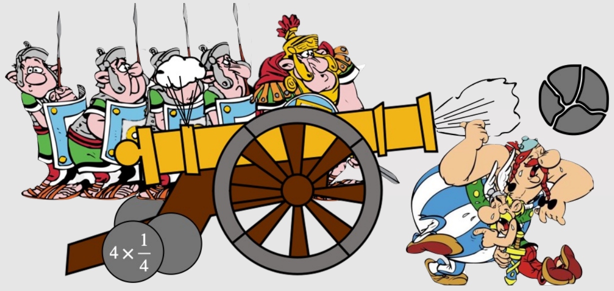
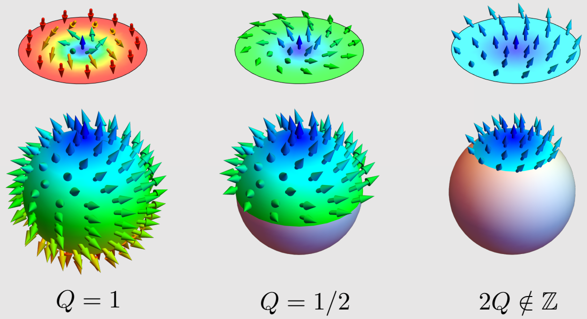
Fractional Topological Charges in 2D Magnets & Aharonov-Bohm scattering

Nina del Ser, Caltech (previously Cologne)



Romans failing to fire their cannon because they used fractional charges



(the talk will be about these charges, don't worry..)

Opening Credits

Collaborators

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Walter Benjamin-Programm

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Paper

"Fractional topological charges in two-dimensional magnets"

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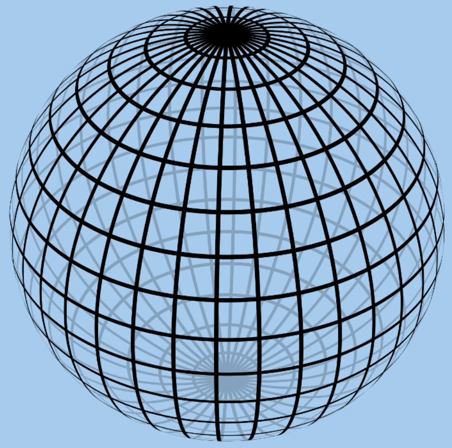
Structure of the talk

1. Background
2. Fractional topological charges in magnets
3. Scattering of magnons from fractional charges (Aharonov-Bohm effect)

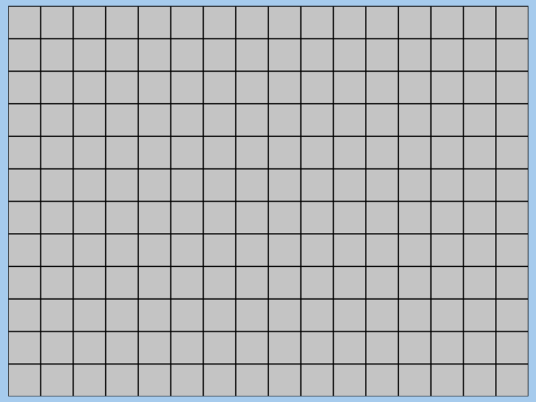
1. Background

Can we always map a **sphere** to a **plane**?

