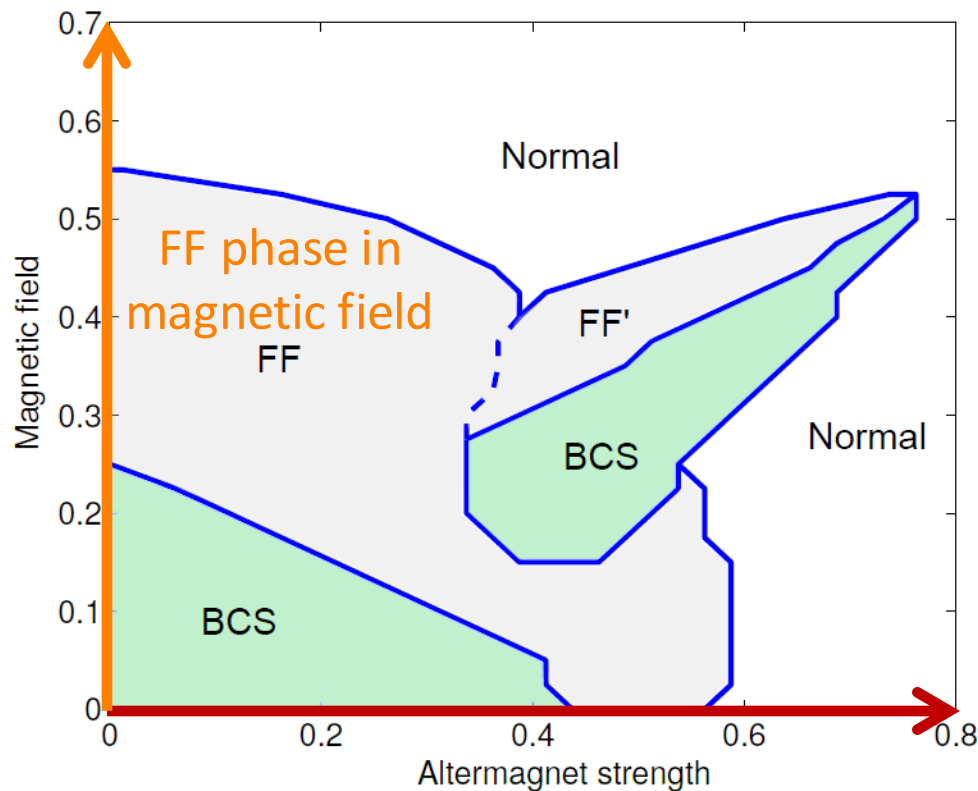
$$\Delta_k = \Delta_0 (\cos k_x - \cos k_y)$$



# Phase diagram – Yoda ear



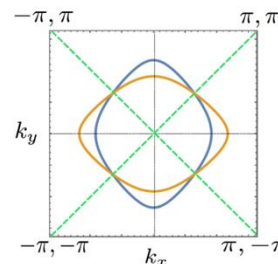
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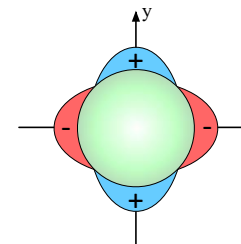
Yoda



