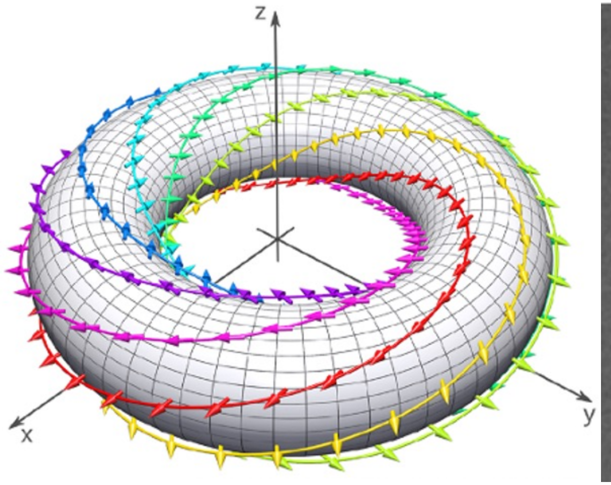
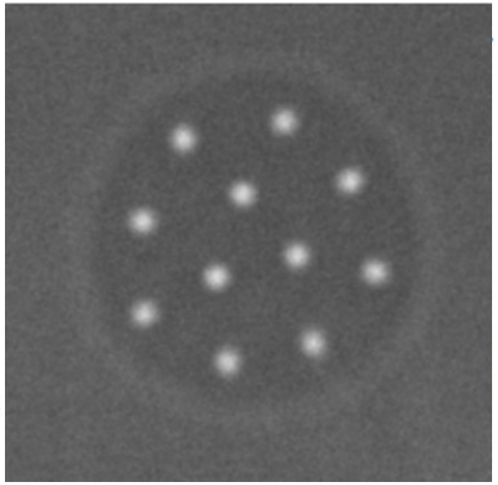
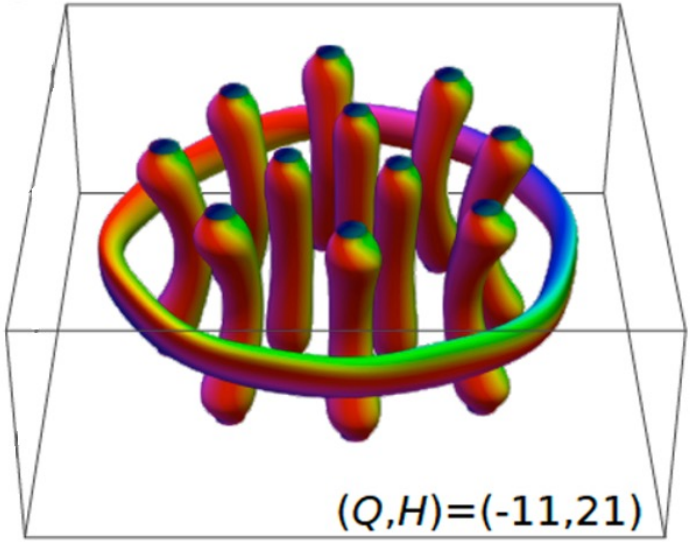
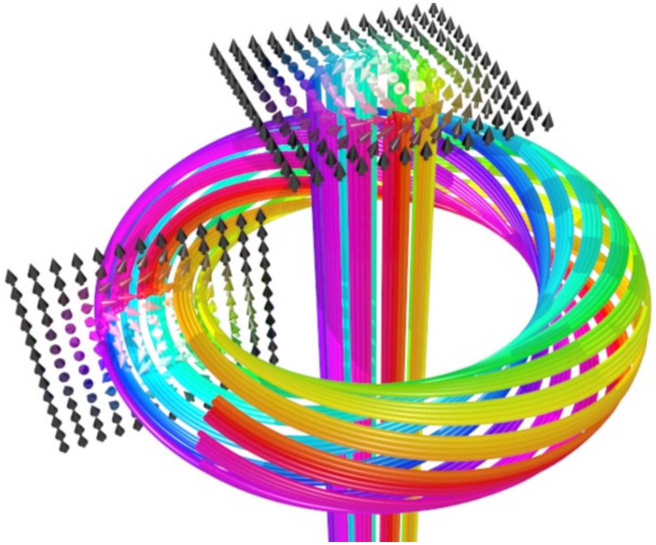




Magnetic hopfion rings

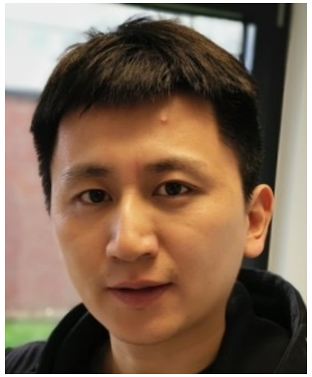
Nikolai Kiselev

Peter Grünberg Institute,
Forschungszentrum Jülich,
Germany

Ernst Ruska-Center for Microscopy and Spectroscopy with Electrons
FZ-Jülich

High Magnetic
Field Lab. Hefei


Fengshan Zheng

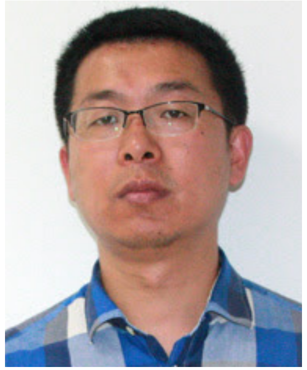

Luyan Yang


Wen Shi


Joseph Vimal Vas


András Kovács


Rafal Dunin-Borkowski


Haifeng Du

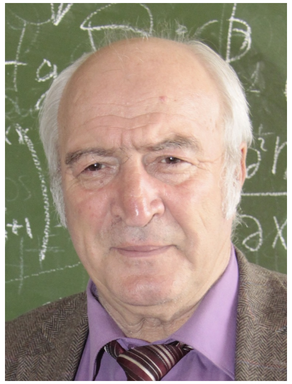
Uppsala
University

IMP
Ekaterinburg

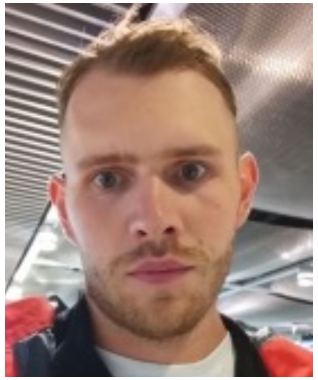
Aachen
University

Peter Grünberg Institute
FZ-Jülich


Filipp Rybakov


Aleksandr Borisov


Christof Melcher


Vladyslav Kuchkin


Moritz Sallermann

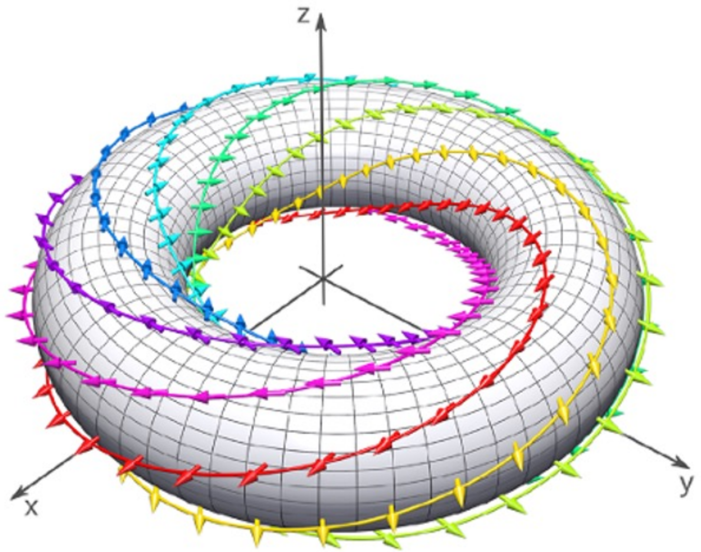

Andrii Savchenko

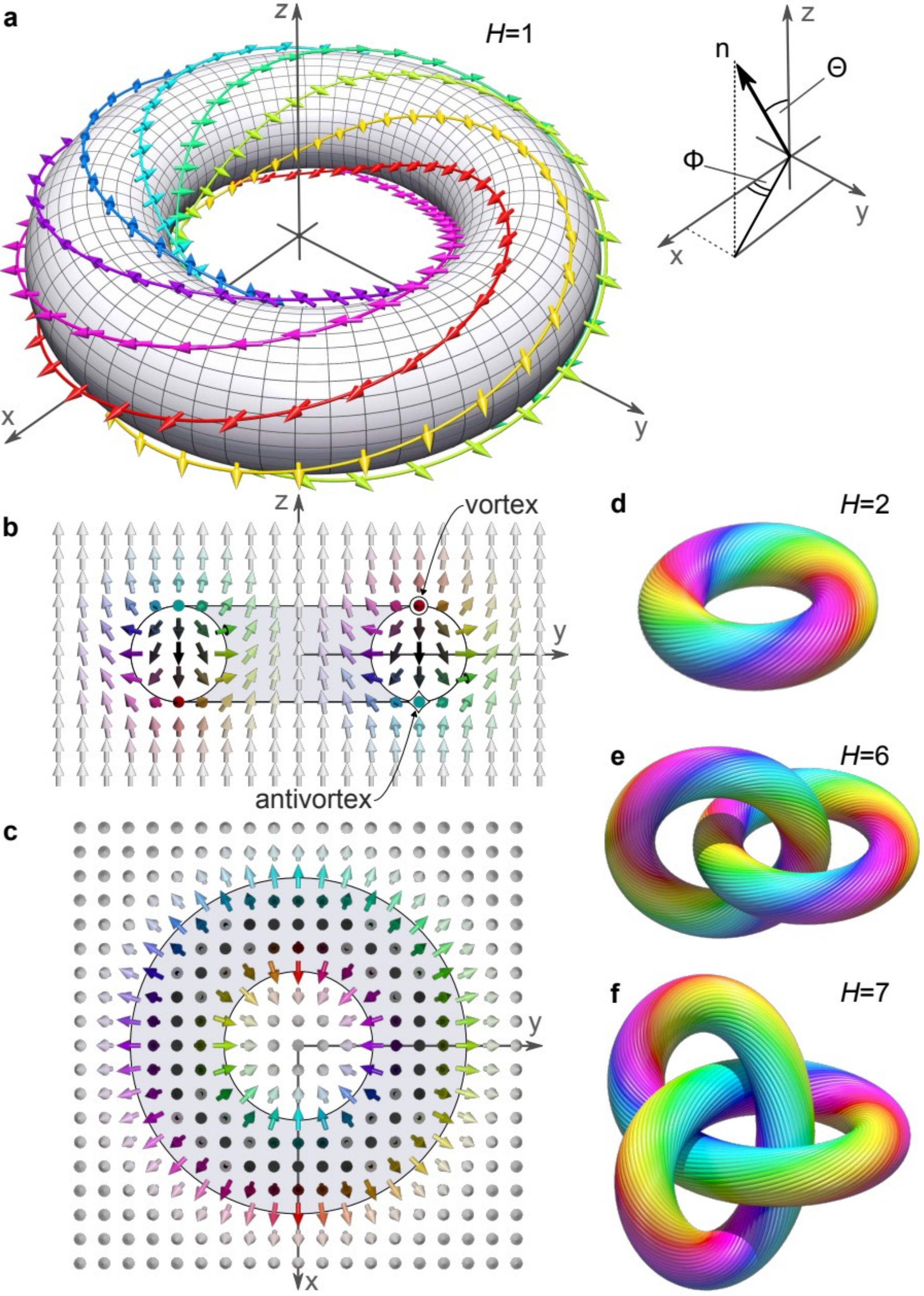

Stefan Blügel

OUTLINE

- Introduction (What are magnetic hopfions?)
- Hopfions in frustarted magnets
- Hopfions in chiral magnets
  - Hopfion rings
  - Heliknotons
- Conclusions

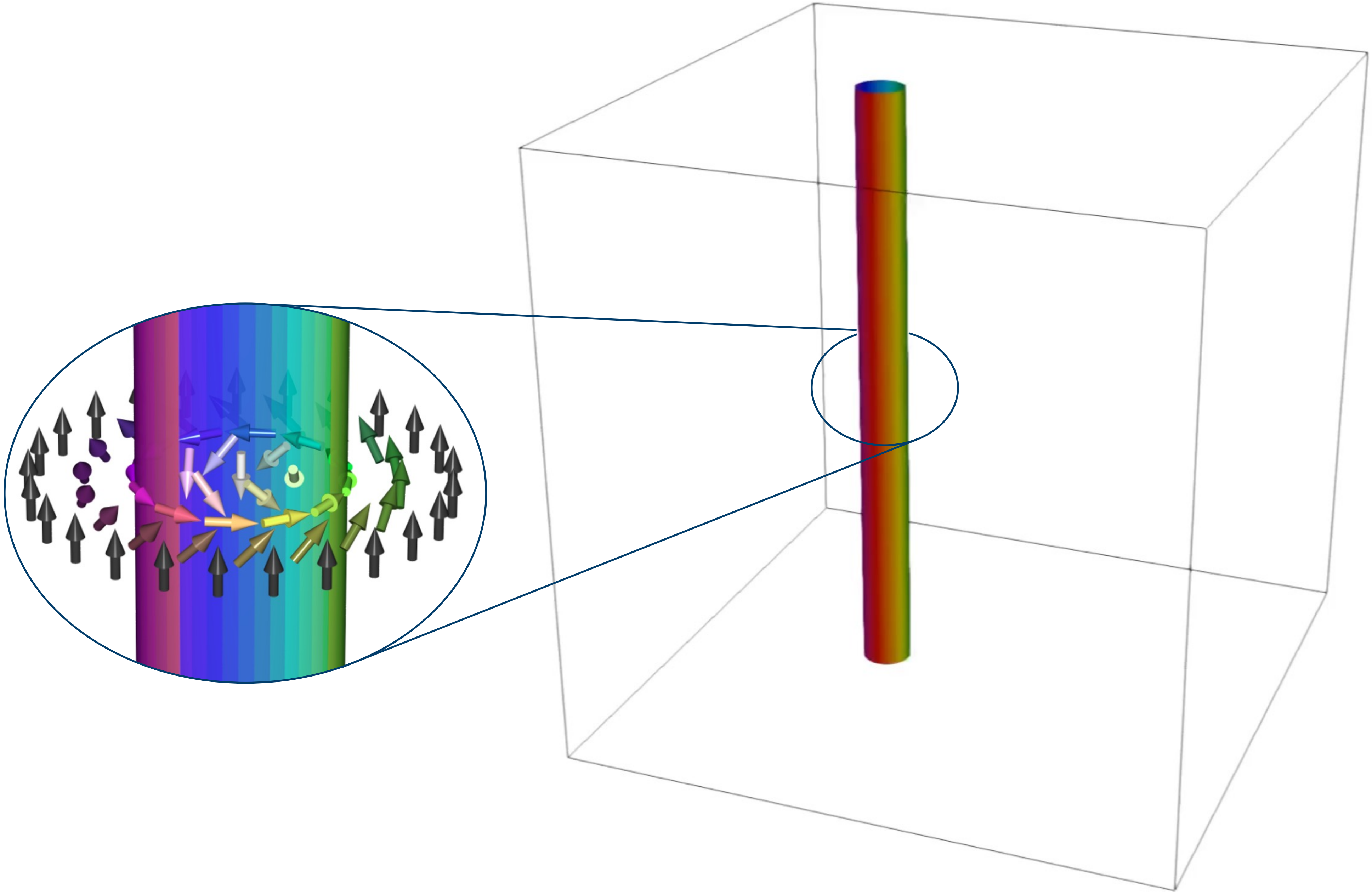
WHAT ARE MAGNETIC HOPFIONS?