2-sphere



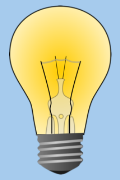
2D plane

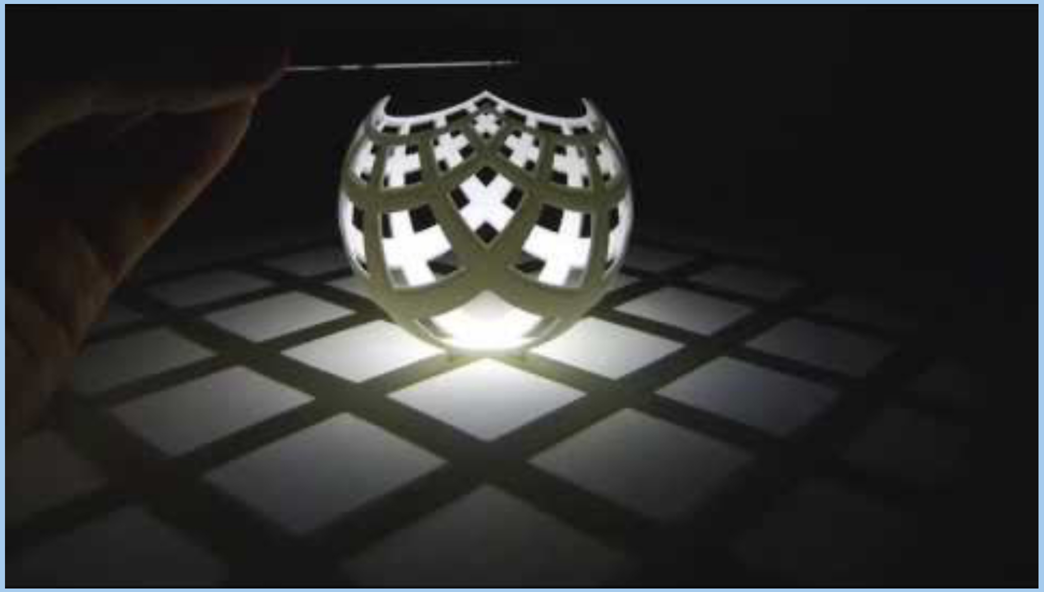


$m(\theta, \phi)$

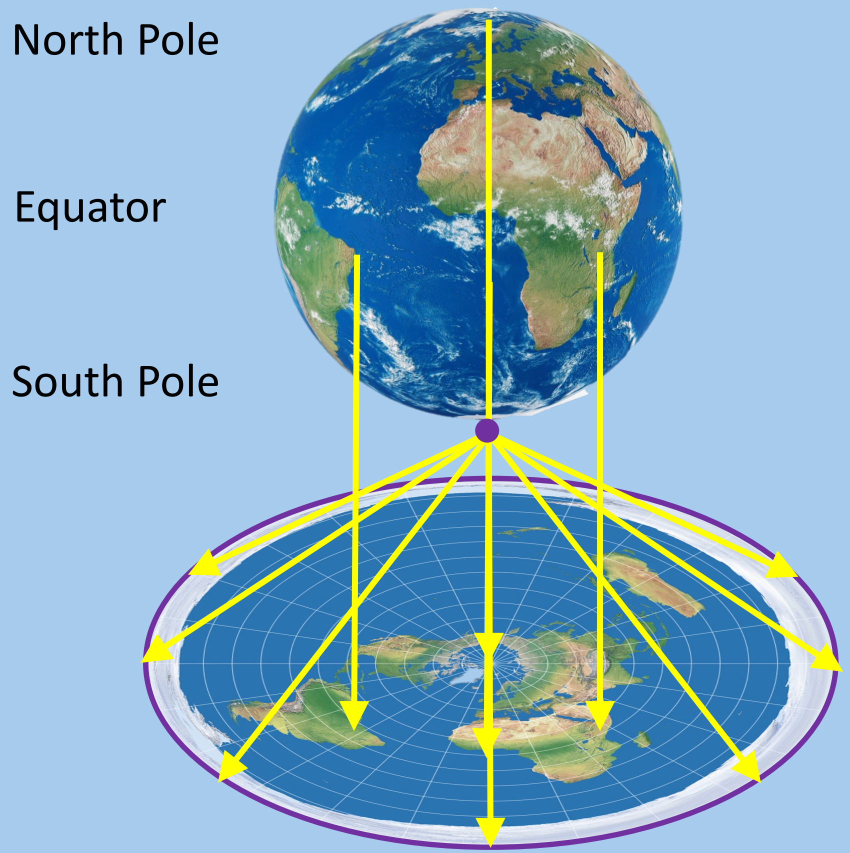
Yes, e.g. by stereographic projection

Stereographic projection





Stereographic projection uses



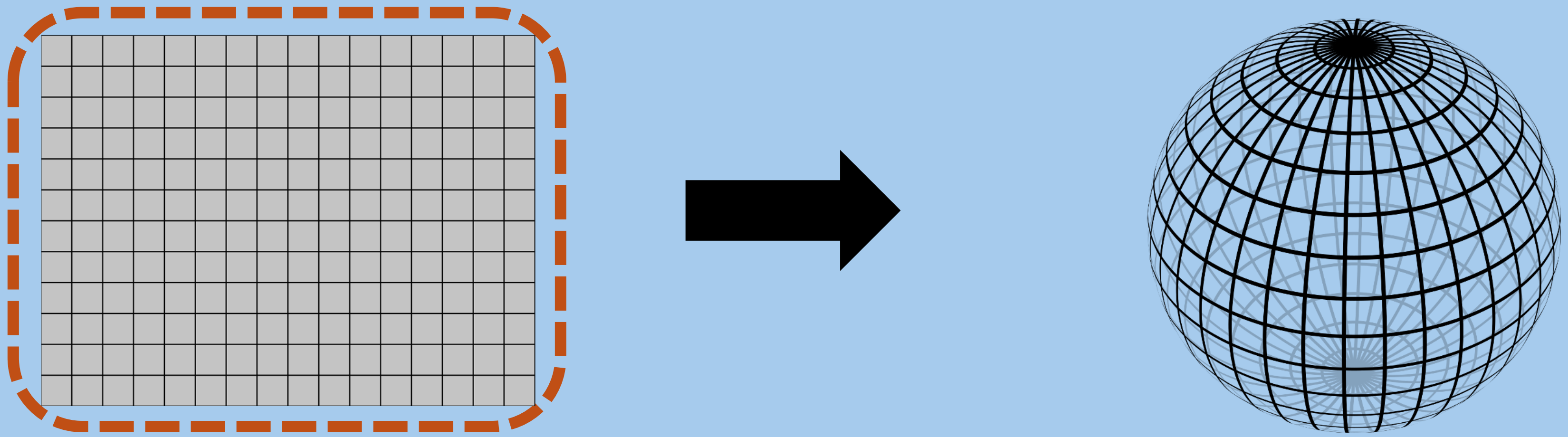
Stereographic technique illustrated by Rubens, 1613

Sanson's Map of the World, 1691

Can we always map a **plane** to a **sphere**?

2D plane

2-sphere

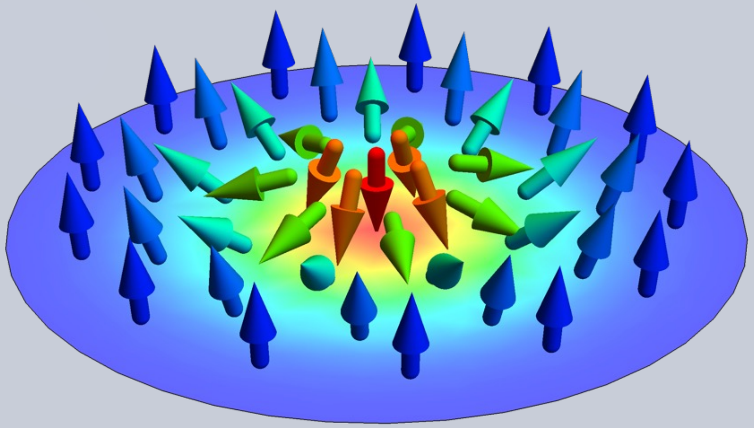


$M($

No, not if $M(x, y)$ **multi-valued** at $x, y = \infty$!

Connection to magnetism

- Slowly varying magnetic textures are well-described by a continuous magnetisation field $\boldsymbol{M}(\boldsymbol{r})$



$$\boldsymbol{M}(\boldsymbol{r}) = \begin{pmatrix} M_x(\boldsymbol{r}) \\ M_y(\boldsymbol{r}) \\ M_z(\boldsymbol{r}) \end{pmatrix} \qquad \text{In 2D, } \boldsymbol{r} = \begin{pmatrix} x \\ y \end{pmatrix}$$

A

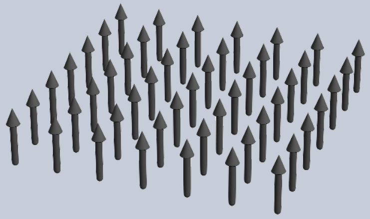
$\boldsymbol{M}(\boldsymbol{r})$ determined by minimising the **free energy** of the magnet