FS AM nodes



SC nodes

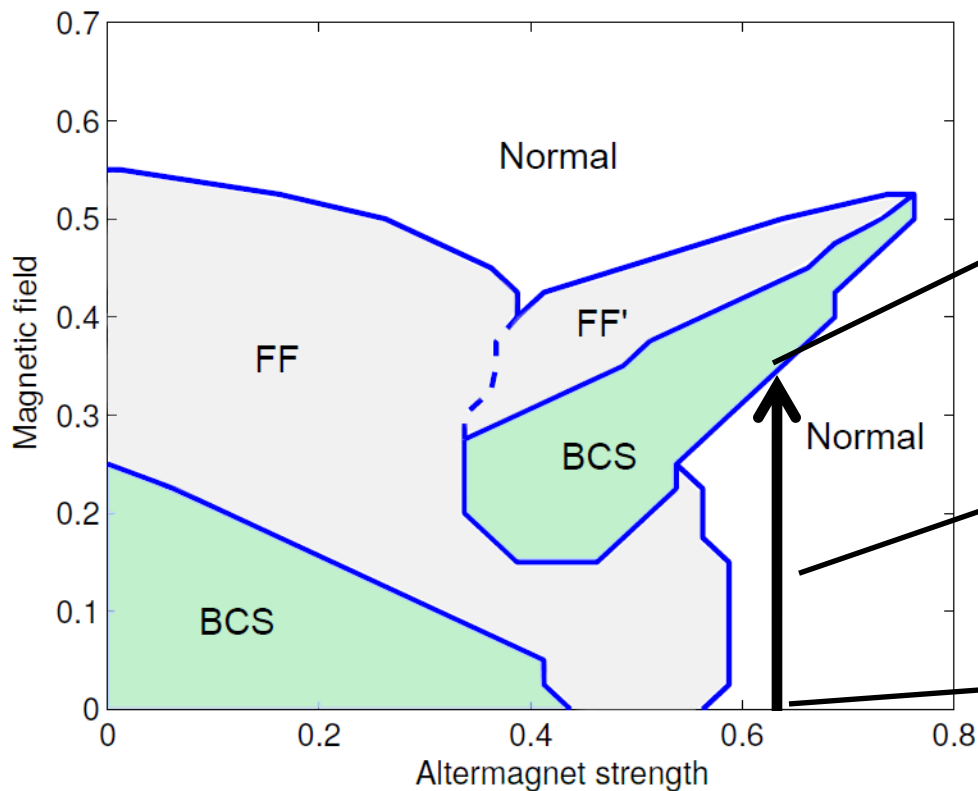


FF phase extends SC region for  
aligned AM and SC nodes

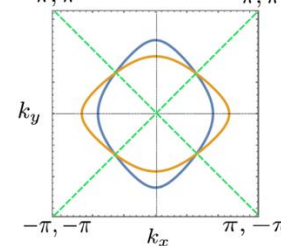
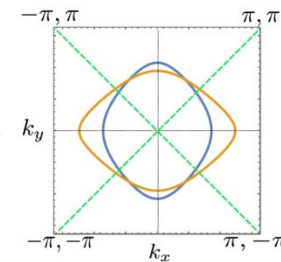
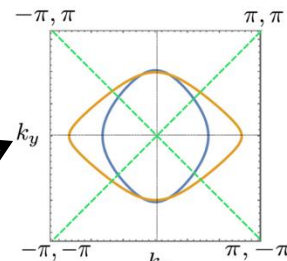
# Field-induced superconductivity



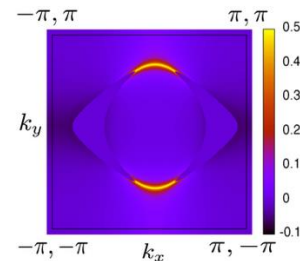
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Normal state  
Fermi surface



Pair density



Spin-degenerate  
BCS pairs

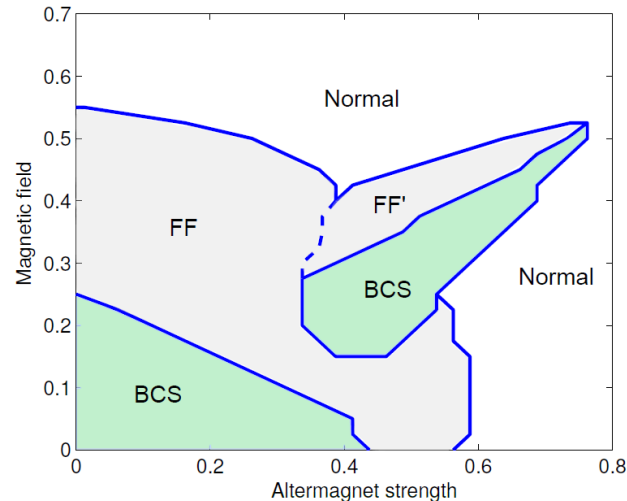
# Summary



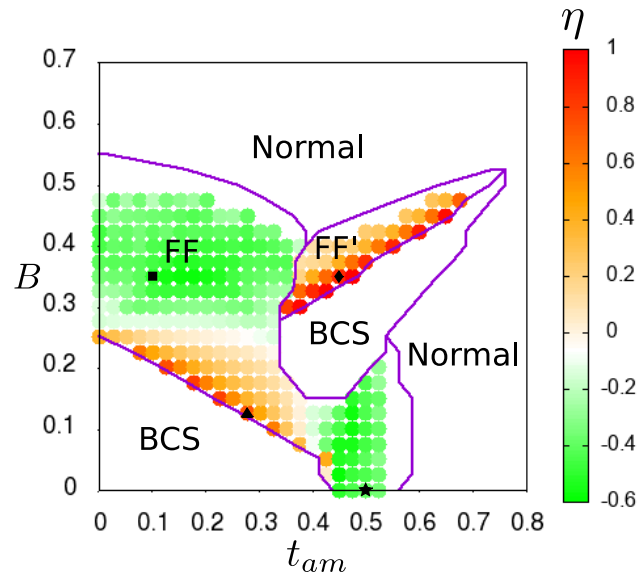
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## Superconductivity in altermagnets

- Finite-momentum (FF) pairing, when SC and AM nodes align
- Field-induced superconductivity at high fields