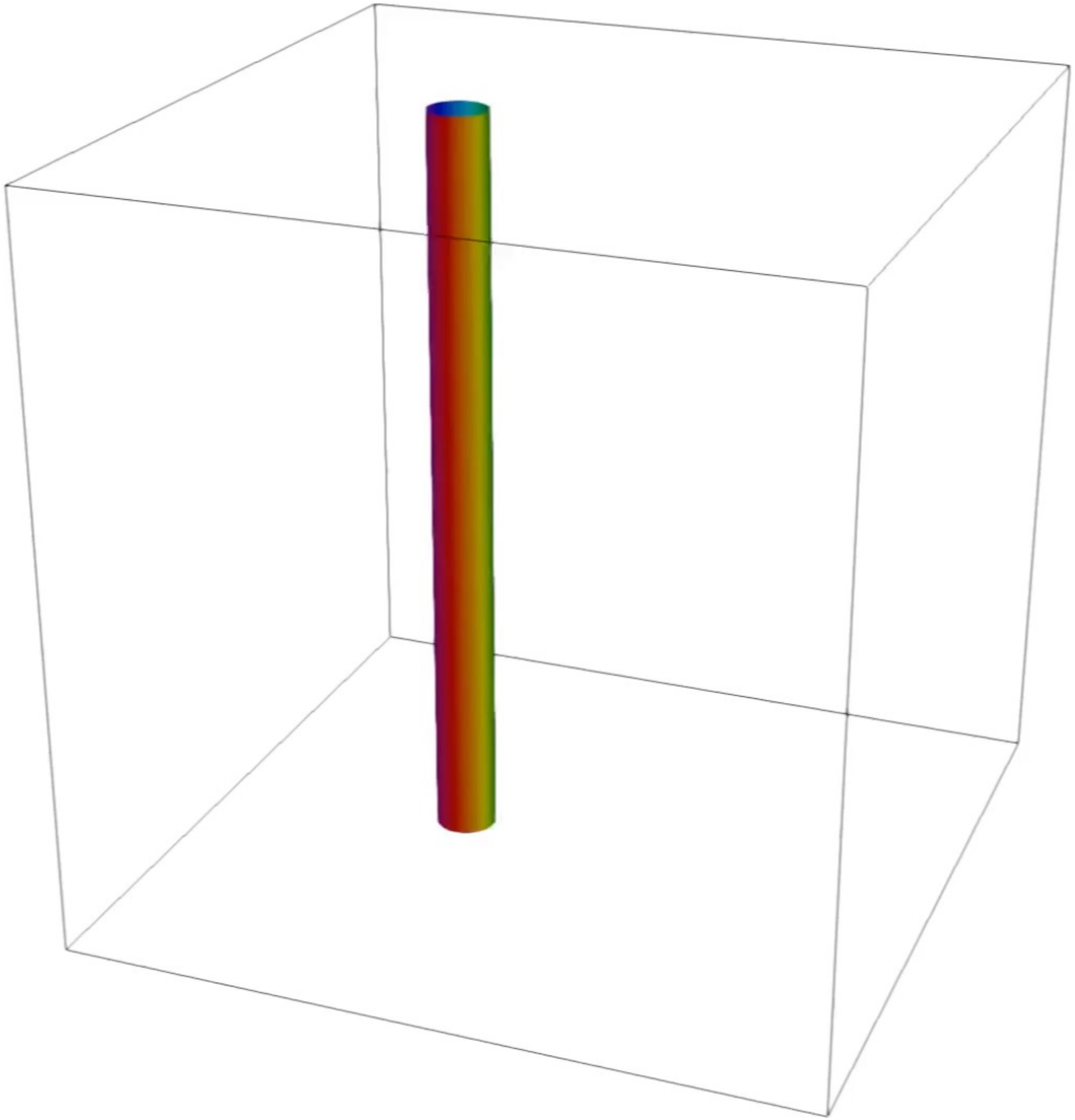


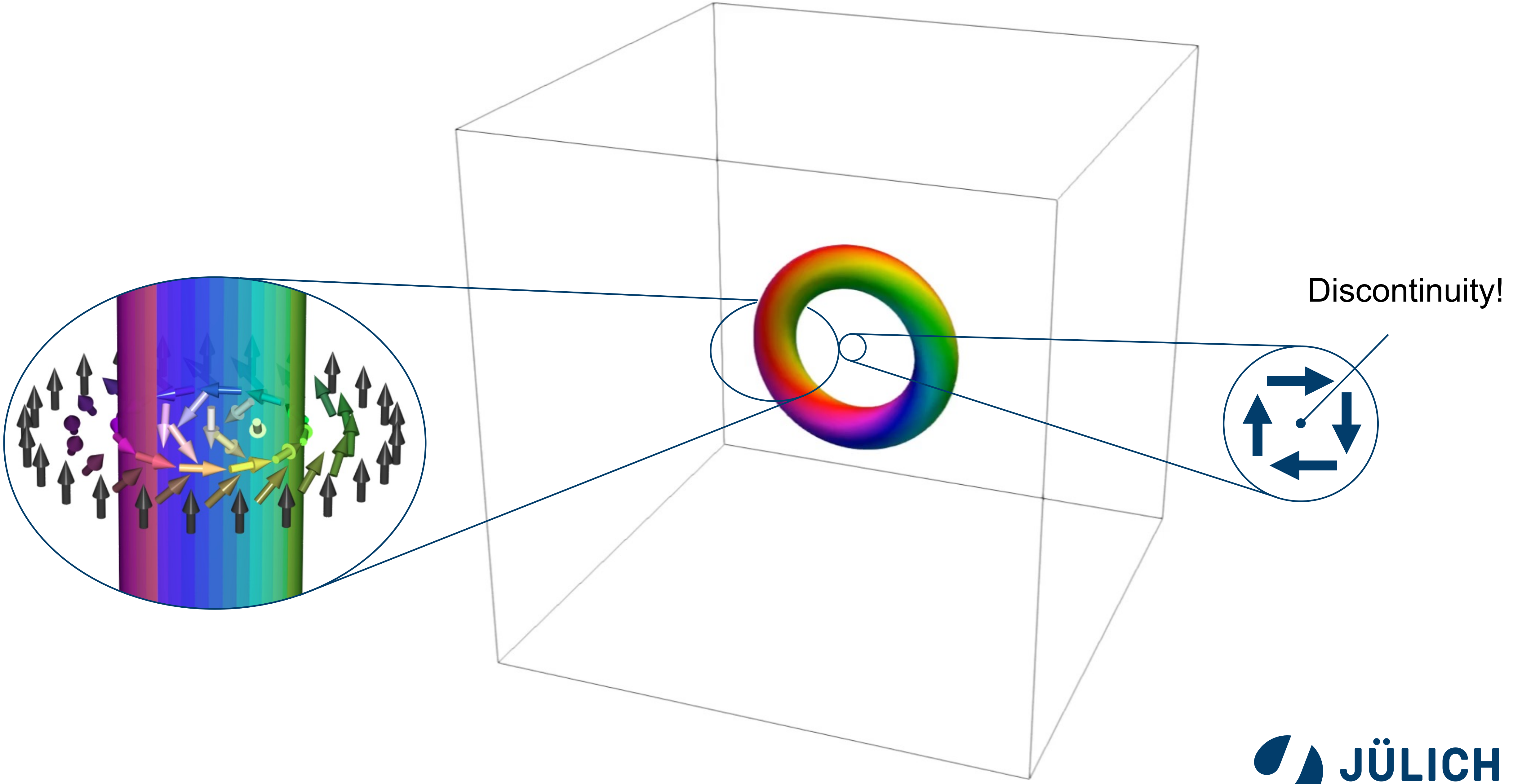


Magnetic hopfions are
(i) 3D topological solitons,
(ii) countable particles in magnetization field,
(iii) topological vortex rings
(iv) vortex-like closed strings (knots)

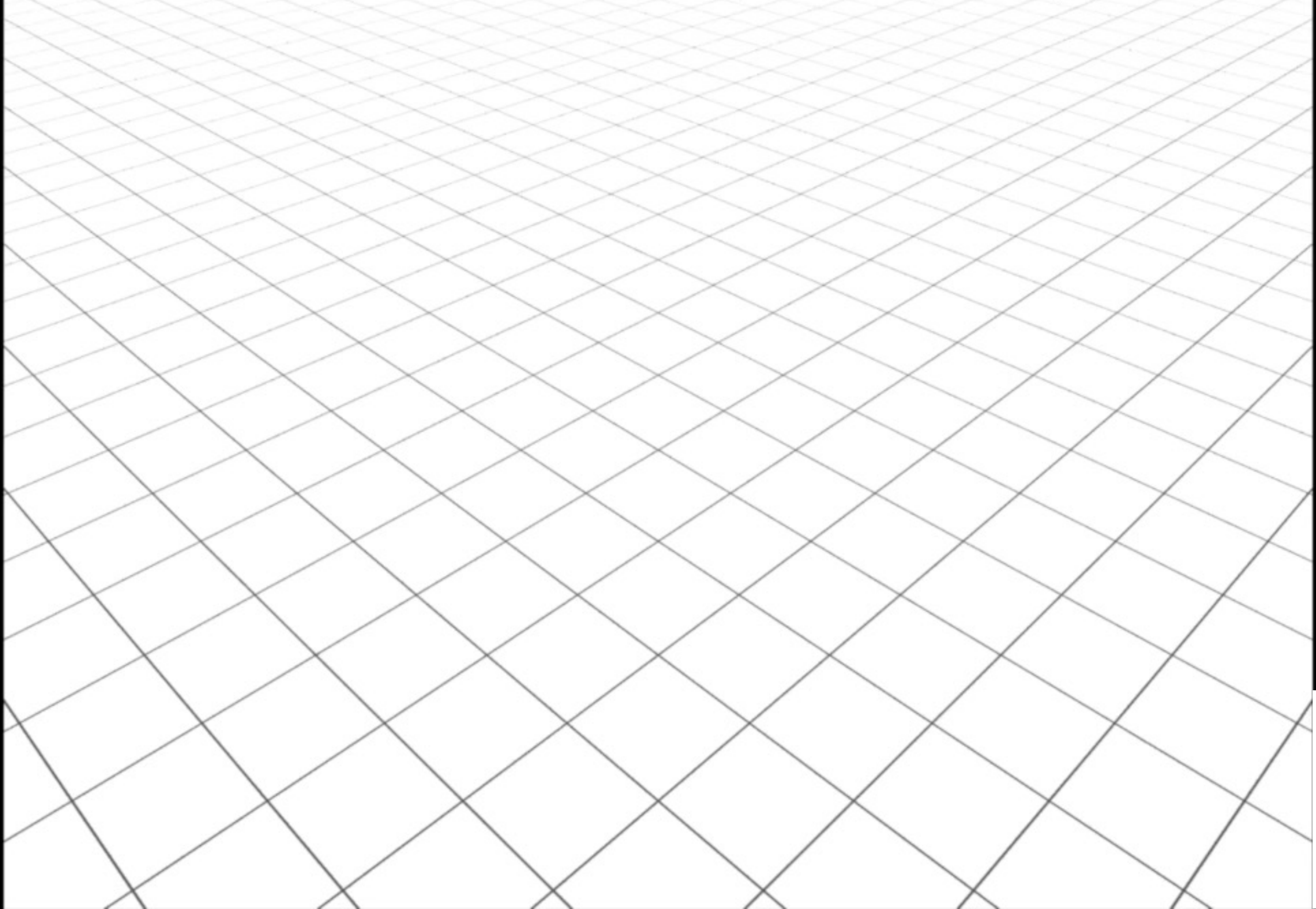
In case of magnetic hopfions, the relevant order parameter of the system is a unit vector field $\mathbf{n}(\mathbf{r}) = (n_x, n_y, n_z)$, $|\mathbf{n}(\mathbf{r})| = 1$, defined at any point $\mathbf{r} \in \mathbb{R}^3$. Field configurations $\mathbf{n}$ attaining a uniform background state at infinity $\mathbf{n}(\mathbf{r}) \to \mathbf{n}_0$ as $|\mathbf{r}| \to \infty$ can be classified according to the linkage of their fibers $\{\mathbf{n} = \mathbf{p}\}$, which, for regular values $\mathbf{p} \in \mathbb{S}^2$, are collections of closed loops in $\mathbb{R}^3$.