Free energy I

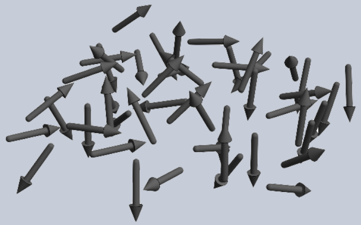
simplest case

$$F = \int \frac{J}{2} \boldsymbol{M} \cdot \nabla^2 \boldsymbol{M} \, dA$$

add some chiral, anisotropy,
external *B*-field terms
for more excitement…



ferromagnet

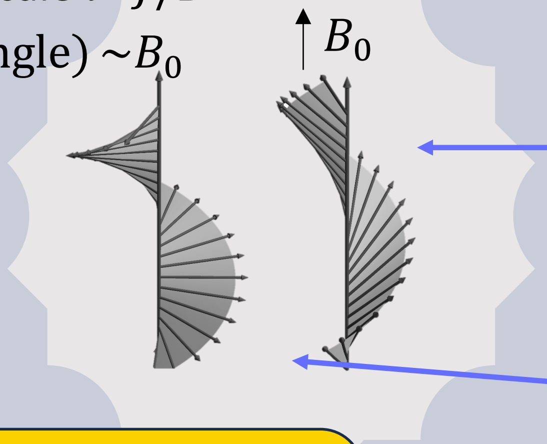


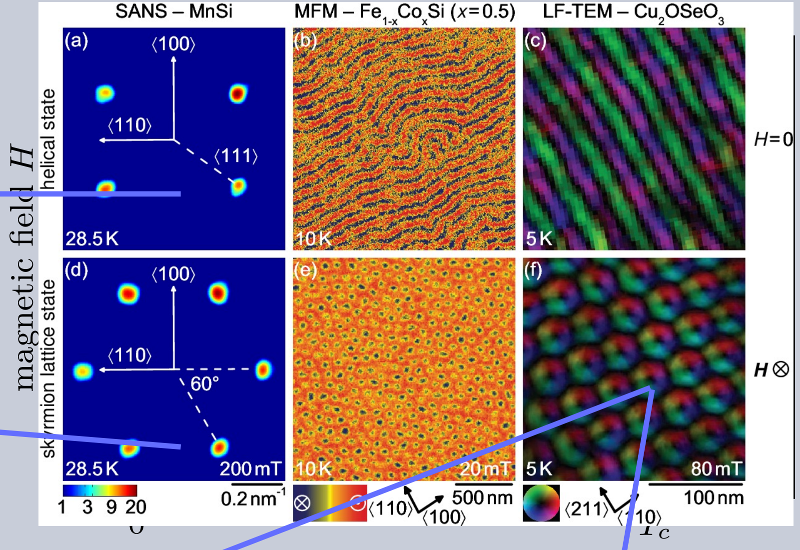
paramagnet

Free energy II

length scale $l \sim J/D$

$\cos(\text{cone angle}) \sim B_0$

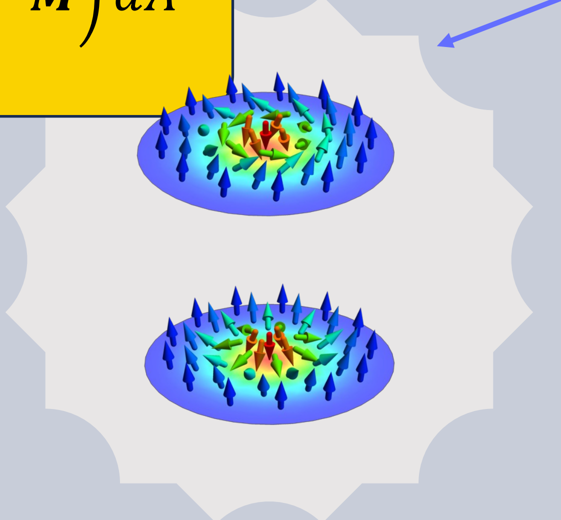


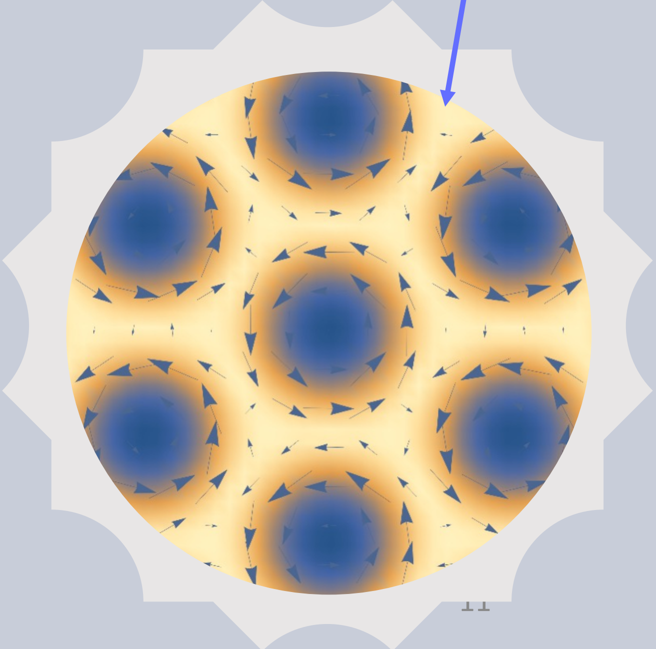


Exist at low&room temp.

$l \sim 1\text{nm} - 1\mu m$

$$F = \int \left(\frac{J}{2} \boldsymbol{M} \cdot \nabla^2 \boldsymbol{M} \pm D\boldsymbol{M} \cdot \nabla \times \boldsymbol{M} + \kappa_u M_z^2 - \boldsymbol{B}_0 \cdot \boldsymbol{M} \right) dA$$





Dzyalozhinskii-Moriya Interaction (DMI)

-**breaks inversion** symmetry
-favours **twisting**

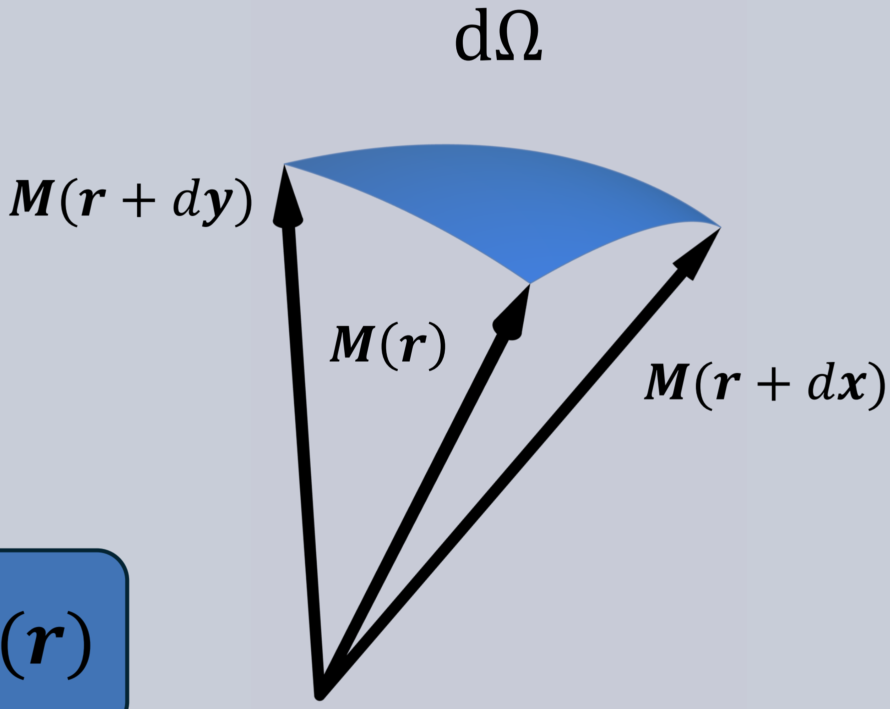
Quantifying “twisting”

- Define topological charge density

$$\rho_{\text{top}}(\boldsymbol{r}) = \boldsymbol{M} \cdot (\nabla_x \boldsymbol{M} \times \nabla_y \boldsymbol{M})$$

$(M = 1)$

$$\mathrm{d}\Omega = \mathrm{d}x\,\mathrm{d}y\,\rho_{\text{top}}(\boldsymbol{r})$$



Quantifying “twisting” – topological charge

- Define total topological charge Q

$$Q = \frac{1}{4\pi} \int \mathrm{d}x\, \mathrm{d}y\, \rho_{\mathrm{top}}\,(\boldsymbol{r}) = \frac{1}{4\pi} \int \mathrm{d}\Omega$$

Q preserved under smooth local deformations (no tears, no singularities)

Q mod Z determined entirely by boundary magnetisation!

Calculating Q for skyrmions

$$Q = \frac{1}{4\pi}\int \mathrm{d}x\,\mathrm{d}y\,\rho_{\mathrm{top}}(\boldsymbol{r}) = \frac{1}{4\pi}\int \mathrm{d}\Omega$$

Option 1) brute force: insert $\boldsymbol{M} = \begin{pmatrix} \sin\ \theta(\boldsymbol{r})\cos\phi(\boldsymbol{r}) \\ \sin\ \theta(\boldsymbol{r})\sin\phi(\boldsymbol{r}) \\ \cos\ \theta(\boldsymbol{r}) \end{pmatrix}$ with $\theta(r=0)=\pi, \theta(r=\infty)=0, \phi=\chi+h$

into Q & calculate

Option 2) just visualise it!

Calculating Q for skyrmions

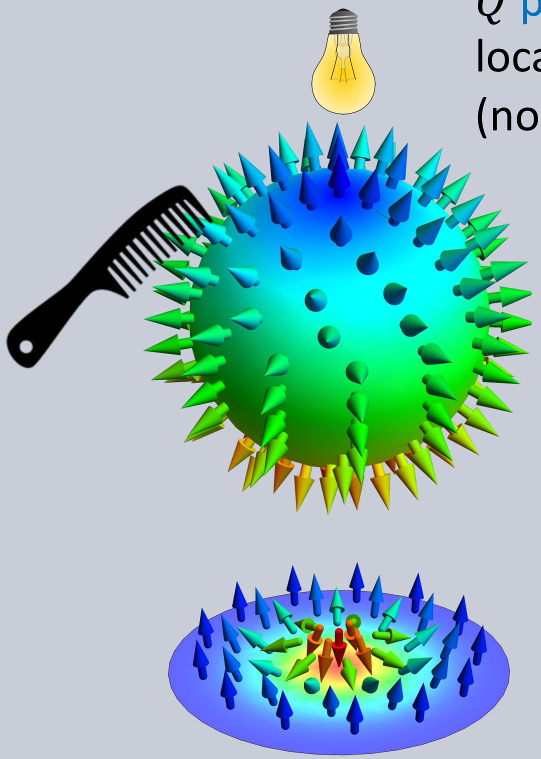
$$Q = \frac{1}{4\pi}\int \mathrm{d}x\,\mathrm{d}y\,\rho_{\mathrm{top}}(\boldsymbol{r}) = \frac{1}{4\pi}\int \mathrm{d}\Omega$$

Option 2) just visualise it!