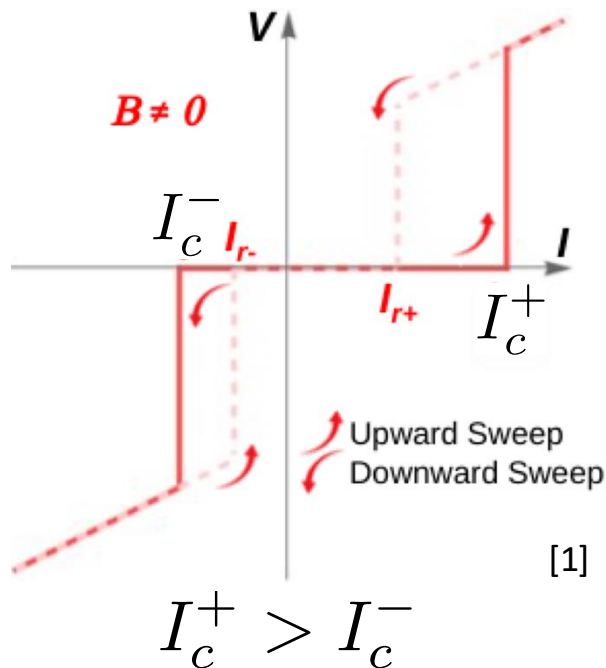


# Perfect superconducting diode effect



# Superconducting diode effect (SDE)

Dissipationless current in only one direction



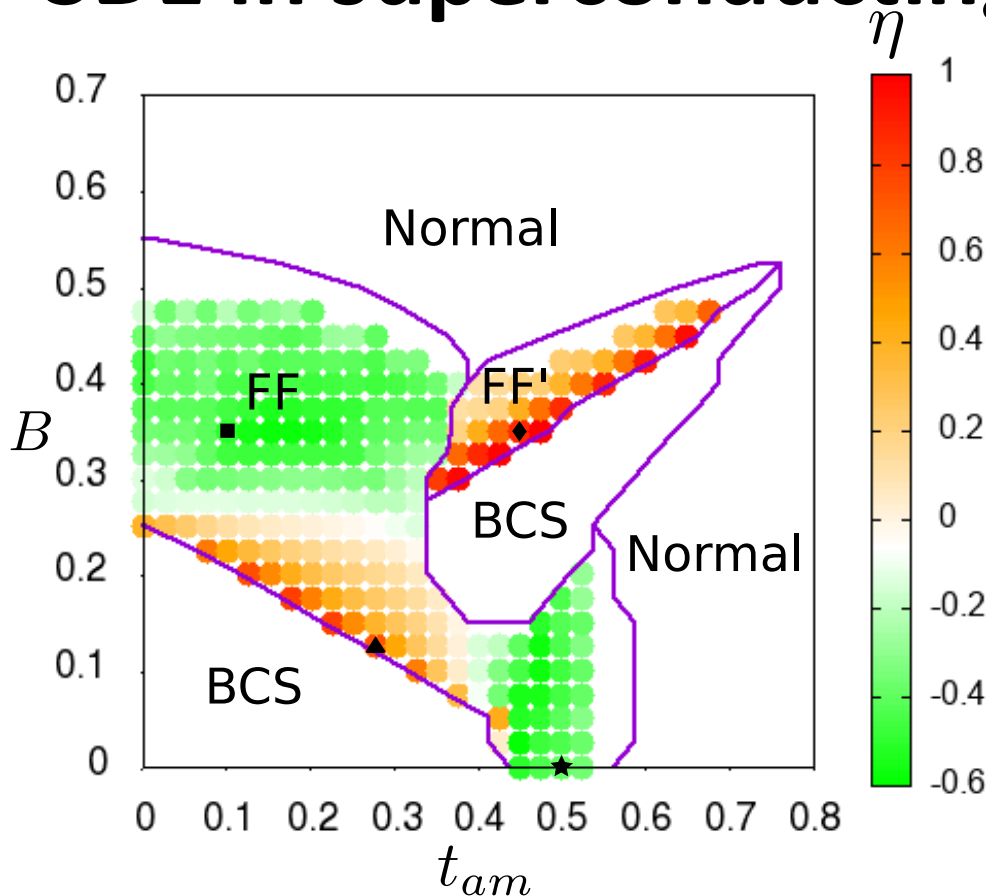
- Requires broken time-reversal and inversion symmetries
- FF states natural candidates [2]
- Efficiency  $\eta = \frac{I_c^+ - I_c^-}{I_c^+ + I_c^-}$  ( $\lesssim 60\%$  [3])

[1]: Nadeem et al, arXiv:2301.13565 (Nat. Rev. Phys. 5, 558 (2023));

[2] Yuan and Fu, PNAS 119, e2119548119 (2022); [3]: Ghosh et al, Nat. Mater. 23, 612 (2024)



# SDE in superconducting altermagnet



Spin-singlet  $d$ -wave pairing

Self-consistent solution for each  $Q$ ,  
ground state at  $Q^*$

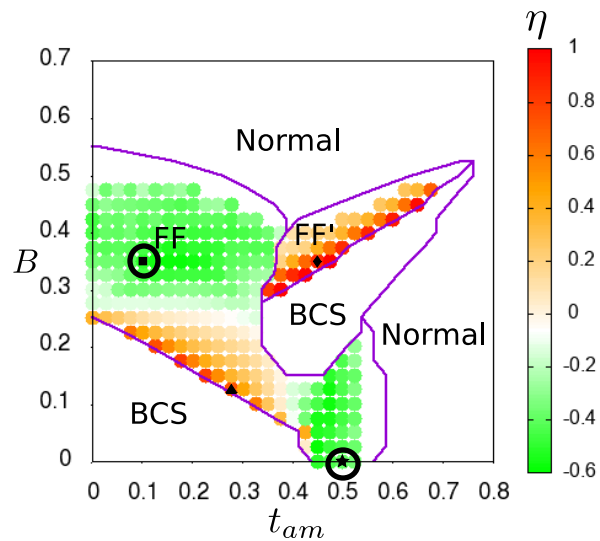
Extract current at each  $Q$ :