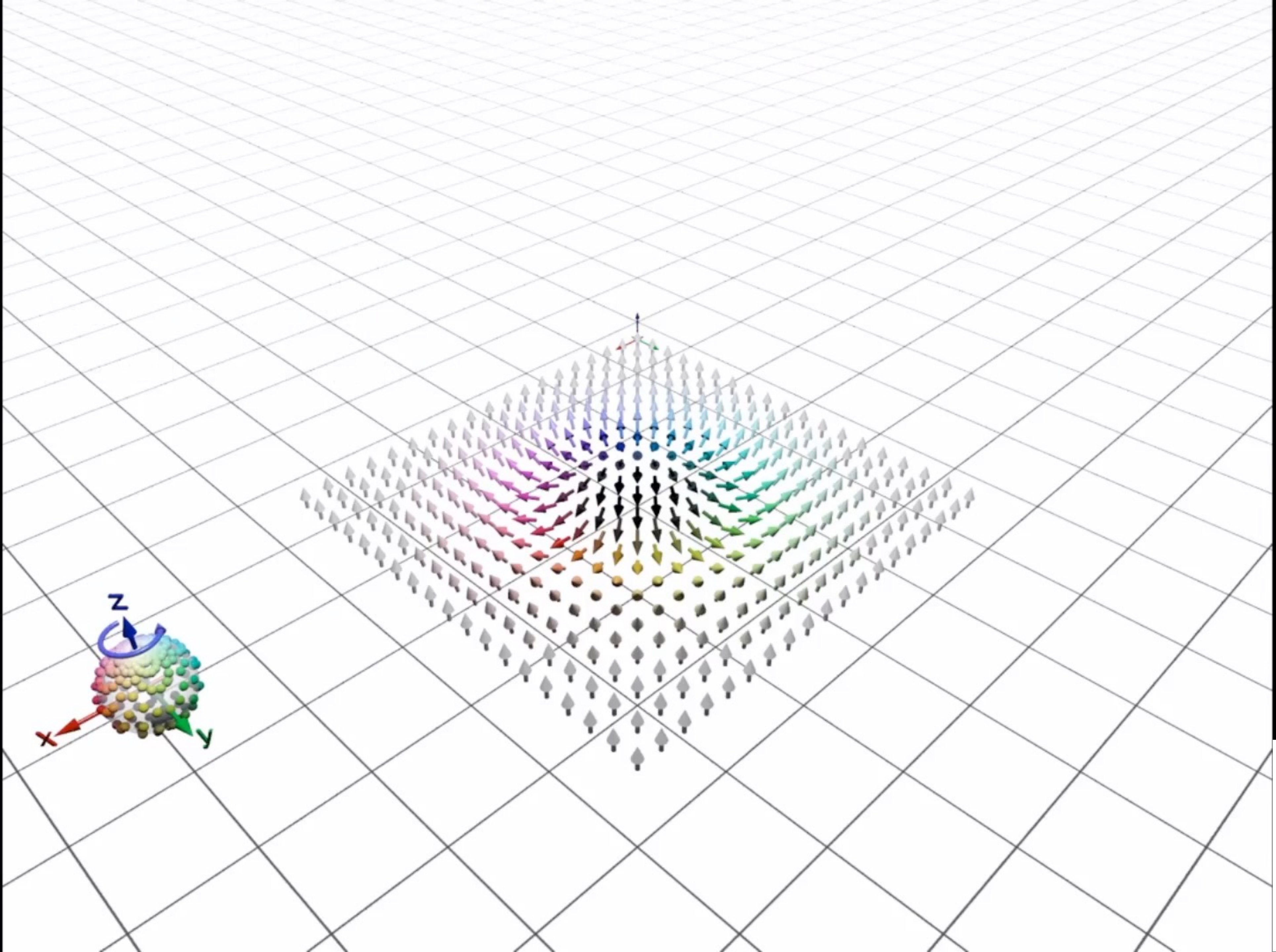


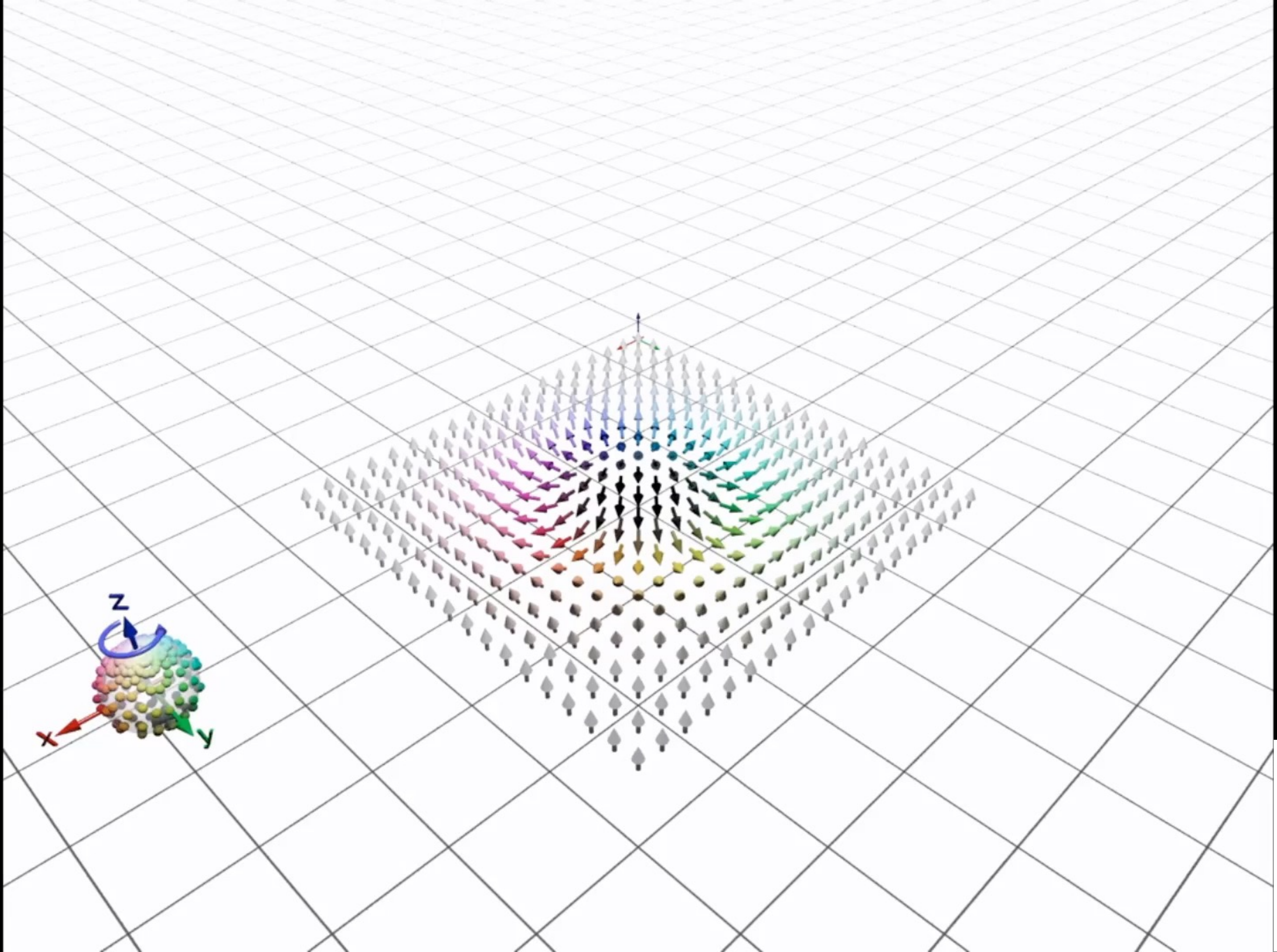


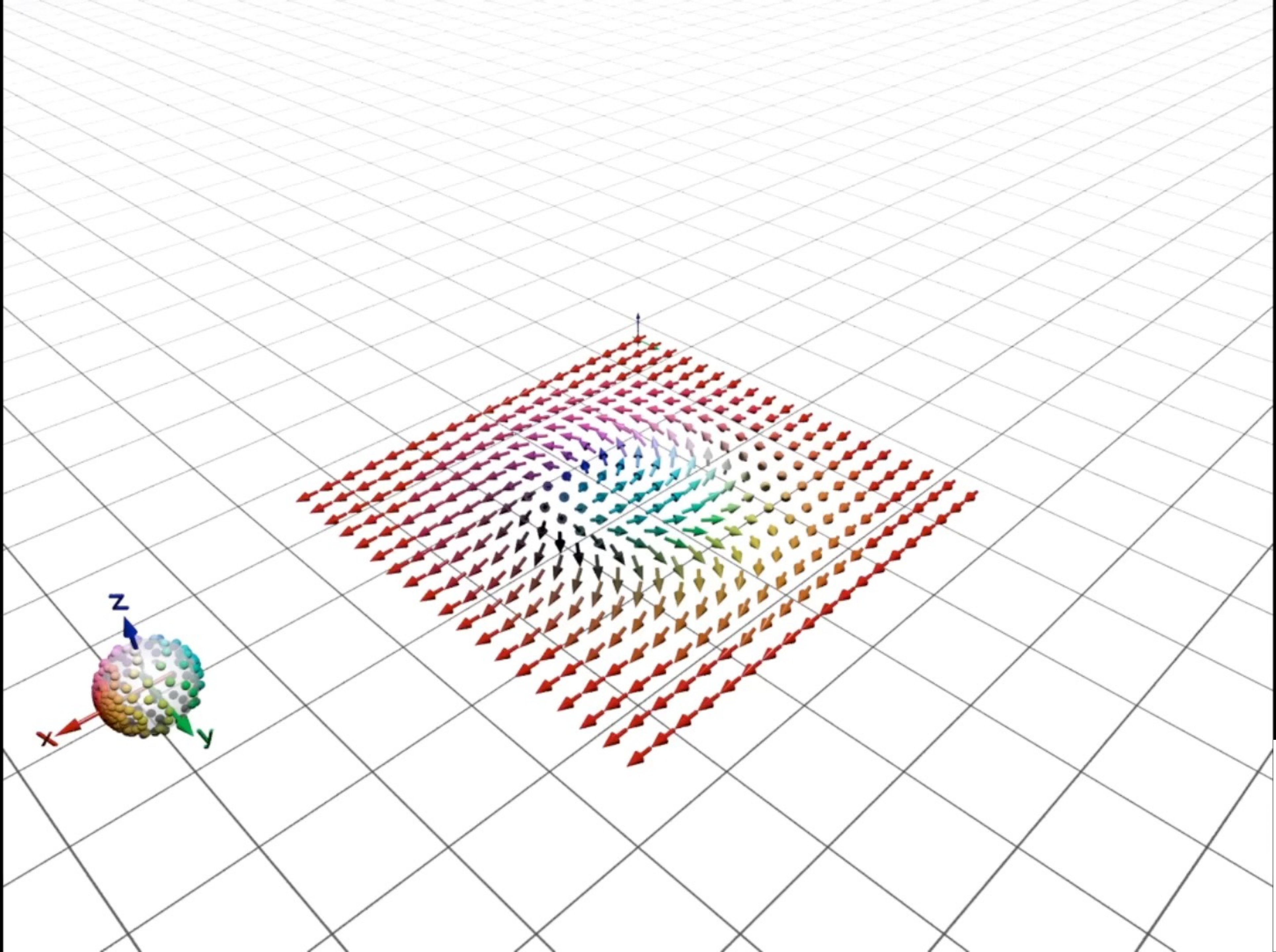

Discontinuity!

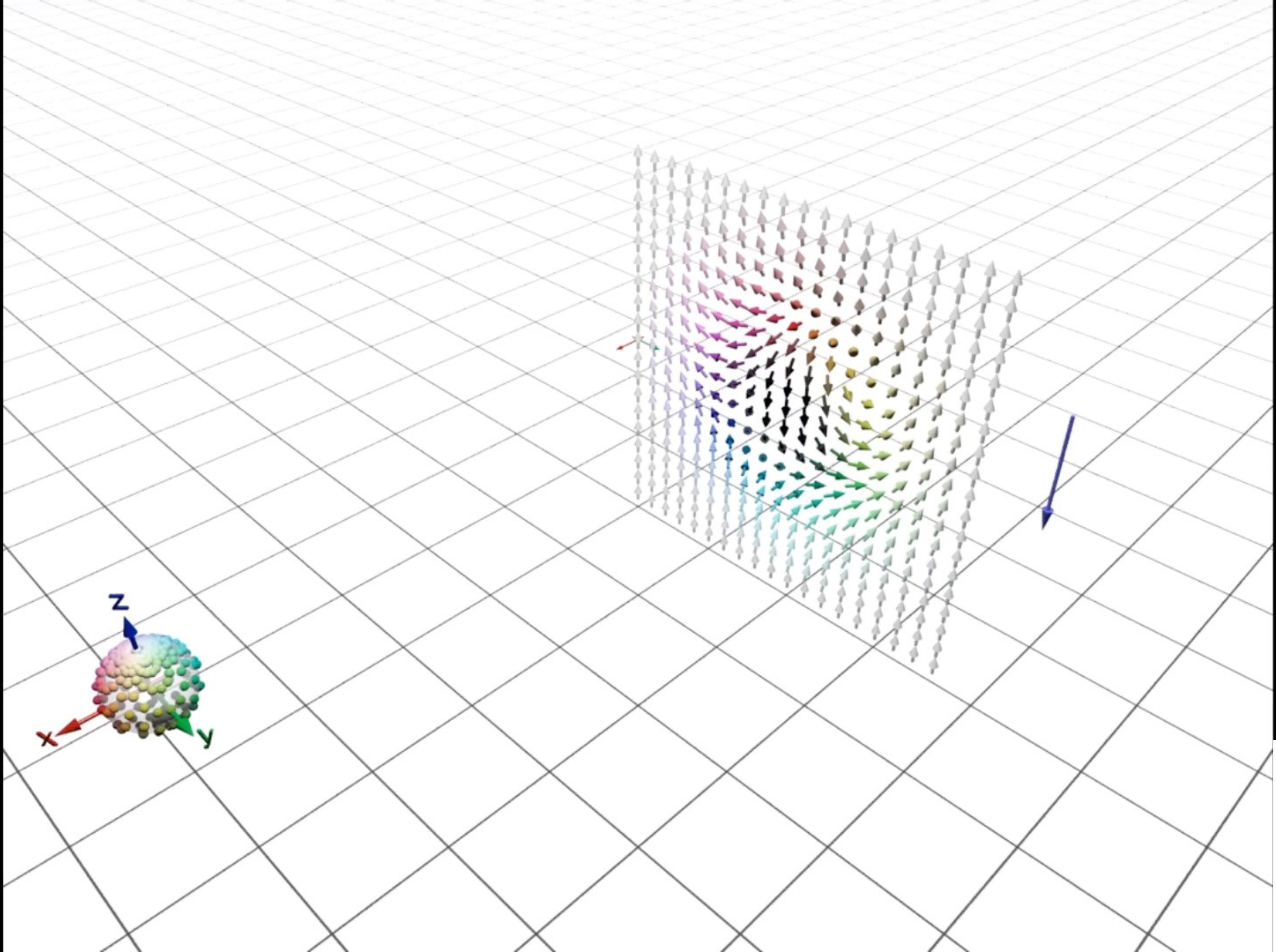
Vector field and isosurface for the hopfion with hopf index, H=1