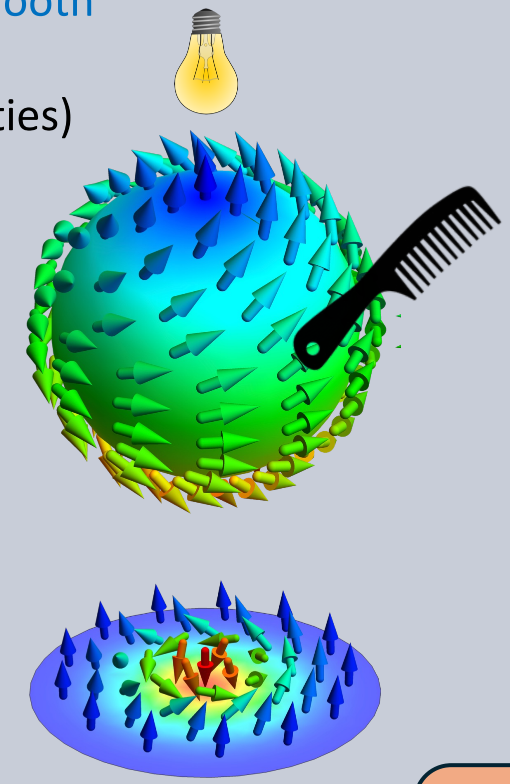
Q preserved under smooth local deformations (no tears, no singularities)

$\boldsymbol{M}(\boldsymbol{r})$ wraps **once** around the sphere

$\rightarrow |Q| = 1$



Néel skyrmion



Bloch skyrmion

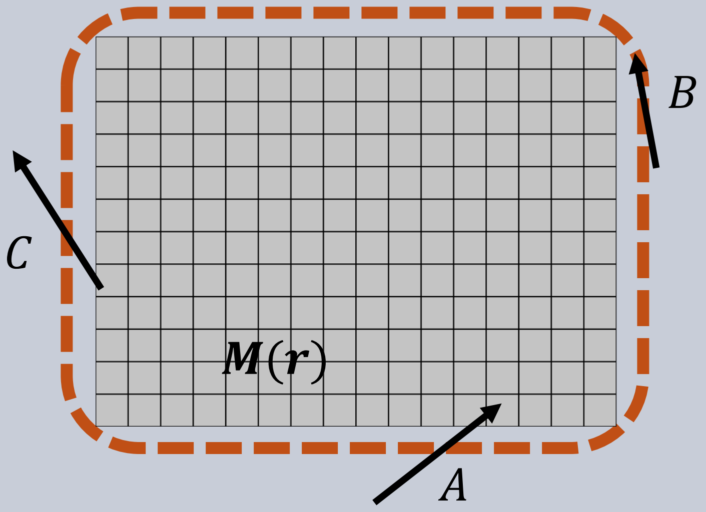
Argument valid for any smooth magnetic texture with **single-valued** $\lim_{\boldsymbol{r}\to\infty} M(\boldsymbol{r})$

$\rightarrow$ all such textures have $\boldsymbol{Q} \bmod \mathbf{1} = \mathbf{0}$

What about **multi-valued** $\lim_{\boldsymbol{r}\to\infty} M(\boldsymbol{r})$?

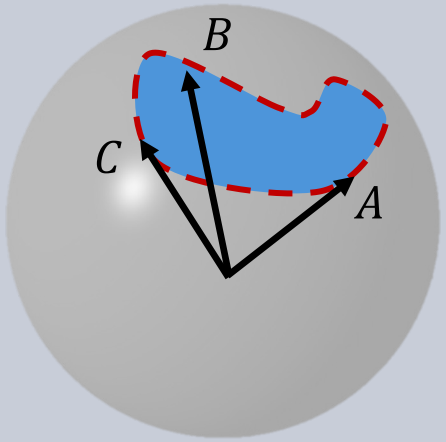
Calculating Q when $\lim_{\boldsymbol{r}\to\infty} \boldsymbol{M}(\boldsymbol{r})$ is multi-valued

2D plane



Plot all boundary values of $\boldsymbol{M}(\boldsymbol{r})$ in spin space

(note: **not** stereographic projection)



Always get **closed** loop

$$Q = \frac{\text{blue shaded area}}{\text{total spherical area}} < 1$$

For multi-valued $\lim_{\boldsymbol{r}\to\infty} M(\boldsymbol{r})$, Q is **fractional**!

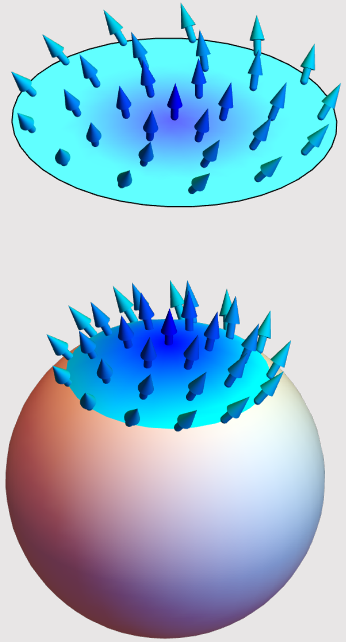
2. Examples of fractional topological charges in magnets

Fractional Vortex

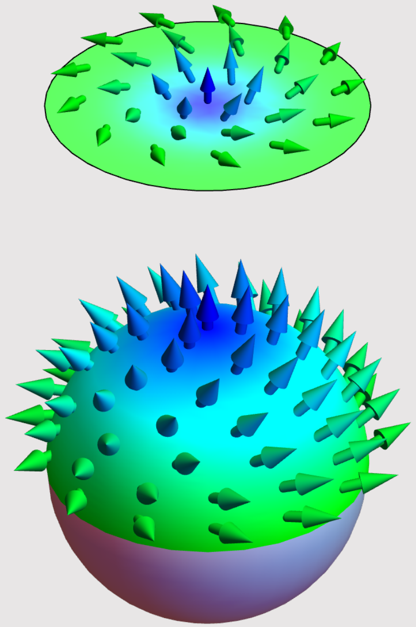
$$F = \int J\left(\frac{1}{2}\nabla_i M_j\,\nabla_i M_j - b_0 M_z + \kappa_u M_z^2\right)\mathrm{d}A$$

F minimised when $\boldsymbol{M}$ lies on a cone with $M_z = \frac{b_0}{2\kappa_u}$

b_0 $\uparrow$



$b_0 = 0$



$2Q \notin \mathbb{Z}$ $\qquad$ $Q = 1/2$

$$Q = \frac{1}{2}\left(1 - \frac{b_0}{2\kappa_u}\right)$$

But, hard to realise experimentally…

Exception: skyrmion decaying into two (bi)merons, see e.g.