- $I(Q^*) = 0$
- Max currents gives  $\eta$

- Large  $\eta$  in FF regions

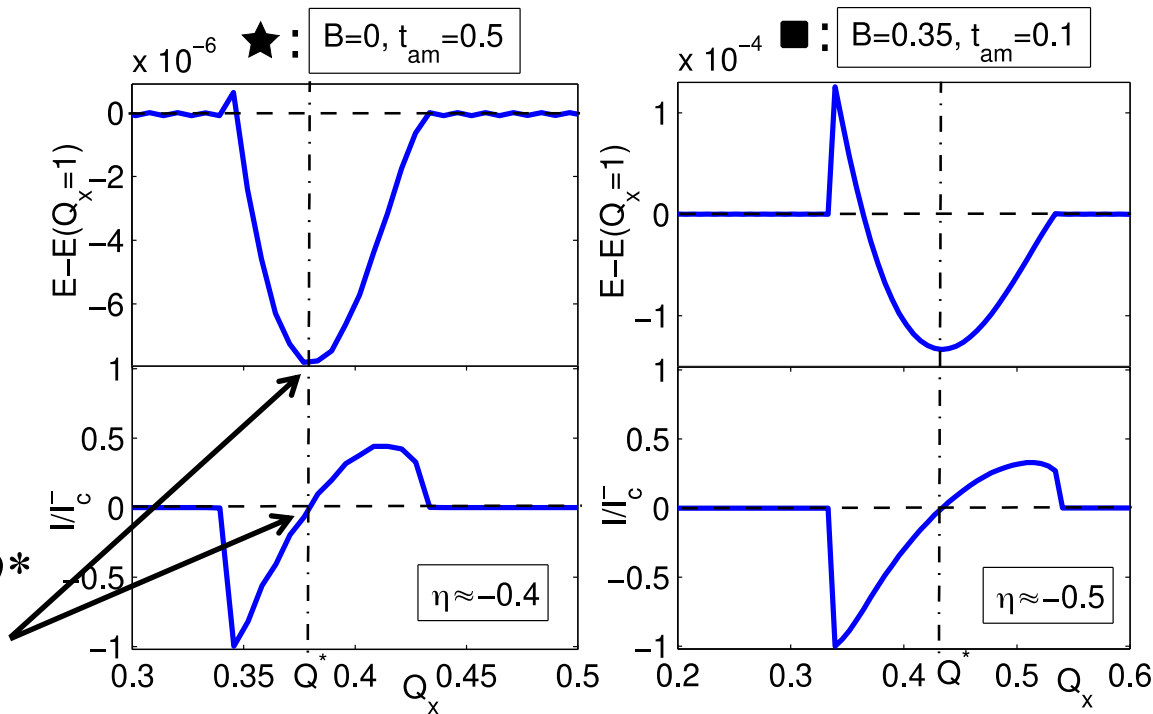
- **$\eta = 100\%$  at FF-BCS  
transitions**

# SDE deep in FF phase



- No current in ground state  $Q^*$

- $I = -e \frac{\partial E}{\partial Q}$

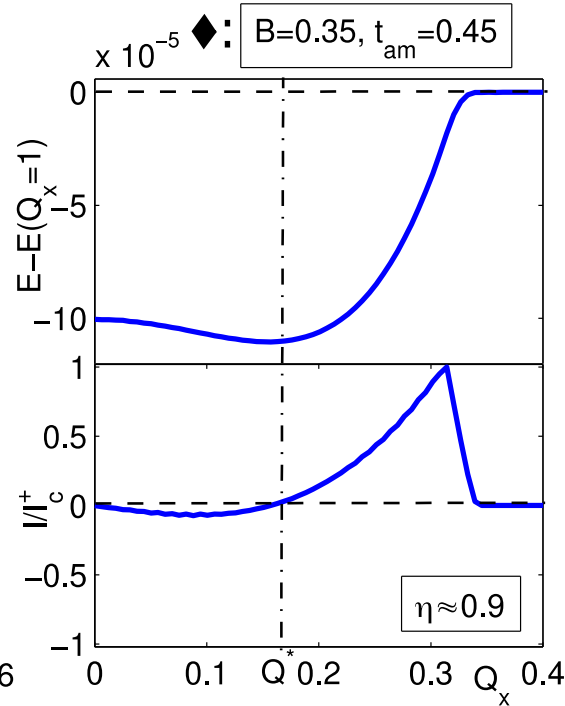
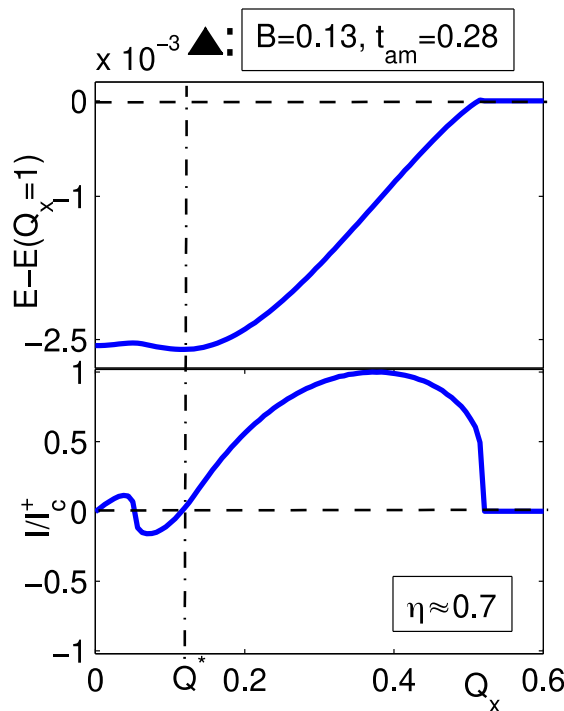
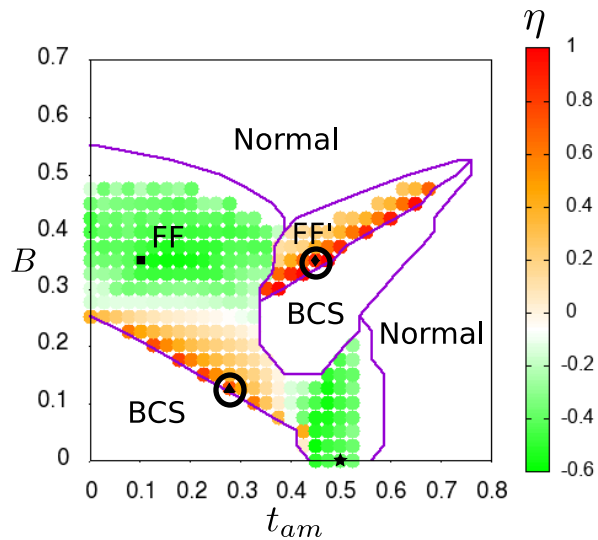