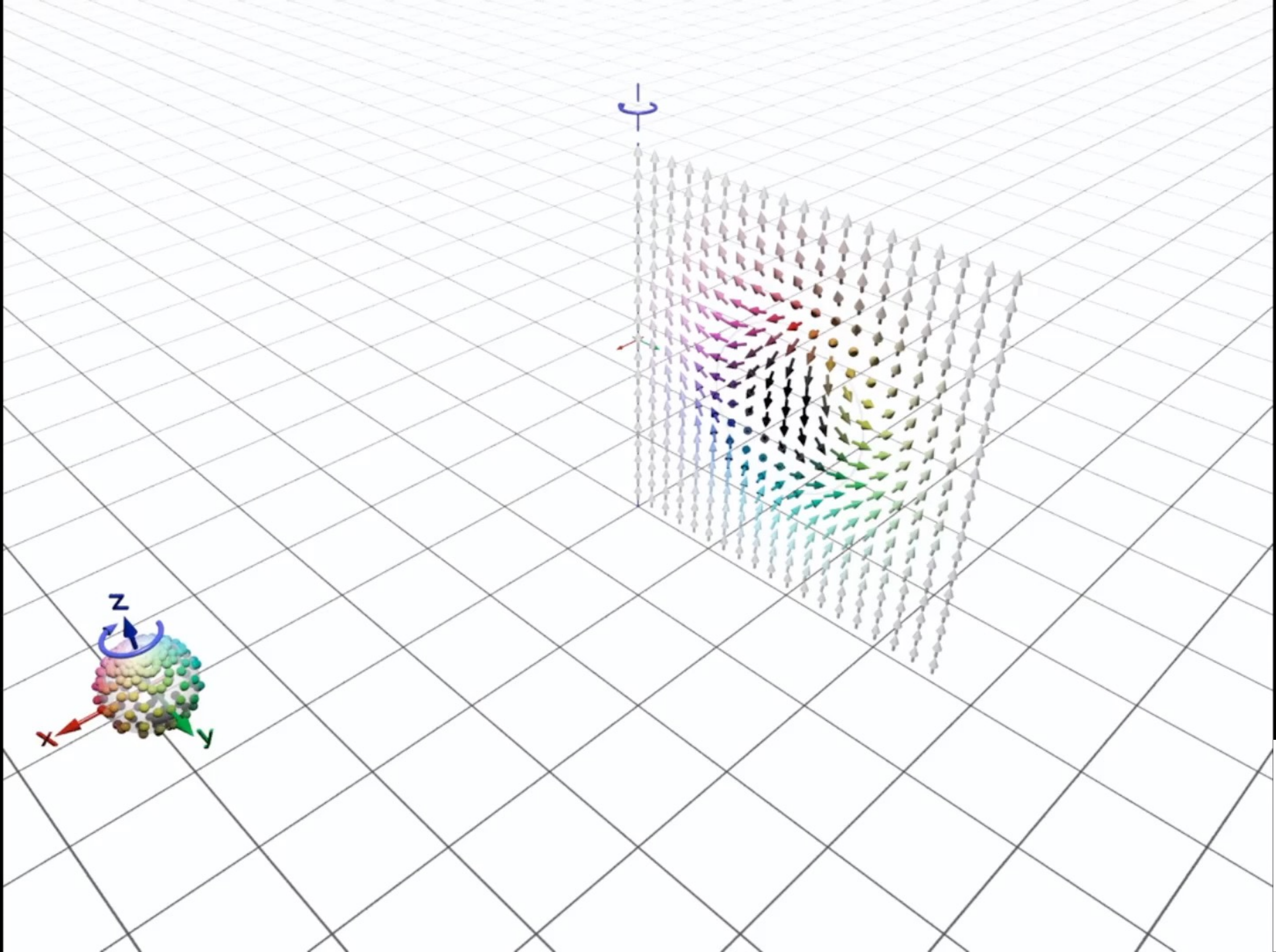


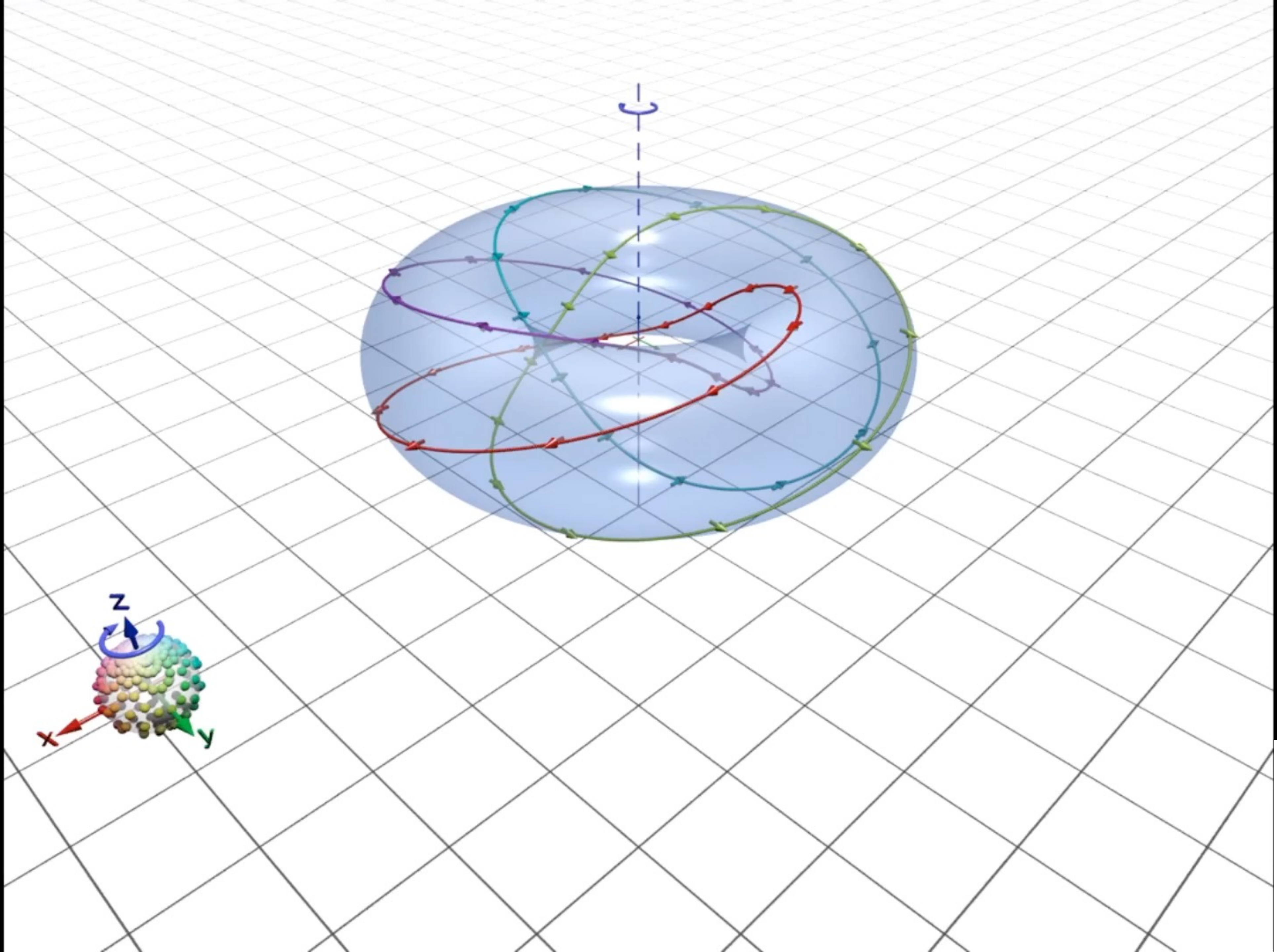

z
x
y

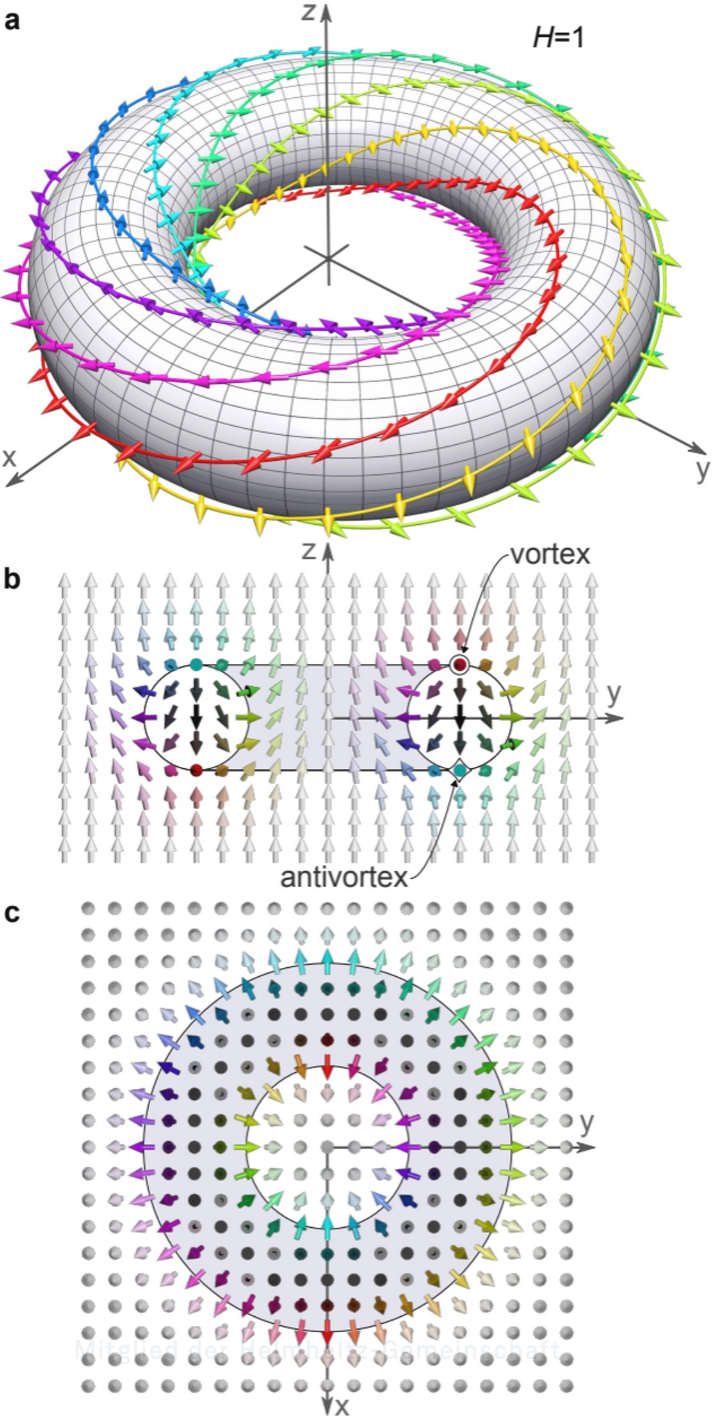

z
x
y


z
x
y


z
x
y


z
x
y


z
x
y



The classifying group is the third homotopy group of the 2-sphere:

$$\pi_3(\mathbb{S}^2, \mathbf{m}_0) = \mathbb{Z}$$

It is isomorphic to the group of integers with respect to addition.

Hopfion topological charge:

$$H = -\frac{1}{(8\pi)^2} \int_{\mathbb{R}^3} (\mathbf{F} \cdot \mathbf{V})\, d\mathbf{r}.$$

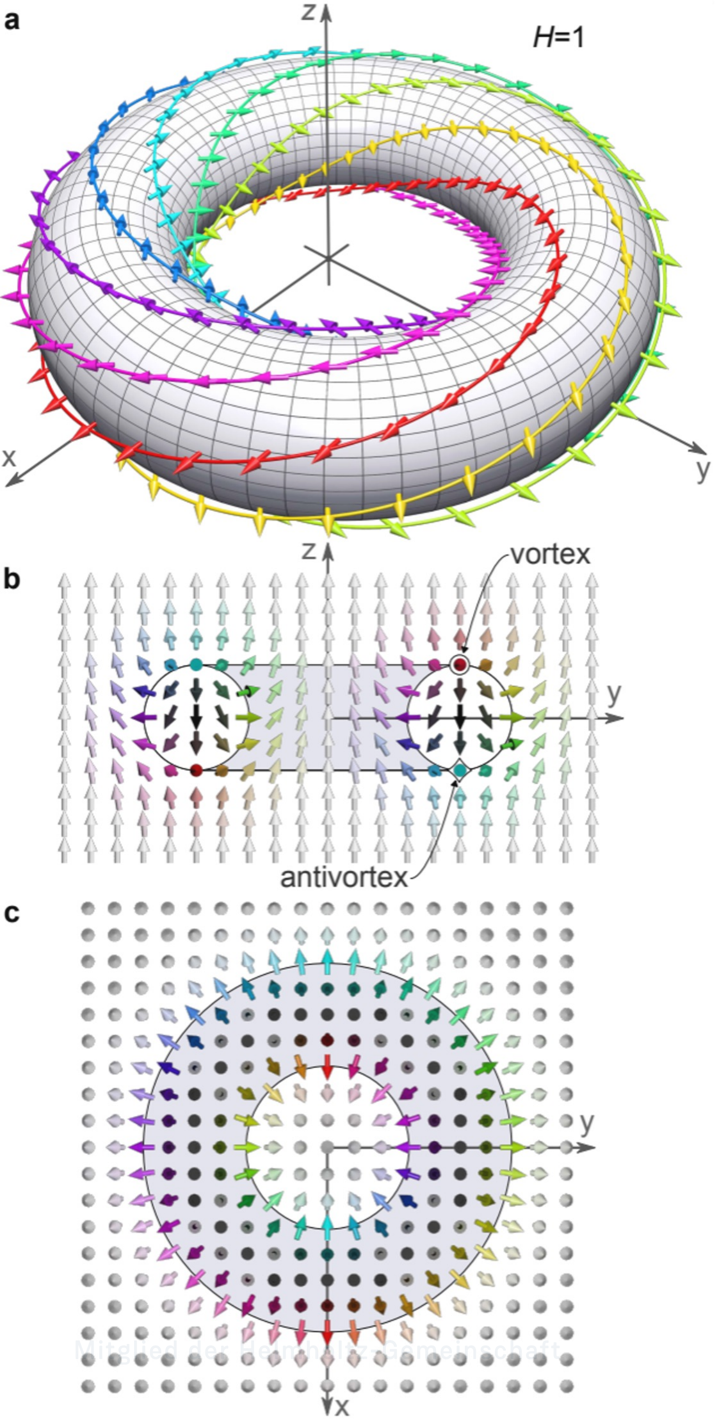
where

$$F_i = \varepsilon_{ijk} \mathbf{n} \cdot \left[\frac{\partial \mathbf{n}}{\partial r_j} \times \frac{\partial \mathbf{n}}{\partial r_k} \right]$$

are components of the solenoidal gyro-vector field **F**,
and **V** is an appropriate vector potential of **F**:

$$\nabla \times \mathbf{V} = \mathbf{F}$$

J. H. C. Whitehead, Proc. Nat. Acad. Sci. U.S.A. 33, 117 (1947).



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$$\pi_3(\mathbb{S}^2, \mathbf{m}_0) = \mathbb{Z}$$

It is isomorphic to the group of integers with respect to addition.