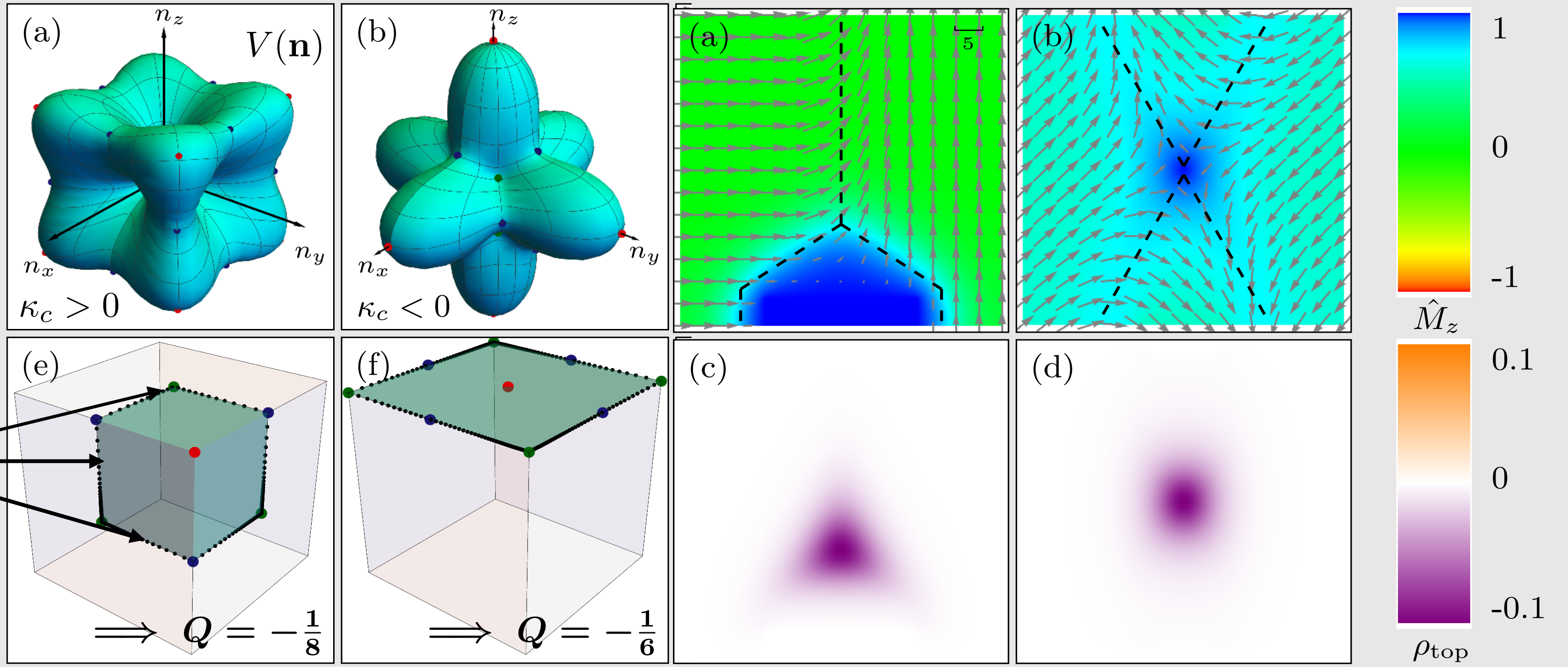
“Spontaneous Voretx-Antivortex Pairs and their topological transitions in a chiral-lattice magnet”Adv.Mater.2024, 36, 2306441

“Reversible Transformation between Isolated Skyrmions and Bimerons” *Nano Lett.* 2022, 22, 21, 8559–8566

Cubic Magnets

See also *"Non-Abelian Vortices in Magnets"* (arXiv:2205.15264) by F. Rybakov and O. Eriksson

$$F = \int J\left(\frac{1}{2}\nabla_i M_j \nabla_i M_j - \kappa_c\left(M_x^4 + M_y^4 + M_z^4\right)\right) \mathrm{d}A$$



Straight line interpolation guaranteed by additional symmetries

→ Q exactly quantised

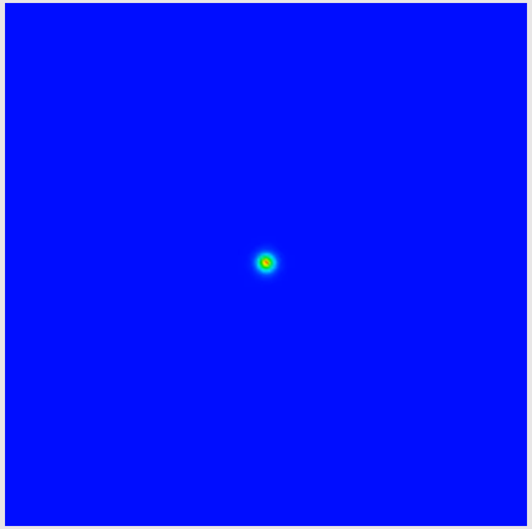
Occurs naturally in cubic magnets!

Exploding Skyrmion

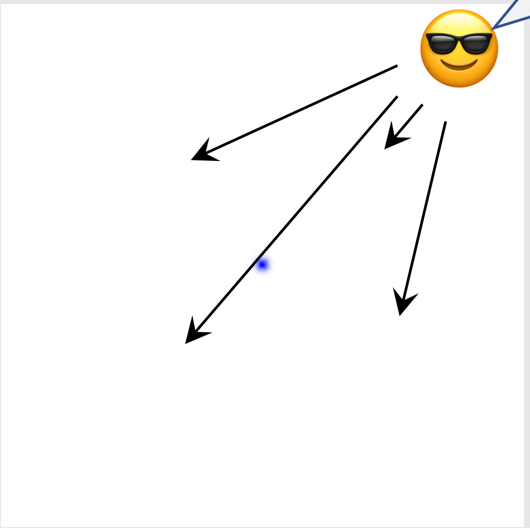
$$F = \int \left(\frac{J}{2}\nabla_i M_j \nabla_i M_j \pm D\boldsymbol{M}\cdot\nabla\times\boldsymbol{M} - B_0 M_z - \kappa_c\left(M_x^4 + M_y^4 + M_z^4\right)\right)\mathrm{d}A \quad \kappa_c = 3.2$$

Each sub-particle has charge $Q = -\frac{1}{4}$!

Easy axes
$\mathrm{P}(1,0,0)$



Magnetisation $M_z(\boldsymbol{r})$

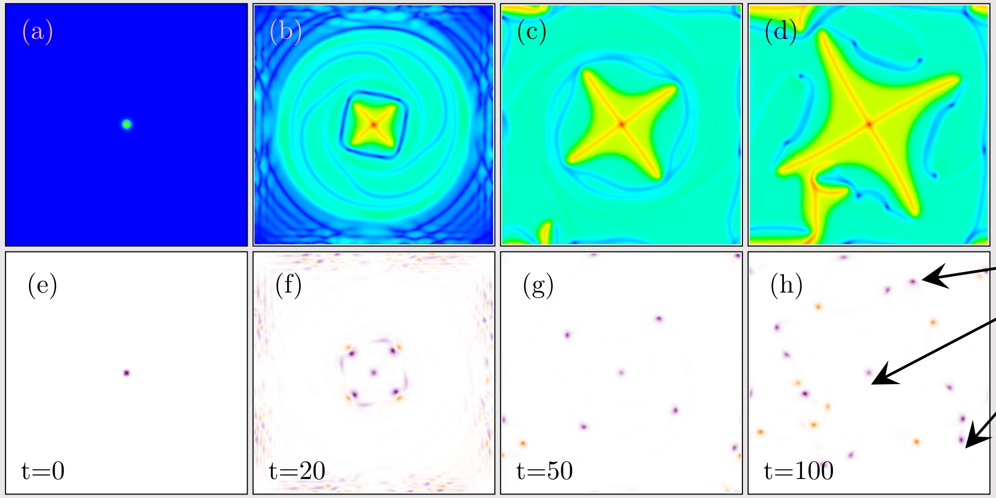


Topological charge density $\rho_{top}(\boldsymbol{r})$

Exploding Skyrmion – negative κ_c

$$F = \int \left(\frac{J}{2} \nabla_i M_j \nabla_i M_j \pm D\boldsymbol{M} \cdot \nabla \times \boldsymbol{M} - B_0 M_z - \kappa_c \left(M_x^4 + M_y^4 + M_z^4 \right) \right) \mathrm{d}A \quad \kappa_c = -3.2$$