**Deep in FF phase:  $\eta \sim -50\%$**

# SDE at FF-BCS transitions



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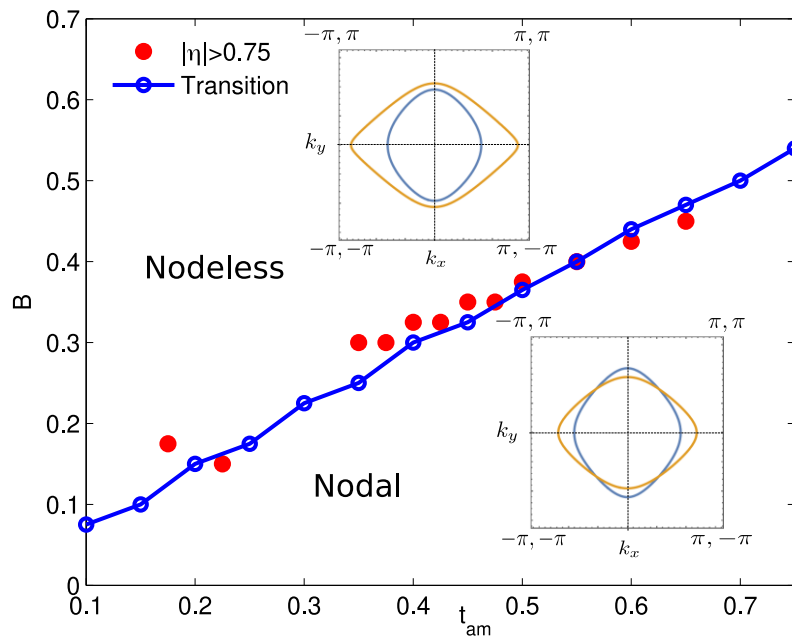
BCS and FF minima:  
large  $\eta$

BCS maximum, FF minimum:  
 $\eta \rightarrow 100\%$

# Topological protection of perfect SDE



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**Topologically protected nodal-to-nodeless transition [1]**  
**→ large- $\eta$  regions**

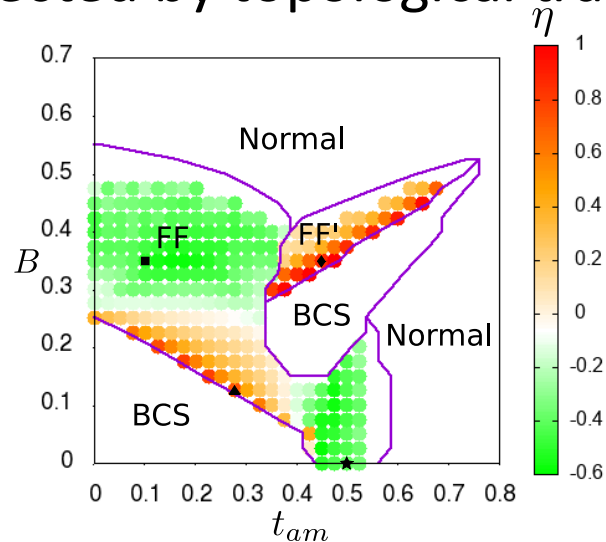
# Summary



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## Superconducting diode effect (SDE) in altermagnets

- Large throughout FF regions
- Perfect SDE, protected by topological transition



Chakraborty and Black-Schaffer, PRL 135, 026001 (2025)



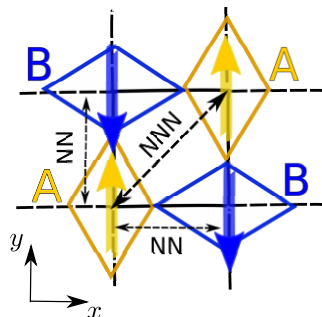
# Constraints on superconducting pairing

Are there constraints on superconducting symmetries in altermagnets?