Hopfion topological charge:

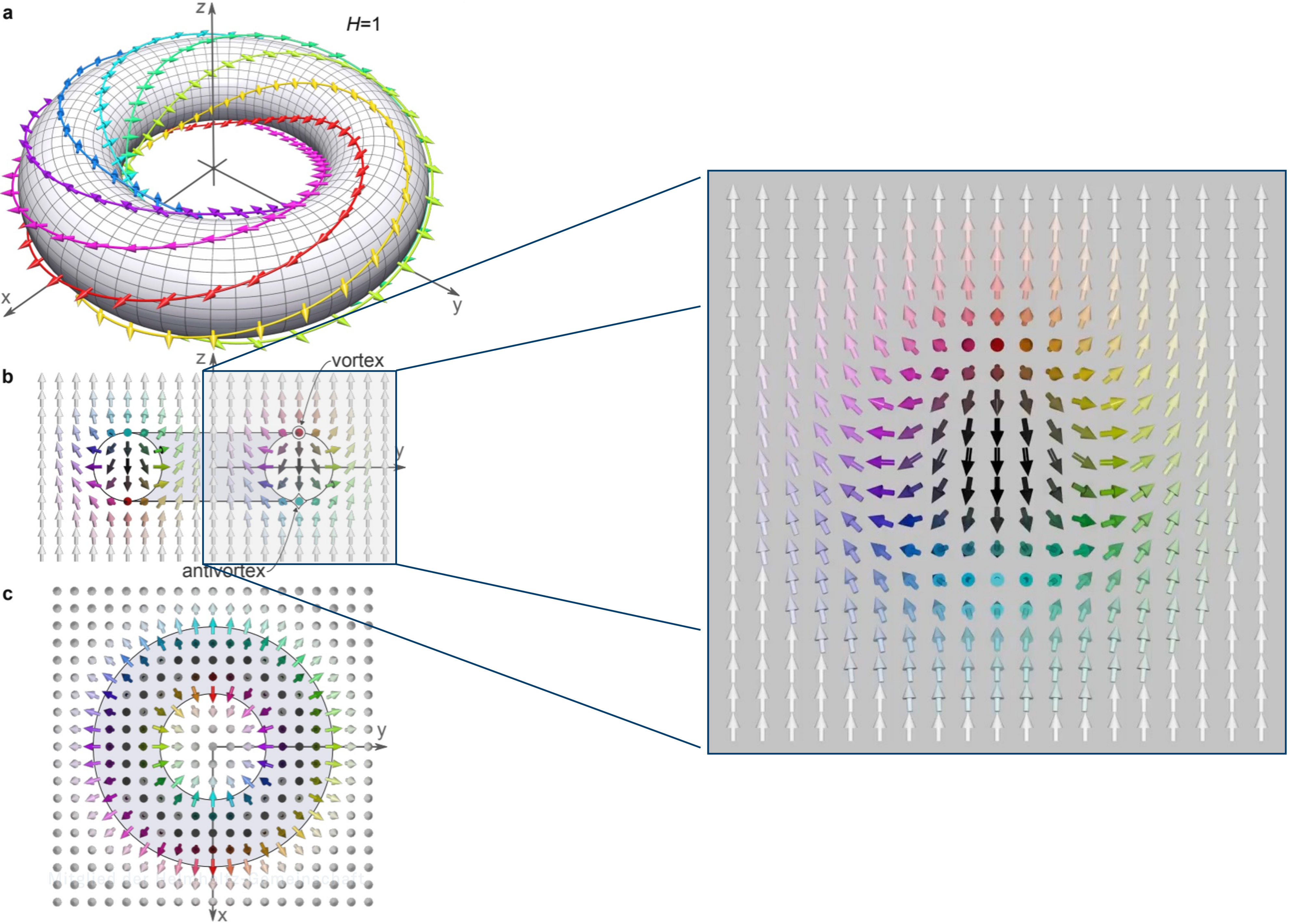
$$H = -\frac{1}{16\pi^2}\int_\Omega \mathrm{d}r_1\mathrm{d}r_2\mathrm{d}r_3\ \mathbf{F}\cdot[(\nabla\times)^{-1}\mathbf{F}].$$

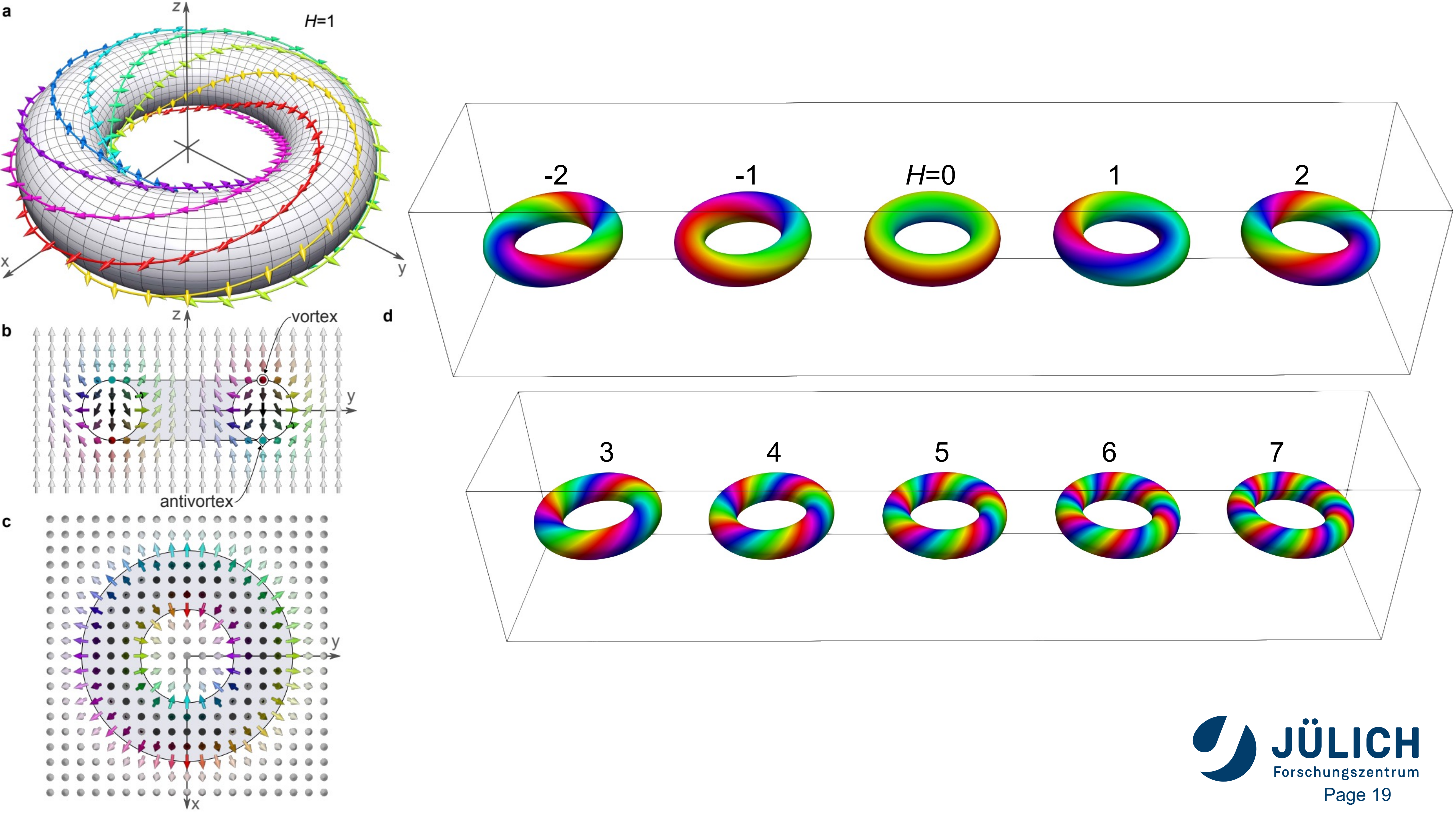
where

$$\mathbf{F} = \begin{pmatrix} \mathbf{m}\cdot[\partial_{r_2}\mathbf{m}\times\partial_{r_3}\mathbf{m}] \\ \mathbf{m}\cdot[\partial_{r_3}\mathbf{m}\times\partial_{r_1}\mathbf{m}] \\ \mathbf{m}\cdot[\partial_{r_1}\mathbf{m}\times\partial_{r_2}\mathbf{m}] \end{pmatrix}$$

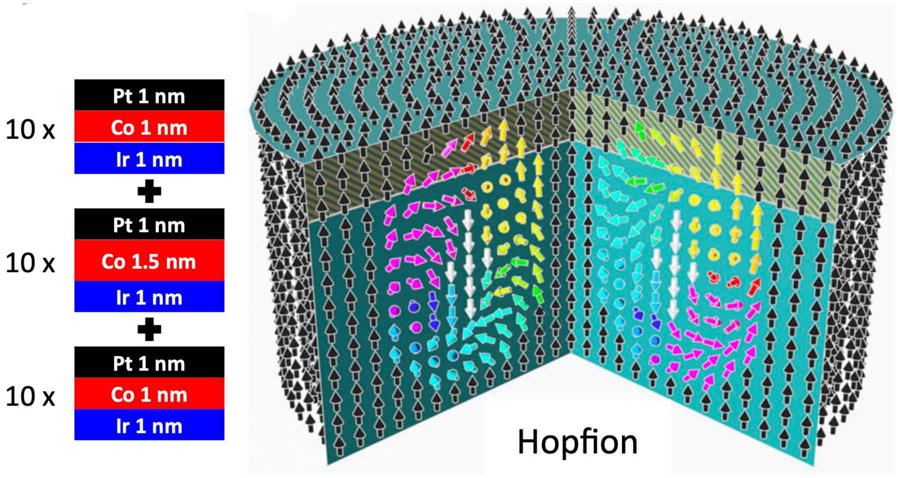
is the vector of curvature and r_1, r_2, r_3 are local right-handed Cartesian coordinates

J. H. C. Whitehead, Proc. Nat. Acad. Sci. U.S.A. 33, 117 (1947).

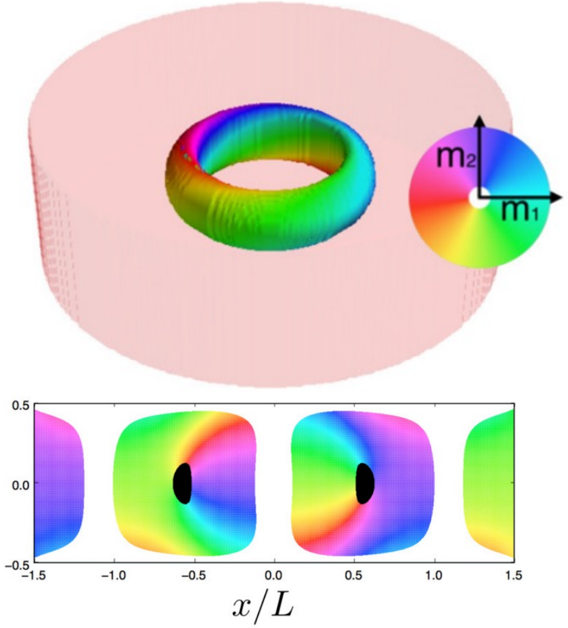

a
H=1
z
x
y
b
vortex
antivortex
c


a
H=1
z
x
y
b
vortex
antivortex
c
d
-2
-1
H=0
1
2
3
4
5
6
7

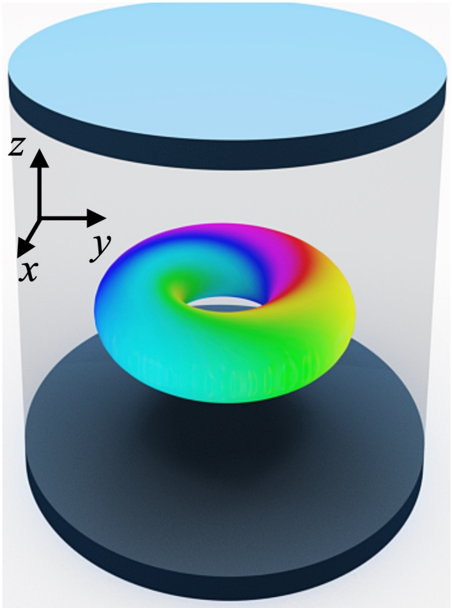
HOPFIONS IN GEOMETRICAL CONFINEMENT



N. Kent, et al.,
Nature Commun. 12, 1562 (2021).

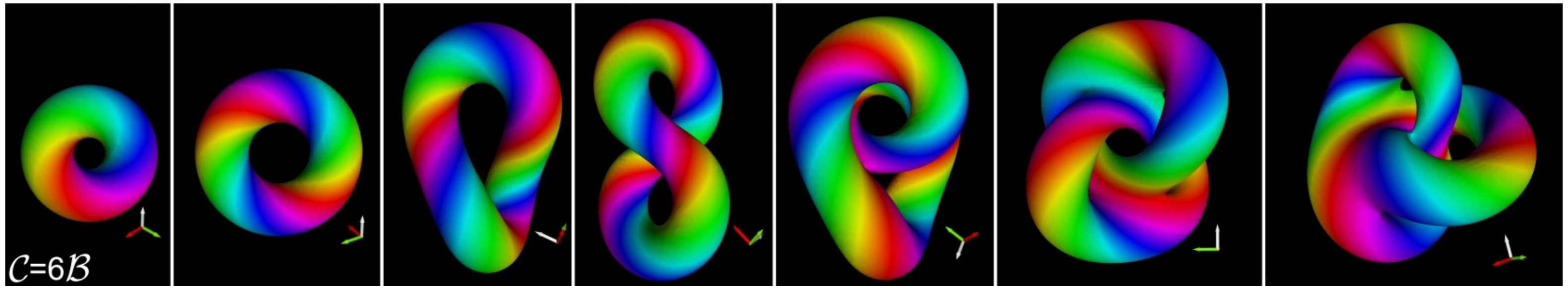


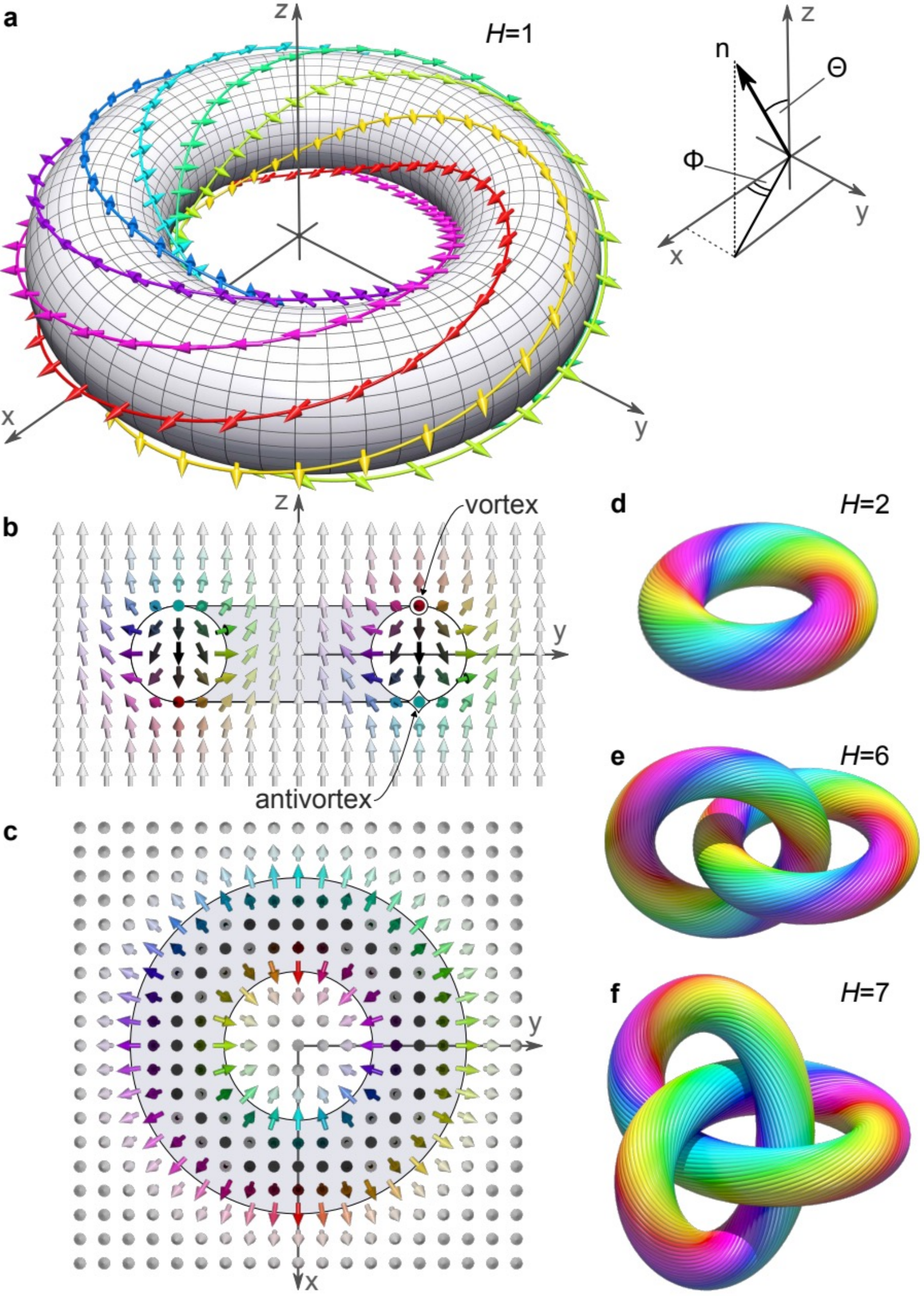
P. Sutcliffe
J. Phys. A: Math. Theor. 51, 375401 (2018).