Easy axes
$(\pm 1, \pm 1, \pm 1)$



OFF

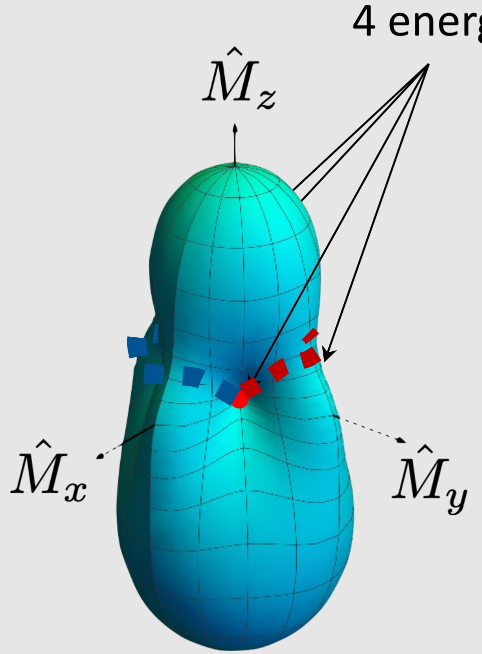
Each sub-particle has charge $Q = \pm\frac{1}{6}$!

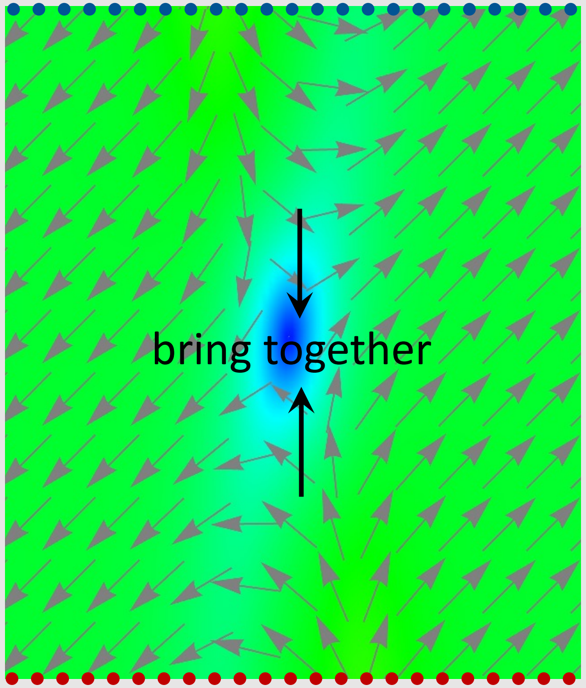
Always six more negatively charged sub-particles to preserve $Q = -1$!

Domain Walls with Broken Symmetry

$$F = \int J\left(\frac{1}{2}\partial_i M_j \partial_i M_j \boxed{+ D(M_z \nabla \cdot \boldsymbol{M} - \boldsymbol{M} \cdot \nabla M_z)} \boxed{+\kappa_p\left(M_x^4 + M_y^4\right) - b_0 M_z)}\right) \mathrm{d}A$$

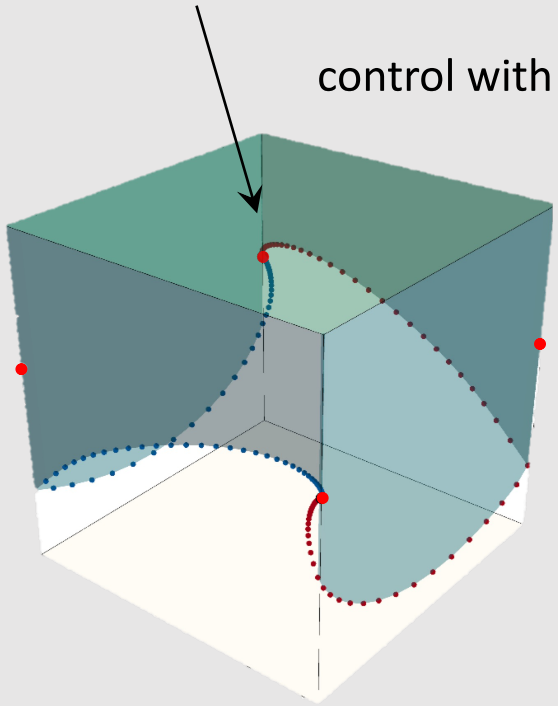
twisting





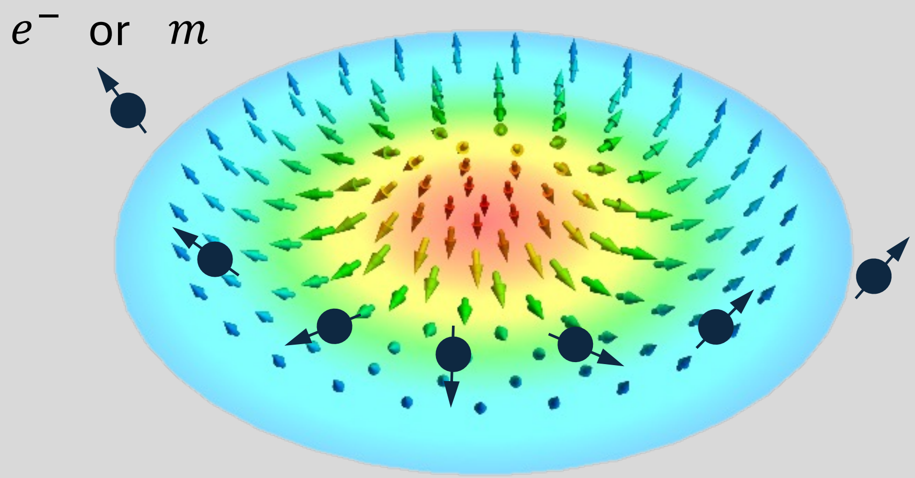
Trapped $Q \approx -0.6$

control with κ_p, b_0



3. Aharonov-Bohm scattering

What makes topological magnetic textures interesting?



- Electrons and magnons traveling through the texture pick up **Berry phases**

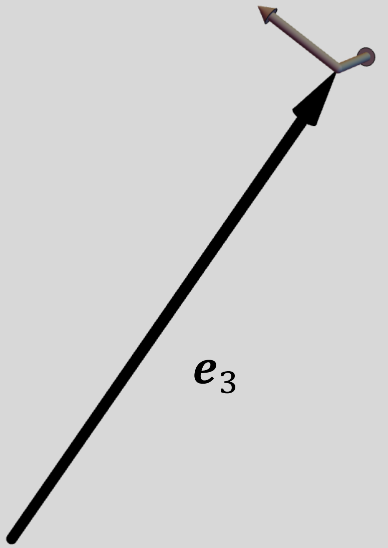
→ Emergent EM fields

Berry phases - magnons

$\{a(\boldsymbol{r}), a^*(\boldsymbol{r}')\} = \delta(\boldsymbol{r}, \boldsymbol{r}')$

enforces spin commutation relation

$\{M_j(\boldsymbol{r}), M_k(\boldsymbol{r}')\} = i\,\epsilon_{ijk} M_i(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}')$



$\boldsymbol{e}_3(\boldsymbol{r}, t), \boldsymbol{e}_\pm(\boldsymbol{r}, t)$

functions of $\boldsymbol{r}, t$!

$$\boldsymbol{M} = U\boldsymbol{M}', \quad U = \begin{pmatrix} \boldsymbol{e}_- & \boldsymbol{e}_+ & \boldsymbol{e}_3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix}, \boldsymbol{M}' = \begin{pmatrix} a \\ a^* \\ 1 - a^* a \end{pmatrix}$$

Substitute into exchange part of free energy F ...

$$\frac{1}{2}J(\nabla_i \boldsymbol{M})^2 = \frac{1}{2}J(U\nabla_i \boldsymbol{M}' + \nabla_i U \boldsymbol{M}')^2$$

$$= \frac{1}{2}J((\boxed{\nabla_i + U^{-1}\nabla_i U})\boldsymbol{M}')^2$$

$$-\frac{1}{\hbar^2}J|(p_i + qA_i)a|^2 \quad \text{with} \quad \boxed{A_i = -\frac{i\hbar}{q}(U^{-1}\nabla_i U)_{1,1}}$$

Only care about number-conserving terms $\sim a^*\, a$

→only keep **diagonal** part of $U^{-1}\nabla_i U$

Berry phases - magnons

$$A_i = -\frac{i\hbar}{q}(U^{-1}\nabla_i U)_{1,1} = \frac{\hbar}{q}\cos(\theta)\nabla_i\phi$$

Can we **distinguish** fractional and integer Φ_m in a magnon scattering experiment?