# Altermagnet normal state



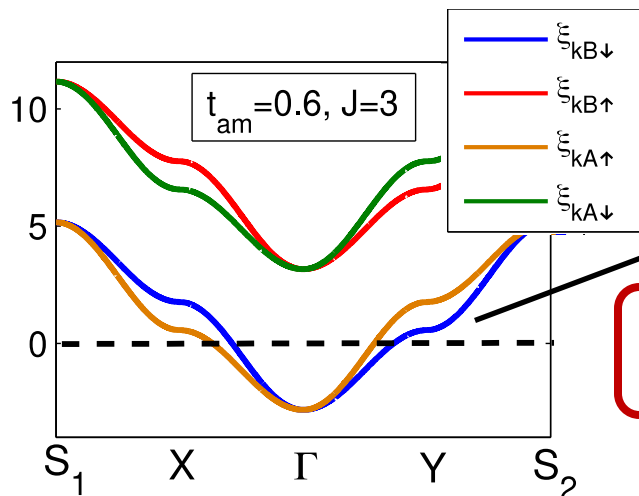
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$$H_{4b} = \sum_k \Psi^\dagger \hat{H}_{4b} \Psi, \quad \Psi^\dagger = \left( c_{kA\uparrow}^\dagger, c_{kB\uparrow}^\dagger, c_{kA\downarrow}^\dagger, c_{kB\downarrow}^\dagger \right)$$

$$\hat{H}_{4b} = (-2t(\cos k_x + \cos k_y) - \mu) \overset{\text{Spin}}{\sigma_0} \overset{\text{Sublattice}}{\tau_0} - \frac{t_{\text{am}}}{2} (\cos k_x - \cos k_y) \sigma_0 \tau_z - J \sigma_z \tau_z$$

Spin-degenerate dispersion                      Crystal field                      Exchange field



$$H_{2b} = \sum_k \Psi_{2b}^\dagger \hat{H}_{2b} \Psi_{2b}, \quad \Psi_{2b}^\dagger = \left( c_{kA\uparrow}^\dagger, c_{kB\downarrow}^\dagger \right)$$

$$\hat{H}_{2b} = (-2t(\cos k_x + \cos k_y) - \mu) \underset{\text{Spin \& sublattice}}{s_0} - \frac{t_{\text{am}}}{2} (\cos k_x - \cos k_y) s_z$$

**Spin & sublattice**

# Superconducting pairing



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| Spin symmetry ( $S$ )     | Momentum symmetry ( $P$ ) | Form factor | Real-space structure | Occurrence |
|---------------------------|---------------------------|-------------|----------------------|------------|
| Spin-singlet ( $S = -1$ ) | $s$ -wave ( $P = 1$ )     | 1           | Onsite               | ✗          |

$$\Psi_{2b}^\dagger = \left( c_{kA\uparrow}^\dagger, c_{kB\downarrow}^\dagger \right)$$

No onsite spin-singlet pairs



**No conventional  $s$ -wave pairing**



# Superconducting pairing



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| Spin symmetry ( $S$ )     | Momentum symmetry ( $P$ )       | Form factor           | Real-space structure | Occurrence |
|---------------------------|---------------------------------|-----------------------|----------------------|------------|
| Spin-singlet ( $S = -1$ ) | $s$ -wave ( $P = 1$ )           | 1                     | Onsite               | ✗          |
|                           | Extended $s$ -wave ( $P = 1$ )  | $\cos k_x + \cos k_y$ | NN                   | ✓          |
|                           | $d_{x^2-y^2}$ -wave ( $P = 1$ ) | $\cos k_x - \cos k_y$ | NN                   | ✓          |
|                           | $d_{xy}$ -wave ( $P = 1$ )      | $\sin k_x \sin k_y$   | NNN                  | ✗          |

$$\Psi_{2b}^\dagger = \left( c_{kA\uparrow}^\dagger, c_{kB\downarrow}^\dagger \right)$$

**Needed for finite-momentum pairing  
& superconducting diode effect**

# Superconducting pairing



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| Spin symmetry ( $S$ )          | Momentum symmetry ( $P$ )       | Form factor           | Real-space structure | Occurrence |
|--------------------------------|---------------------------------|-----------------------|----------------------|------------|
| Spin-singlet ( $S = -1$ )      | $s$ -wave ( $P = 1$ )           | 1                     | Onsite               | ✗          |
|                                | Extended $s$ -wave ( $P = 1$ )  | $\cos k_x + \cos k_y$ | NN                   | ✓          |
|                                | $d_{x^2-y^2}$ -wave ( $P = 1$ ) | $\cos k_x - \cos k_y$ | NN                   | ✓          |
|                                | $d_{xy}$ -wave ( $P = 1$ )      | $\sin k_x \sin k_y$   | NNN                  | ✗          |
| Equal-spin-triplet ( $S = 1$ ) | $p$ -wave ( $P = -1$ )          | $\sin k_x, \sin k_y$  | NN                   | ✗          |
| Mixed-spin-triplet ( $S = 1$ ) | $p$ -wave ( $P = -1$ )          | $\sin k_x, \sin k_y$  | NN                   | ✓          |

Superconductivity in altermagnets:

- **Spin-singlet  $d$ -wave** (or extended  $s$ -wave)
- **Mixed-spin-triplet  $p$ -wave**



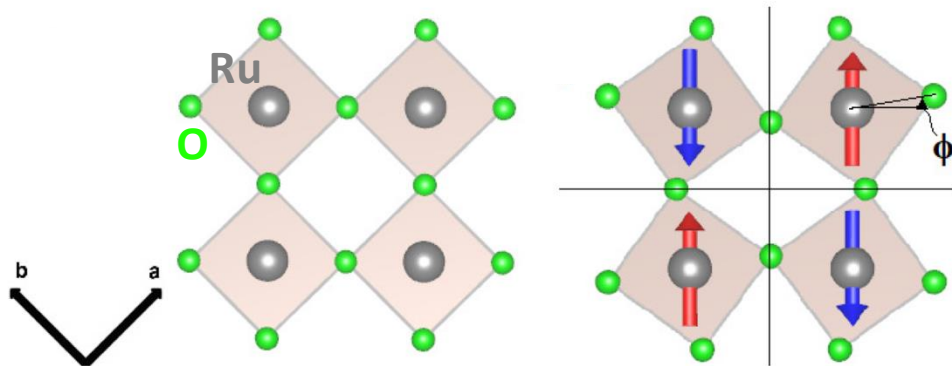
# Altermagnetism in $\text{Sr}_2\text{RuO}_4$

$\text{Sr}_2\text{RuO}_4$ : an intrinsic superconductor  
and altermagnet?