Y. Liu, et al.,
Phys. Rev. B 98, 174437 (2018).

HOPFIONS IN FRUSTRATED MAGNETS

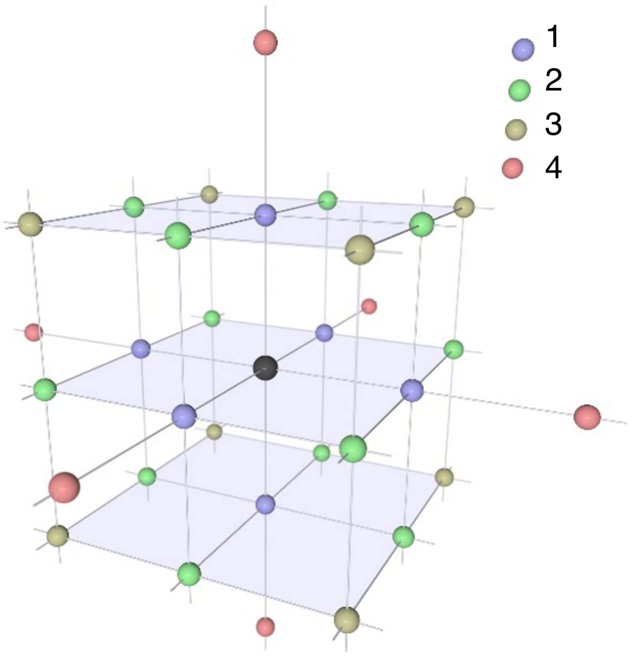


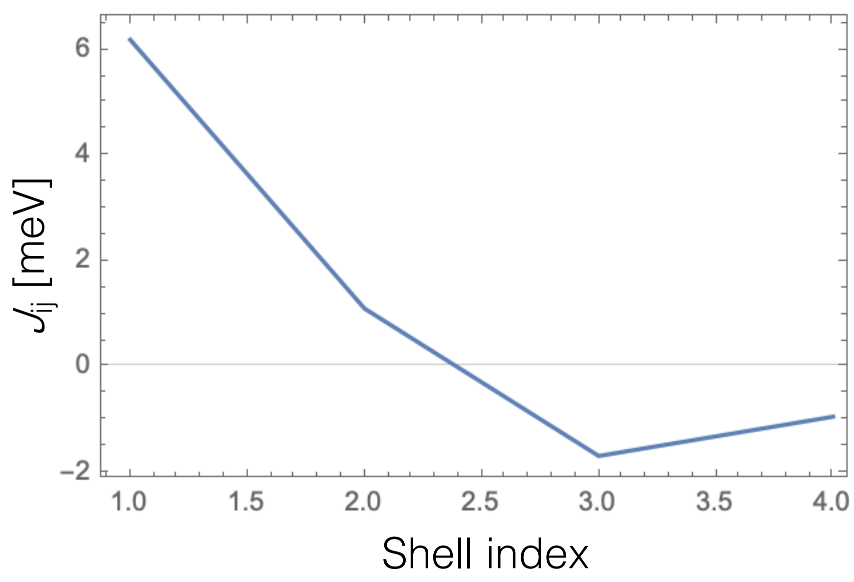


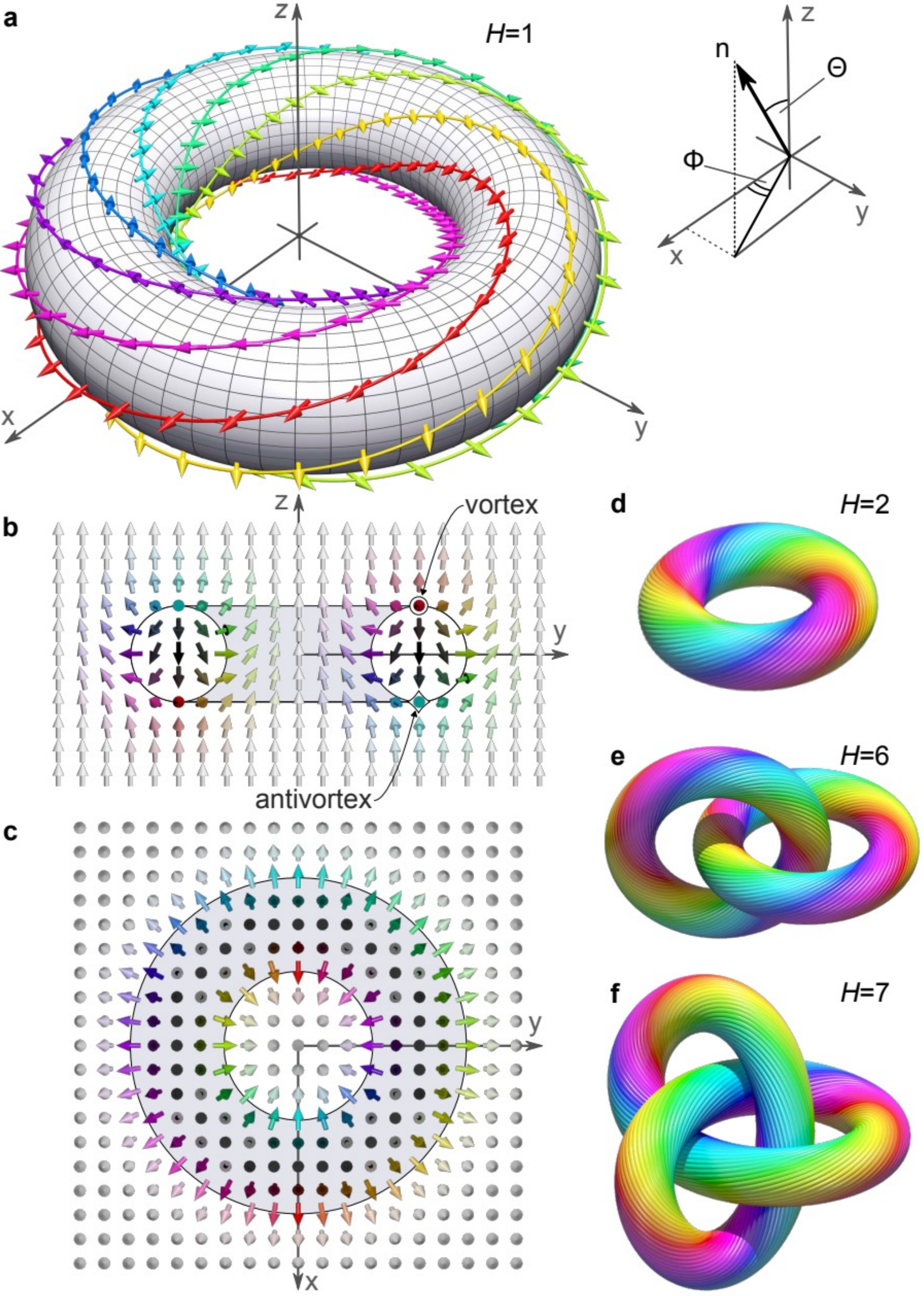
F.N. Rybakov et al, APL Mater. **10**, 111113 (2022)
arXiv:1904.00250 (2019)

$$\mathcal{H} = -\sum_{i>j} \mathcal{J}_{ij}(\mathbf{n}_i \cdot \mathbf{n}_j),$$

$$\mathcal{J}_{ij} \equiv \mathcal{J}(\mathbf{r}_{ij}) = \begin{cases} J_1 & \text{for } |\mathbf{r}_{ij}| = |\mathbf{r}_j - \mathbf{r}_i| = R_1, \\ \dots & \\ J_S & \text{for } |\mathbf{r}_{ij}| = R_S. \end{cases}$$







F.N. Rybakov et al, APL Mater. **10**, 111113 (2022)
arXiv:1904.00250 (2019)

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$$\mathcal{J}_{ij} \equiv \mathcal{J}(\mathbf{r}_{ij}) = \begin{cases} J_1 & \text{for } |\mathbf{r}_{ij}| = |\mathbf{r}_j - \mathbf{r}_i| = R_1, \\ \dots & \\ J_S & \text{for } |\mathbf{r}_{ij}| = R_S. \end{cases}$$

Micromagnetic model for cubic Heisenberg magnet:

$$E = \int_{\mathbb{R}^3} \mathcal{A}\left(\frac{\partial \mathbf{n}}{\partial r_\alpha}\right)^2 + \mathcal{B}\left(\frac{\partial^2 \mathbf{n}}{\partial r_\alpha^2} - \frac{\partial^2 \mathbf{n}}{\partial r_\beta^2}\right)^2 + \mathcal{C}\left(\frac{\partial^2 \mathbf{n}}{\partial r_\alpha \partial r_\beta}\right)^2 d\mathbf{r}$$

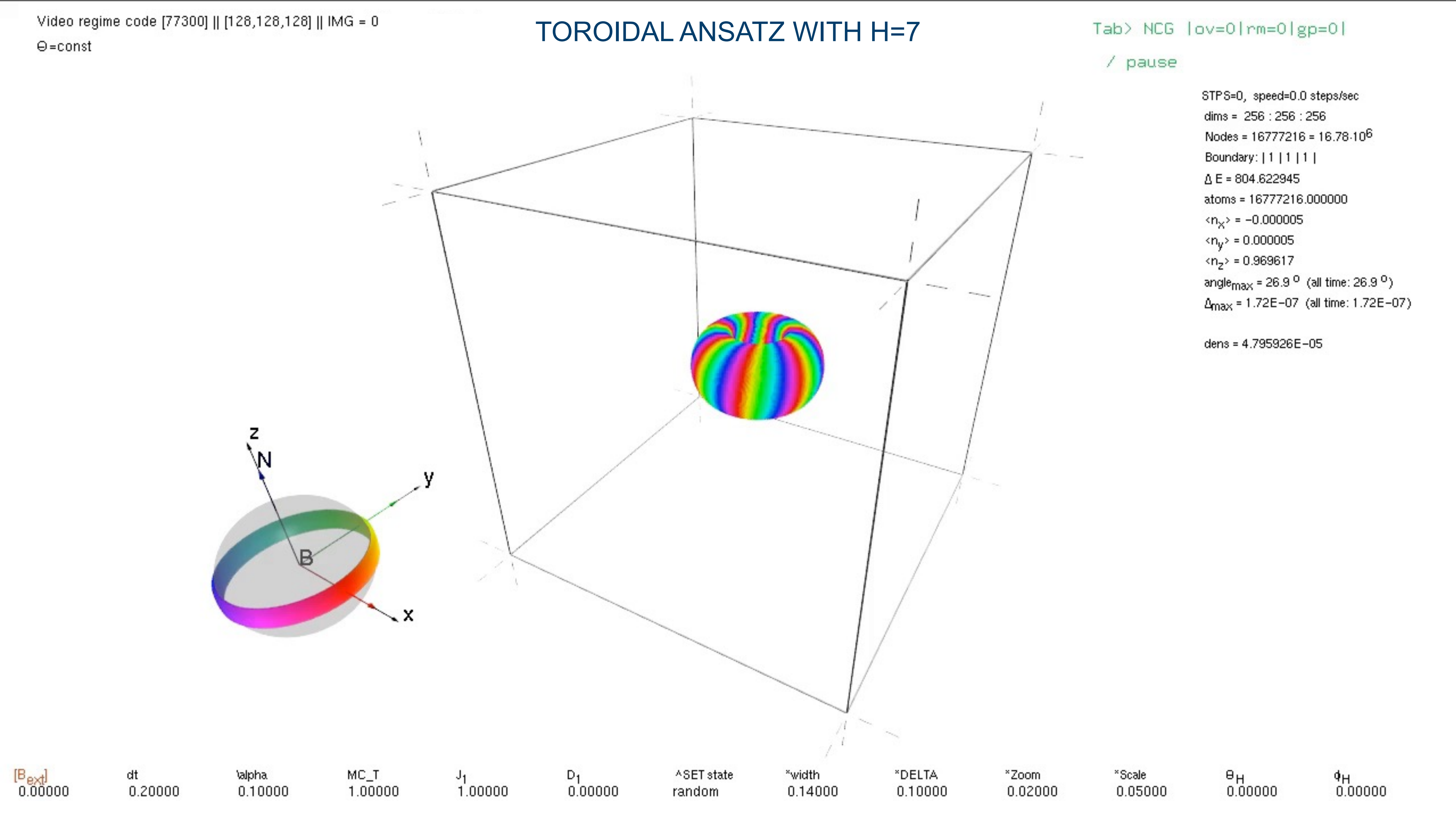
$$\mathcal{A} = (1/a)\textstyle\sum_s \mathsf{a}_s J_s,$$

$$\mathcal{B} = -a\textstyle\sum_s \mathsf{b}_s J_s,$$

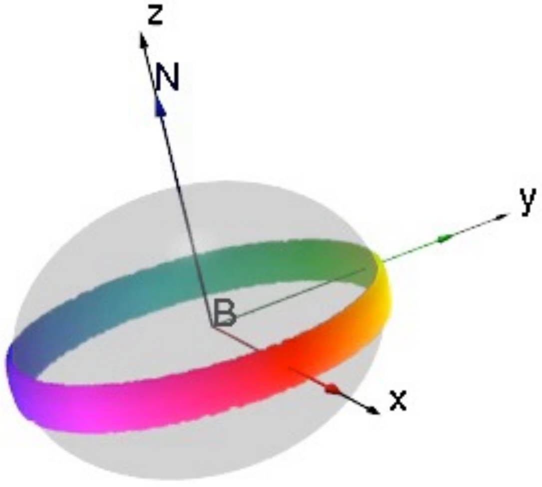
$$\mathcal{C} = -a\textstyle\sum_s \mathsf{c}_s J_s$$