EM tensor $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$

In 2D: $\rightarrow B_z(\boldsymbol{r}) = \frac{\hbar}{q}\sin(\theta)\left(\nabla_x\theta\nabla_y\phi - \nabla_y\theta\nabla_x\phi\right) = \frac{\hbar}{q}\rho_{\text{top}}(\boldsymbol{r})$

In units of flux quantum $\Phi_0 = \frac{2\pi\hbar}{q}, B_z(\boldsymbol{r}) = \frac{1}{2\pi}\rho_{\text{top}}(\boldsymbol{r})$

$$\rightarrow \Phi_m = 2Q$$

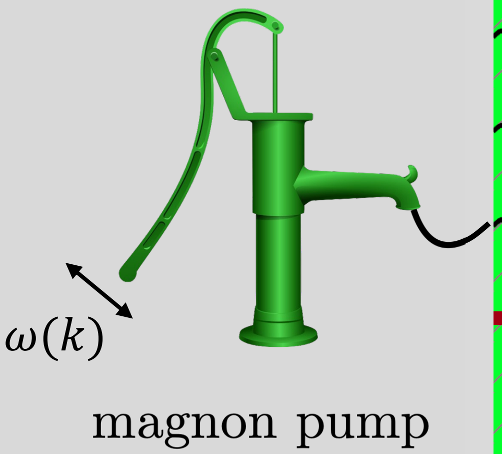
(half-)integer Q seen by magnon as integer flux

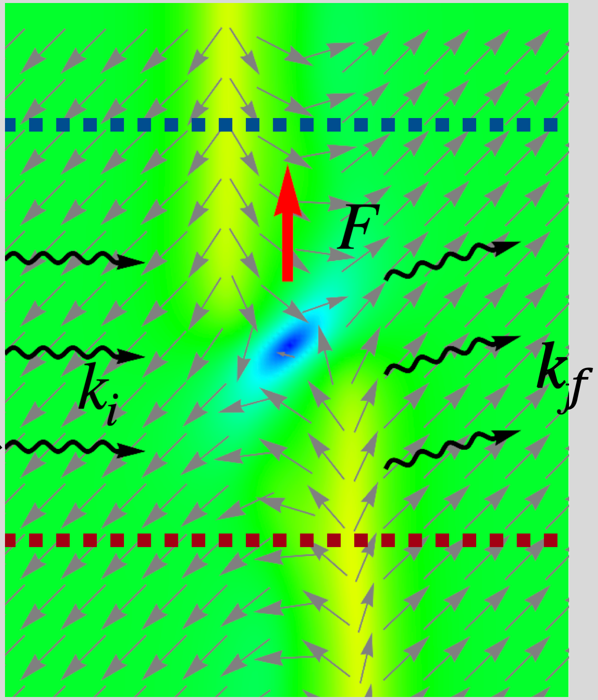
Note: for **electrons**

$$\Phi_e = Q$$

(similar derivation, but U is a 2x2 matrix)

Magnon scattering





$\omega(k) = \omega_g + k^2$

$\Delta \boldsymbol{k} = k \begin{pmatrix} \cos\chi - 1 \\ \sin\chi \end{pmatrix}$

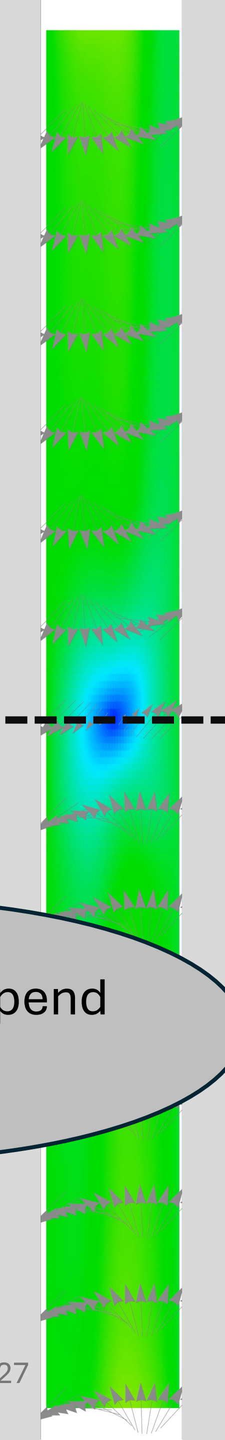
- Incoming magnons are scattered, experiencing change in their total momentum, $\Delta \boldsymbol{P}_m$
- Net force $\boldsymbol{F}$ on fractional charge given by $\boldsymbol{F} = -\frac{\mathrm{d}\boldsymbol{P}_m}{\mathrm{d}t}$

Solve Thiele eq.

$\boldsymbol{G} \times \boldsymbol{V} + \alpha\, D\boldsymbol{V} = \boldsymbol{F}$

How does F_y depend on k, Φ_m?

$\rightarrow V_y = \frac{F_y}{\alpha\, D_{yy}},\ V_x = 0$



Scattering force

$$F_y = \hbar k \; j_m \; \sigma_\perp$$

momentum of 1 magnon

magnon current

perp. scatt. cross-section

$$j_m \sim v_g(k) e^{-\frac{2\alpha\omega(k)}{v_g(k)} L_x}$$

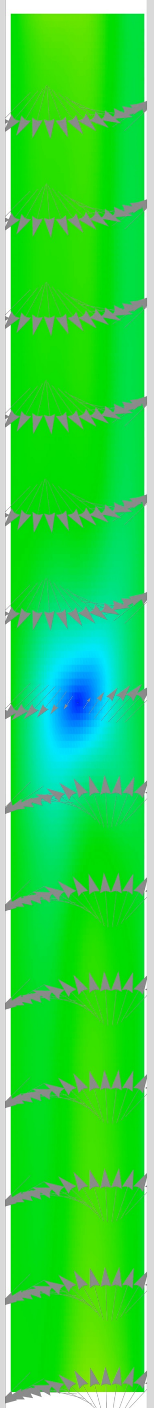
(see App.F of paper for derivation and prefactors)

$\omega(k) = \omega_g + k^2, \; v_g(k) = \frac{\mathrm{d}\omega}{\mathrm{d}k}$,

α Gilbert damping

Predicted by Aharonov & Bohm in 1959, but major source of controversy for years!

$$\sigma_\perp = \begin{cases} \dfrac{\sin(4\pi Q)}{k}, & kR_d \ll 1 \;\; \text{(AB limit)} \\ \dfrac{4\pi Q}{k}, & kR_d \gg 1 \;\text{(classical limit)} \end{cases}$$



Scattering force

$$j_m \sim v_g(k) e^{-\frac{2\alpha\omega(k)}{v_g(k)} L_x}$$

(see App.F of paper for derivation and prefactors)

$$F_y = \hbar k \; j_m \; \sigma_\perp$$

momentum of 1 magnon

magnon current

pe
cro

v_g magnon group velocity,
α Gilbert damping

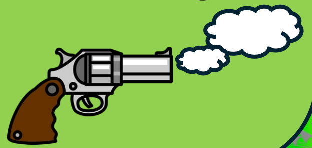
& Bohm in 1959,
ntroversy for years!