# Altermagnetism in $\text{Sr}_2\text{RuO}_4$ ?



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Distortion of oxygen octahedron:

- Soft phonon mode
- Surface distorted ( $\phi > 7^\circ$ ) [1]

DFT+U calculations & tight-binding modeling assuming finite octahedral distortion angle  $\phi$

Single  $\text{RuO}_2$  layer and bulk  $\text{Sr}_2\text{RuO}_4$

[1]: Damascelli et al, PRL 85, 5194 (2004)

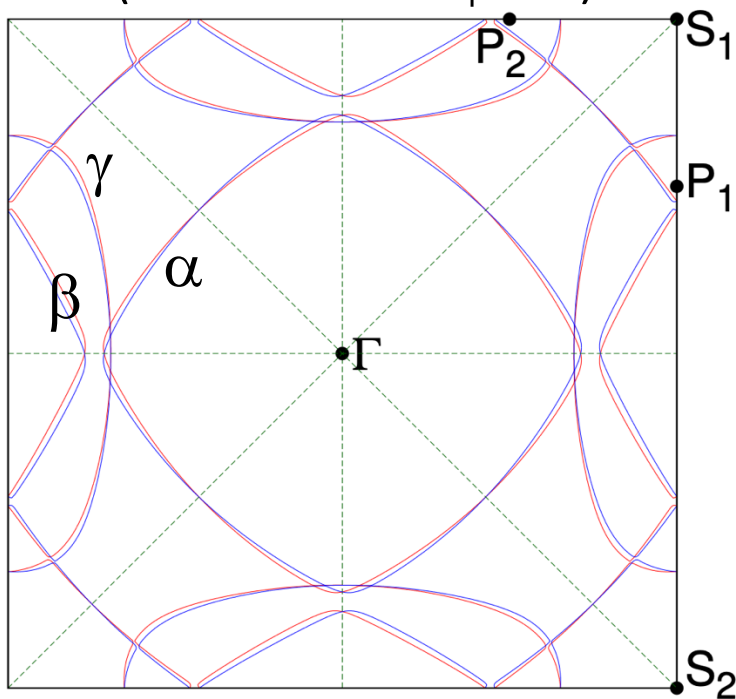
Autieri, Cuono, Chakraborty, Gentile, ABS, PRB 112, 014412 (2025)

# $g$ -wave AM in single layer $\text{Sr}_2\text{RuO}_4$



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Spin-split Fermi surface  
( $U = 1.0$  eV and  $\phi = 8^\circ$ )



**AM with  $g$ -wave symmetry**

Spin-splitting primarily on  $\alpha, \beta$   
bands =  $\text{Ru } d(xz), d(yz)$  orbitals

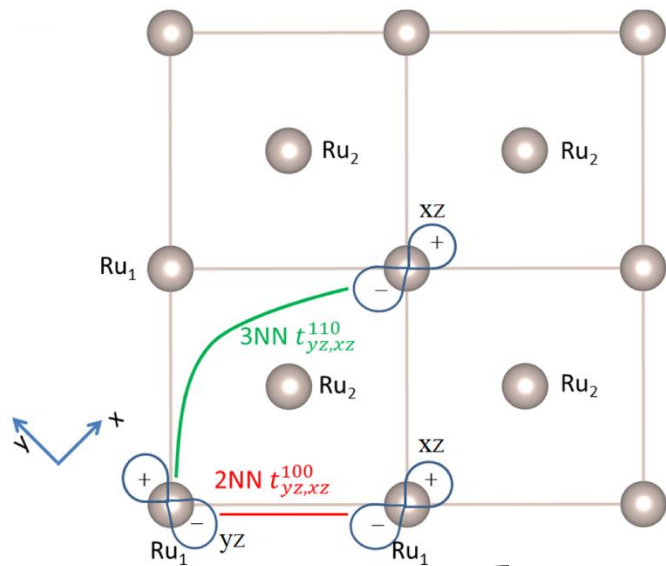
$\underbrace{\hspace{10em}}$

$yz$  orbitals

# Tight-binding modeling



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$$H_{\gamma z} = H_{\gamma z}^0 + H_{\gamma z}^{\text{AM}}$$

$$H_{\gamma z}^{\text{AM}} = \begin{pmatrix} & -\frac{\Delta_{\gamma z}}{2} & \\ 4t_{xz,yz}^{110} \sin k_x \sin k_y & & -\frac{\Delta_{\gamma z}}{2} \end{pmatrix}$$