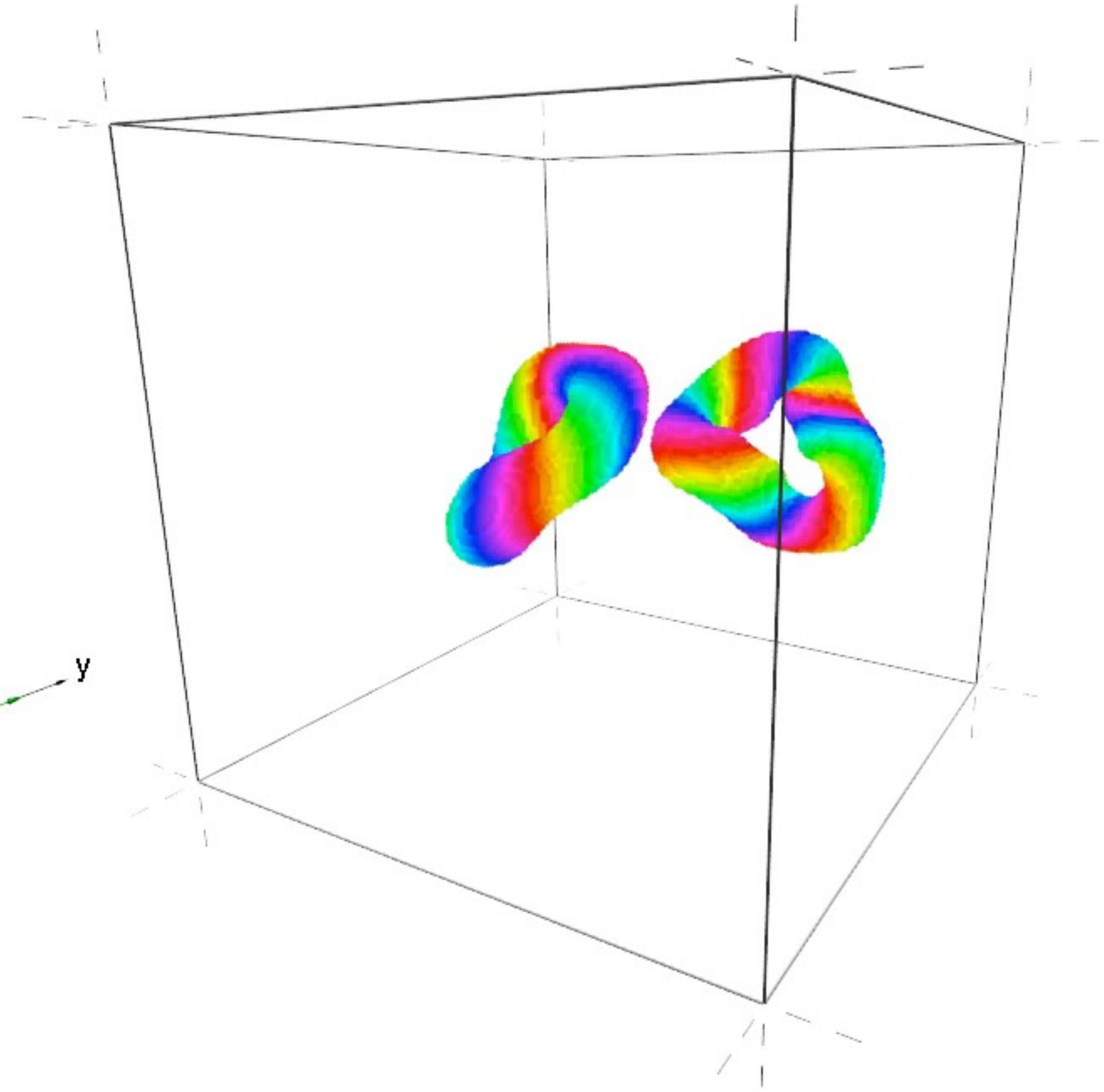
A necessary but not sufficient condition for hopfion existence: A,B,C>0

TOROIDAL ANSATZ WITH H=7

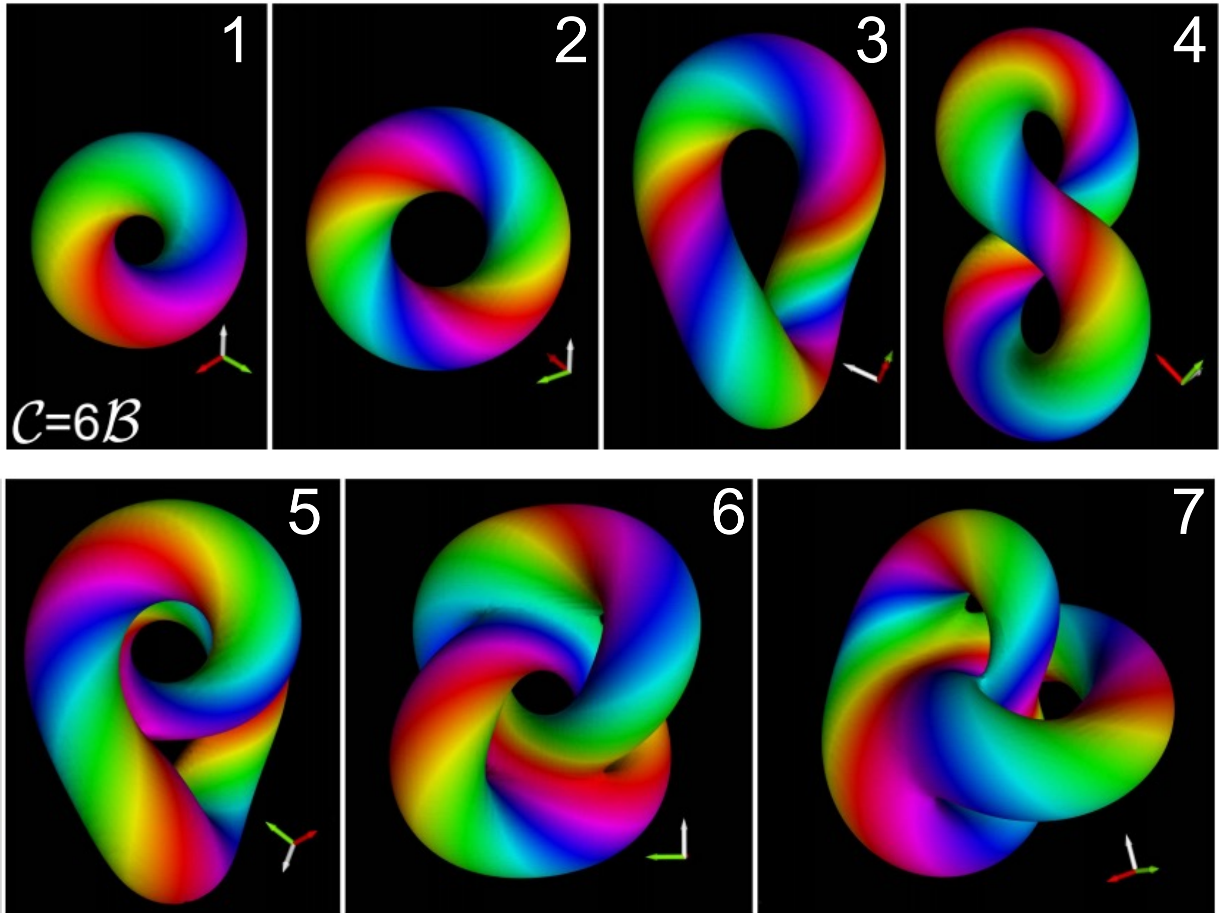


HOPFION FUSION, H=3+4=7

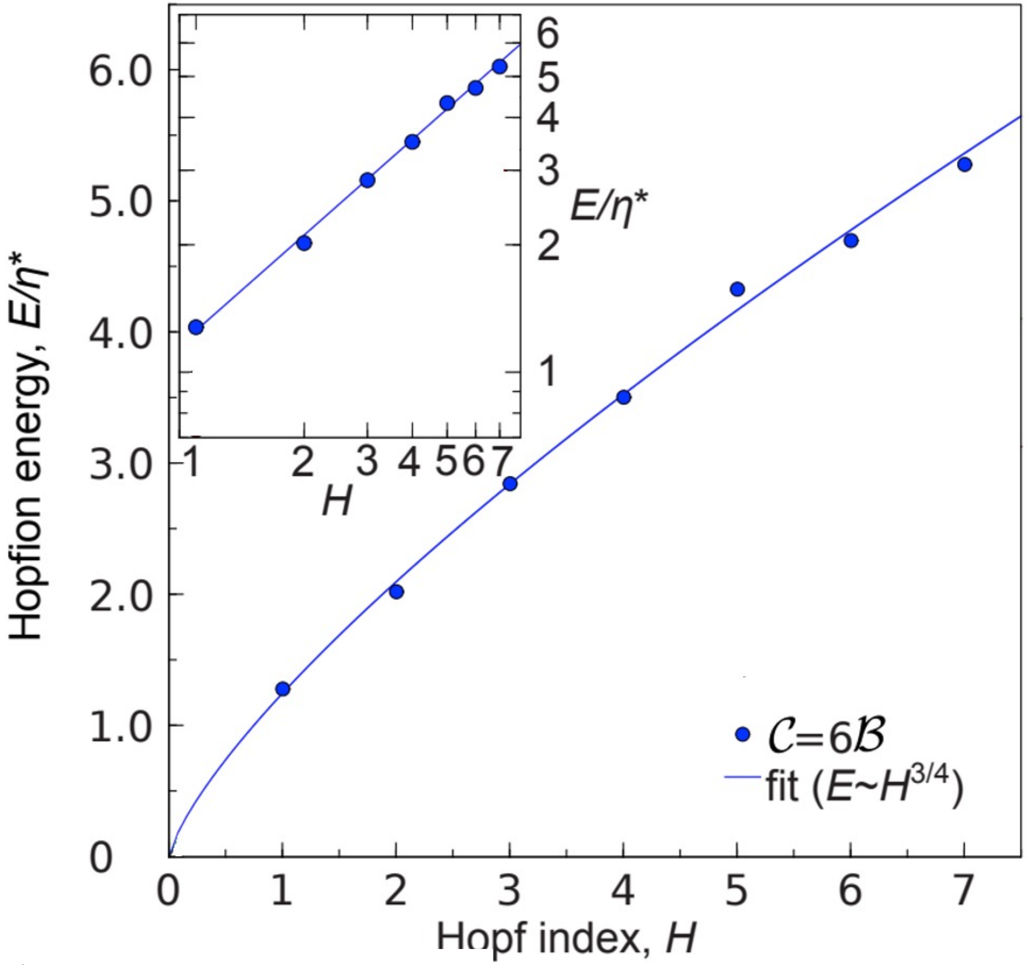




ENERGY AND TOPOLOGICAL CHARGE

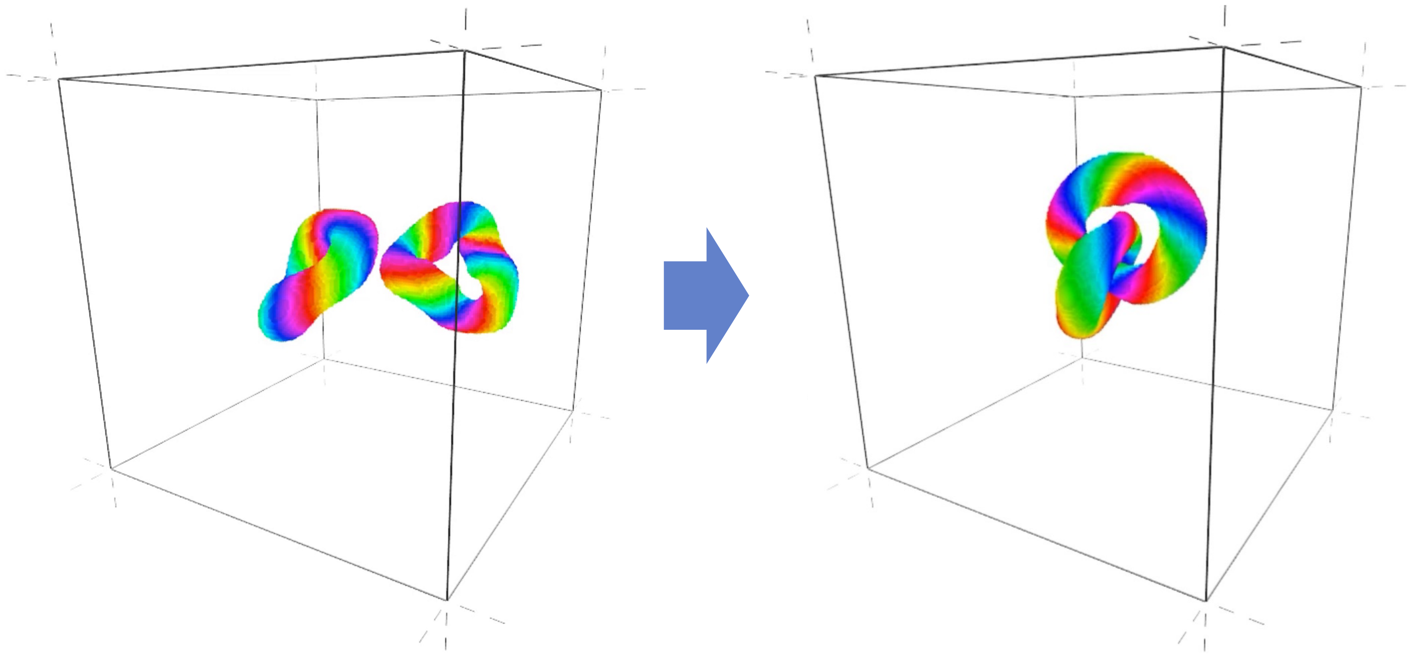


F.N. Rybakov et al, APL Mater. **10**, 111113 (2022).