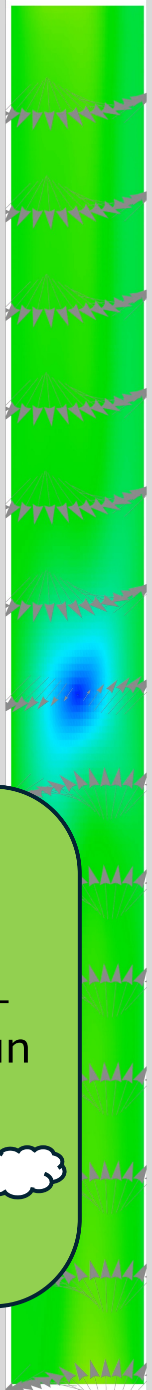
For $Q \neq \text{(half)integer}$, $\sigma_\perp$ is **singular** for small k!

$$\sigma_\perp = \begin{cases} \frac{\sin(4\pi Q)}{k} \\ \frac{4\pi Q}{k}, \end{cases}$$

$$\rightarrow \lim_{k\rightarrow 0} F_y \sim j_m(k) \sim 0 \text{ for } \alpha > 0$$ ☹

Instead, use sign change of $\sigma_\perp$ for $\frac{1}{2} < 2|Q| < 1$ as smoking gun signature of fractional charge





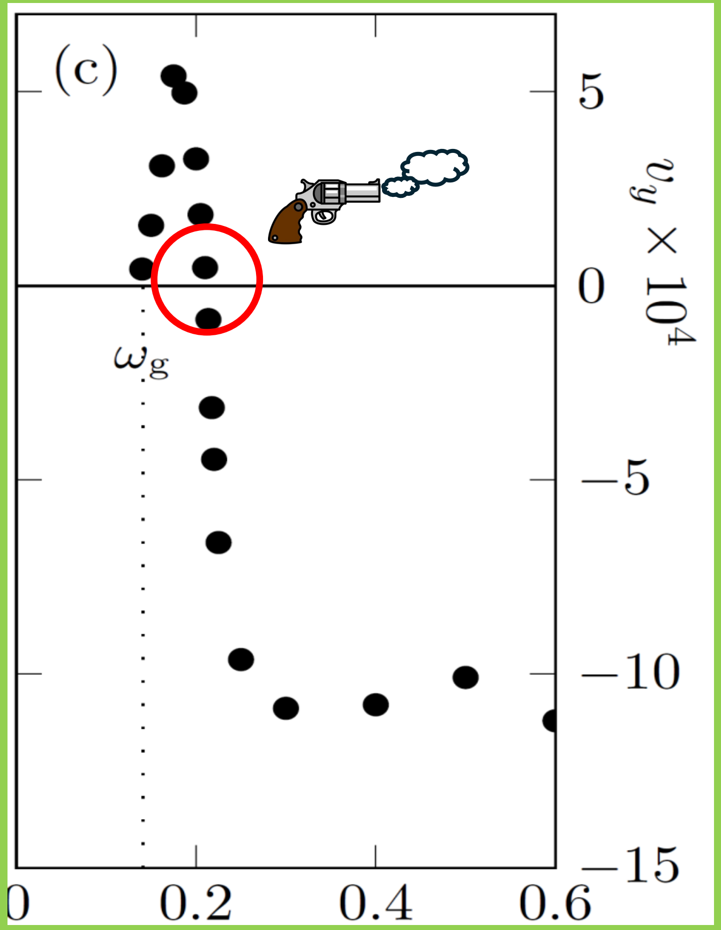
Scattering f

$F_y = \hbar k\, j$

momentum of 1 magnon

ma cur

$\sigma_\perp = \begin{cases} \frac{\sin(4\ldots}{k} \\ \frac{4\pi Q}{k}, \end{cases}$



vertical velocity V_y

magnon frequency $\omega(k)$

$\frac{\ldots(k)}{(k)} L_x$

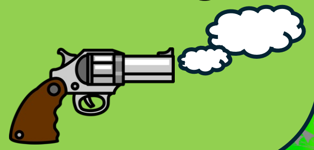
(see App.F of paper for derivation and prefactors)

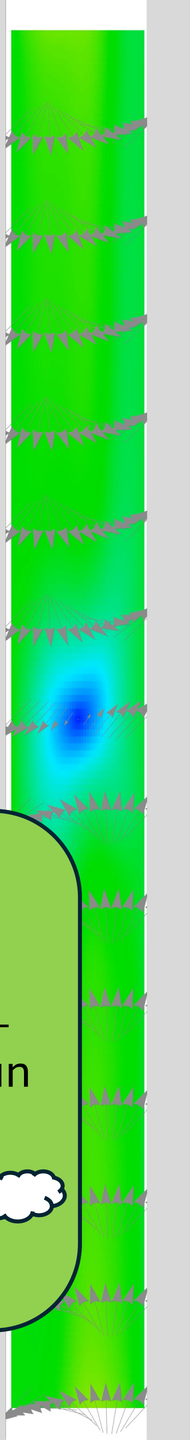
city,

in 1959,
y for years!

0 ☹

Instead, use sign change of $\sigma_\perp$ for $\frac{1}{2} < 2|Q| < 1$ as smoking gun signature of fractional charge



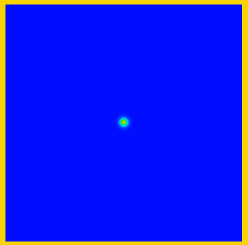


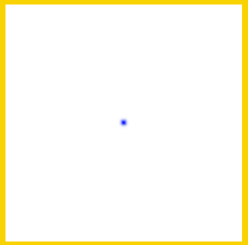
Conclusions & Outlook

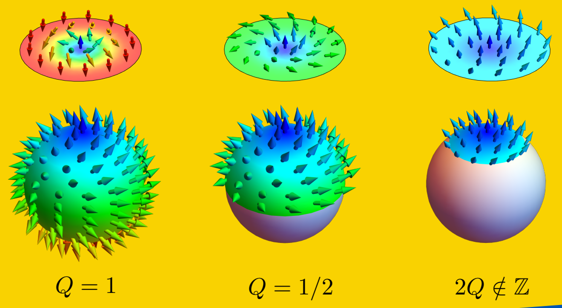
Summary

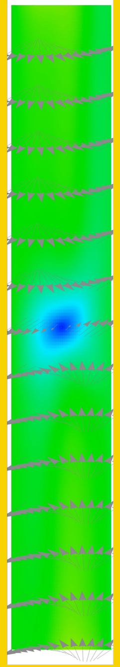
- Magnetic textures with fractional topological charge occur naturally in generic systems

- They **singularly** scatter low energy magnons and electrons







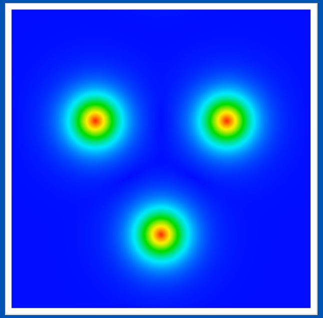


Paper

"Fractional topological charges in two-dimensional magnets"
NdS, I. el Achchi & A.Rosch
DOI:10.1103/PhysRevB.110.094442 (2024)

Future

- Can we stabilize the exploding skyrmion and create **bound states** of fractional charges?
- How would such states respond to driving?



Bound state of 3 driven skyrmions