Interorbital NNN  
hybridization for finite  $\phi$   
 $\sim t_{\text{am}}$

On-site spin-splitting  
 $\sim J$

$$\rightarrow \begin{cases} \mathcal{H}_{\gamma z}^0 = H_{\gamma z}^0 \sigma_0^{\text{spin}} \sigma_0^{\text{site}} \\ \mathcal{H}_{\gamma z}^{\text{AM}} = -\frac{\Delta_{\gamma z}}{2} \sigma_z^{\text{spin}} \sigma_0^{\text{orbital}} \sigma_z^{\text{site}} \\ \quad + 4t_{xz,yz}^{110} \sin k_x \sin k_y \sigma_0^{\text{spin}} \sigma_x^{\text{orbital}} \sigma_z^{\text{site}} \end{cases}$$

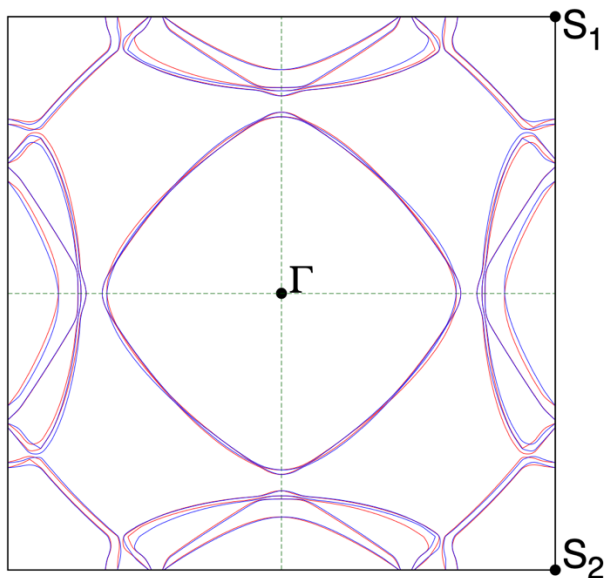
**Orbital-selective  
g-wave altermagnet**

# $d(xy)$ -wave AM in bulk $\text{Sr}_2\text{RuO}_4$



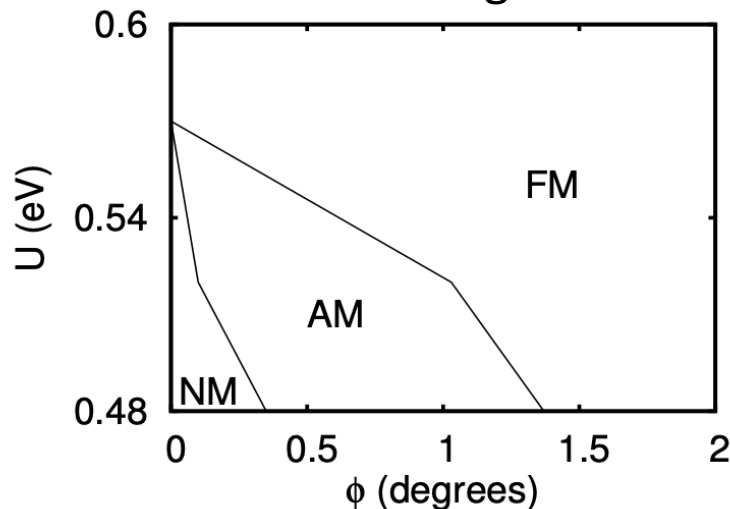
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Spin-split Fermi surface  
( $U = 1.0$  eV and  $\phi = 8^\circ$ )



**$d(xy)$ -wave AM** due to  
interlayer hybridization

Phase diagram



Including SOC: Weak ferromagnetism from  
canted moments out of ab-plane  
(through DM interaction)

# SC and AM in $\text{Sr}_2\text{RuO}_4$



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AM spin-splitting  $\sim 1$  meV at  $\phi = 1^\circ$ , cp to SC gap  $\sim 350$   $\mu\text{eV}$

SC: Likely spin-singlet

AM: Opposite spins on NN, NNNN, ... sites

If spin-singlet, intraorbital,  $\gamma z$  pairing:

→  $d(x^2-y^2)$ -wave or  $g$ -wave SC or combinations thereof

→  $d+ig$ : simplest possibility agreeing with exp [1]

(can also be interorbital and/or  $xy$ -orbital pairing)

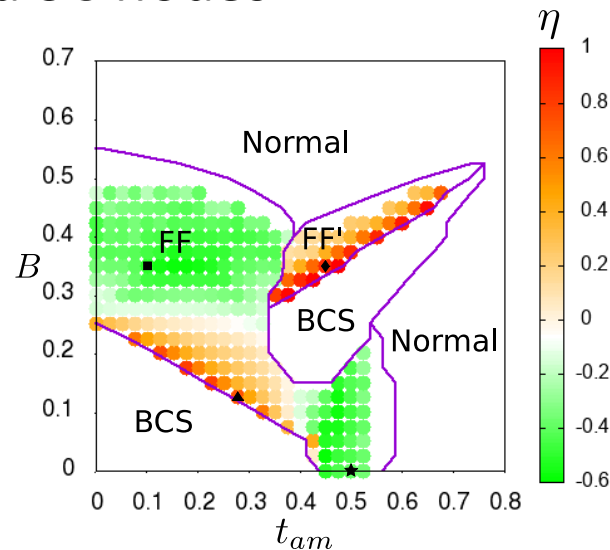


# Superconductivity in altermagnets



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- Finite-momentum SC, when AM and SC nodes align
- Field-induced SC at high fields
- Perfect SDE protected by topology
- No conventional  $s$ -wave SC (or equal-spin  $p$ -wave SC)
- $\text{Sr}_2\text{RuO}_4$ : native SC and AM with octahedral distortion





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