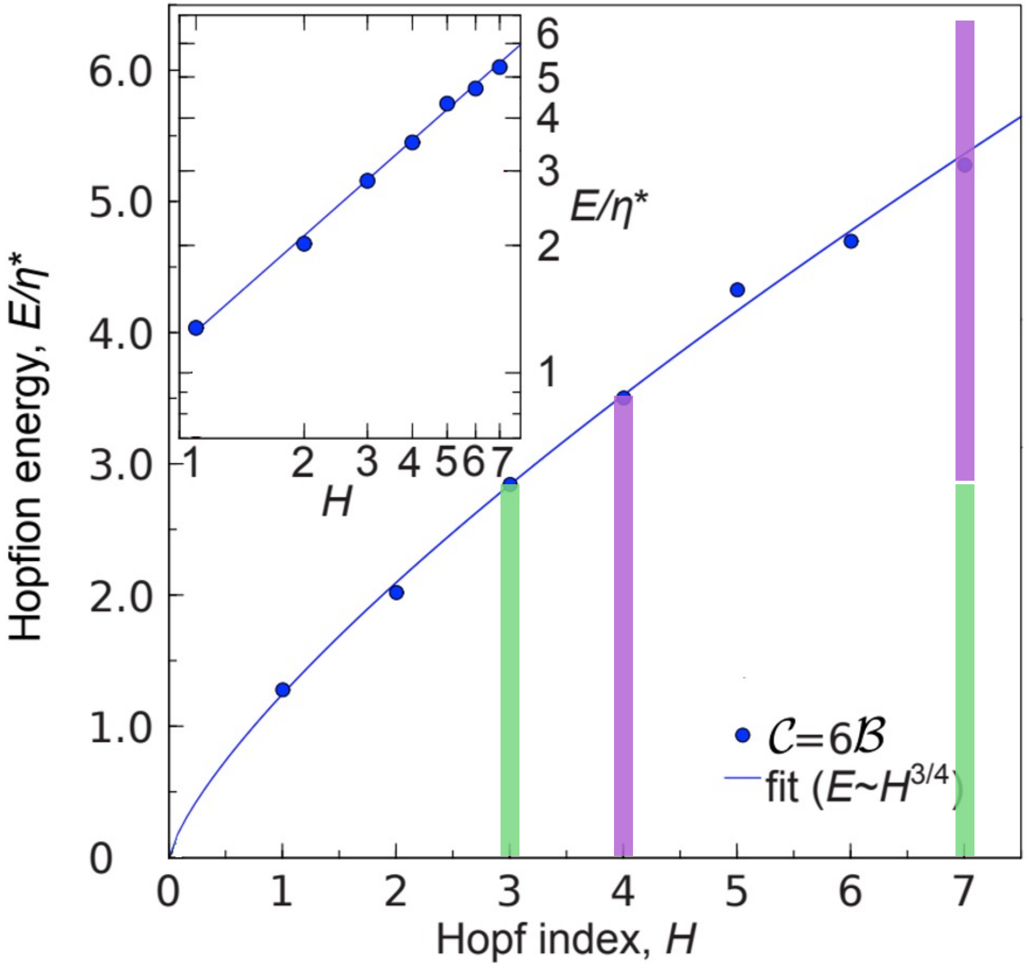


L.D. Faddeev, Lett. Math. Phys. 1, 289 (1976).
L.D. Faddeev, A.J. Niemi, Nature 387, 58 (1997).
A.F. Vakulenko & L.V. Kapitanski, Sov. Phys. Dokl. 24, 432 (1979).

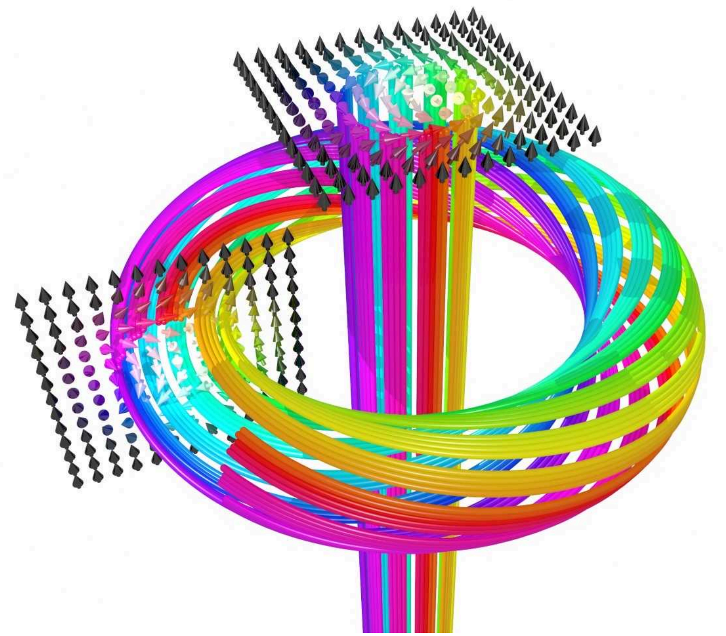
ENERGY AND TOPOLOGICAL CHARGE



F.N. Rybakov et al, APL Mater. **10**, 111113 (2022).

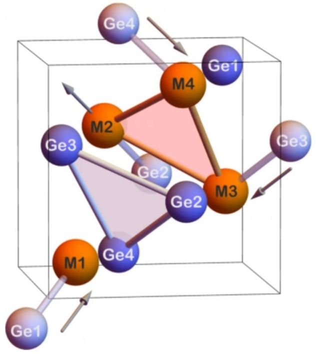


HOPFIONS IN CHIRAL MAGNETS

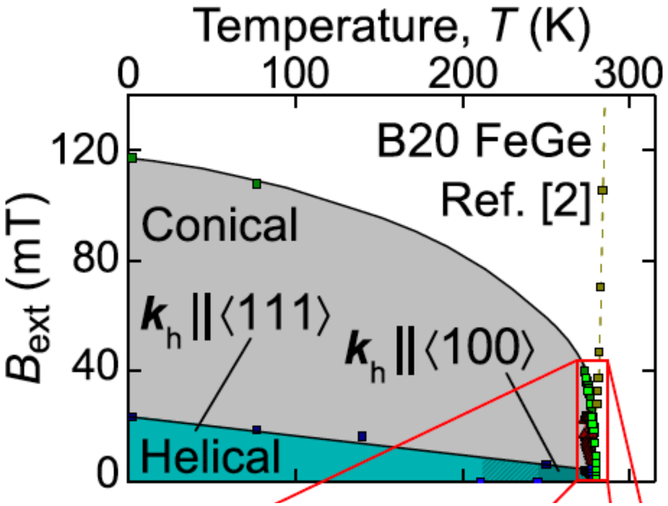


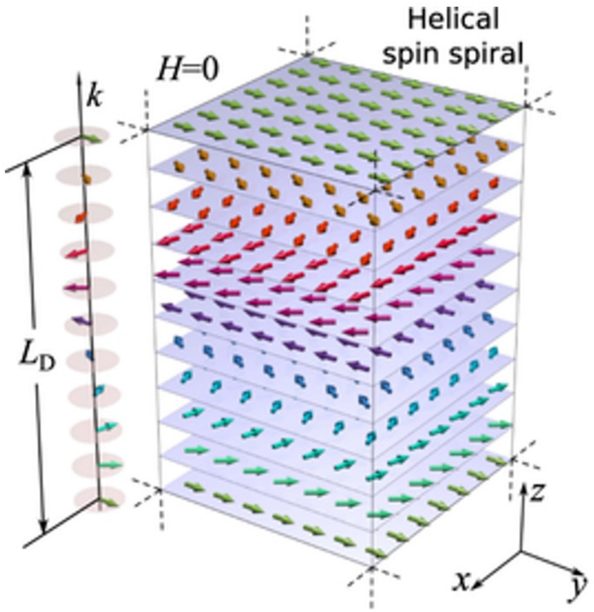
CHIRAL MAGNETS

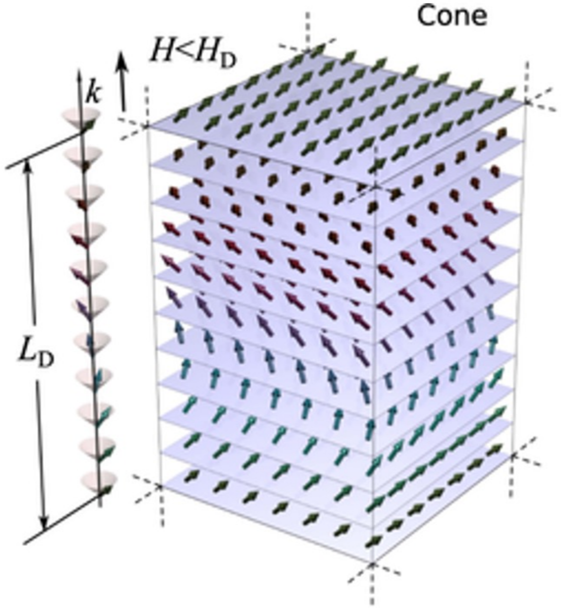
$$\mathcal{E} = \int_{V_m} d\mathbf{r}\, \mathcal{A}\, |\nabla \mathbf{m}|^2 + \mathcal{D}\, \mathbf{m} \cdot (\nabla \times \mathbf{m}) - M_s\, \mathbf{m} \cdot \mathbf{B} + \frac{1}{2\mu_0} \int_{\mathbb{R}^3} d\mathbf{r}\, |\nabla \times \mathbf{A}_d|^2, \quad \mathbf{B} = \mathbf{B}_{ext} + \nabla \times \mathbf{A}_d\,,$$

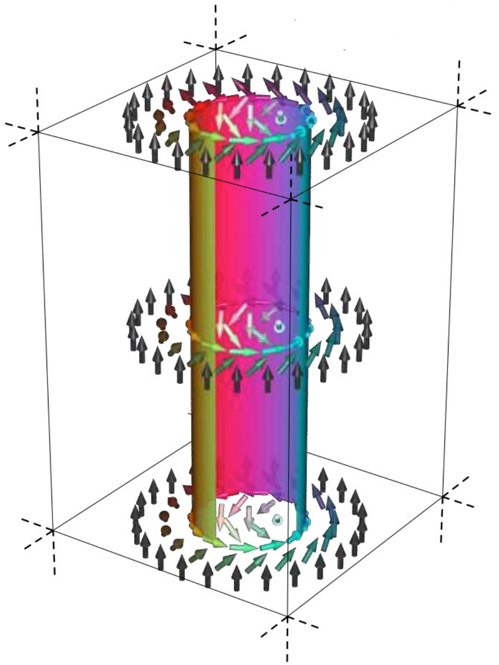


FeGe, MnSi, $Fe_{1-x}Co_xSi$, etc.





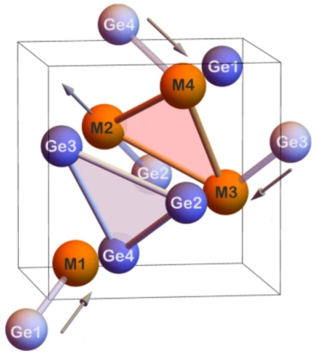




A. Bauer, C. Pfleiderer,
B. Springer Series in Materials Science, 228, 1 (2016)

B20-TYPE CRYSTALS

$$\mathcal{E} = \int_{V_m} d\mathbf{r}\, \mathcal{A}\, |\nabla \mathbf{m}|^2 + \mathcal{D}\, \mathbf{m} \cdot (\nabla \times \mathbf{m}) - M_s\, \mathbf{m} \cdot \mathbf{B} + \frac{1}{2\mu_0} \int_{\mathbb{R}^3} d\mathbf{r}\, |\nabla \times \mathbf{A}_d|^2, \quad \mathbf{B} = \mathbf{B}_{ext} + \nabla \times \mathbf{A}_d,$$



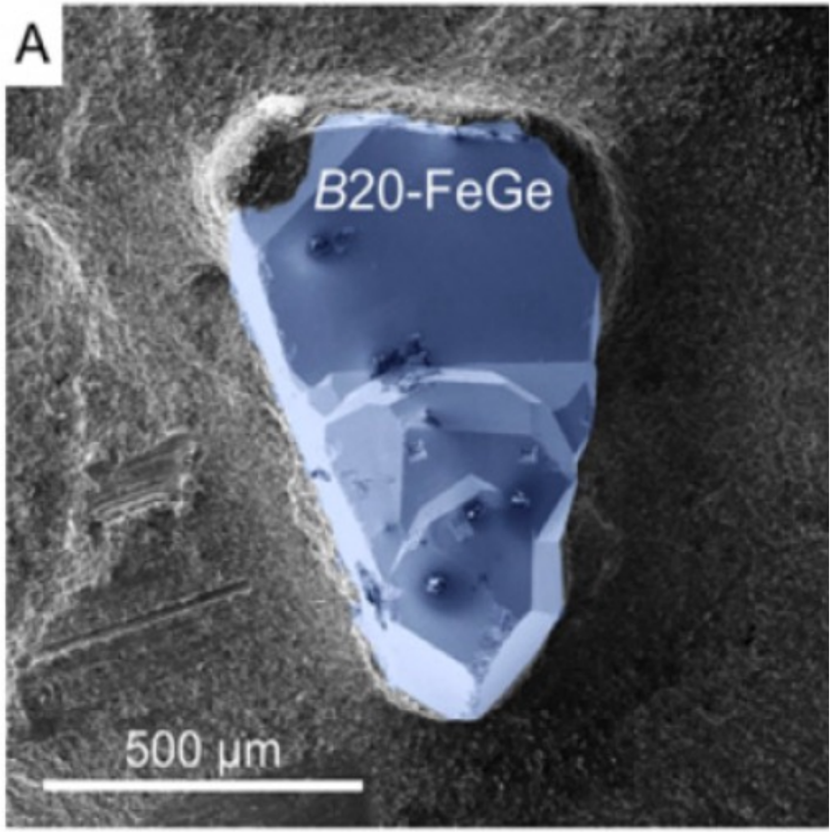
B20-type FeGe material parameters:

$\mathcal{A} = 4.75\ \mathrm{pJm^{-1}}$,

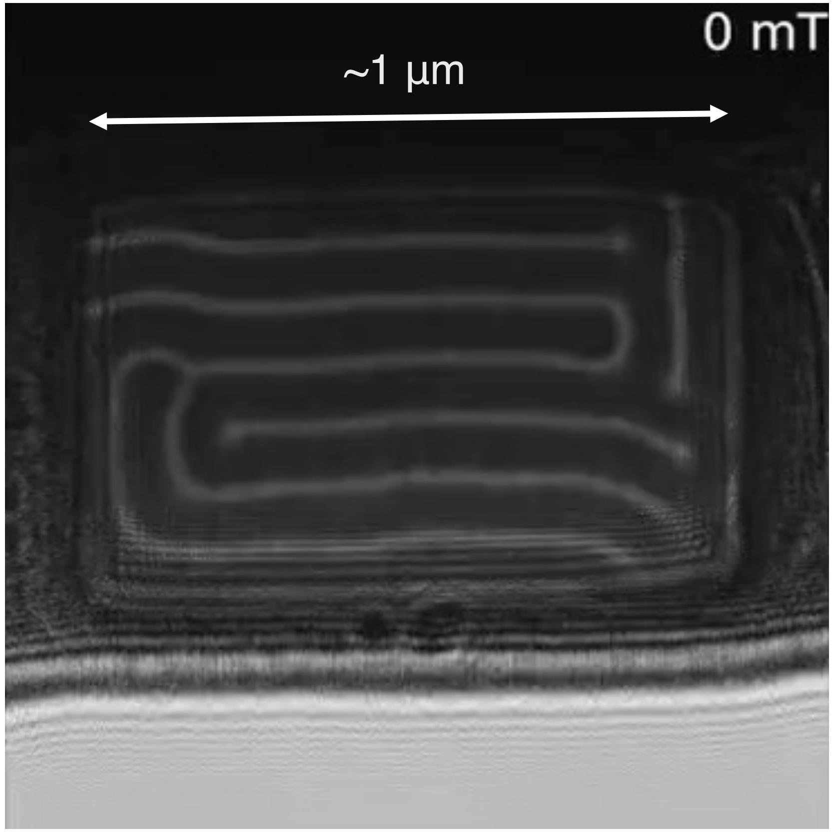
$\mathcal{D} = 0.853\ \mathrm{mJm^{-2}}$,

$M_s = 384\ \mathrm{kAm^{-1}}$.

FeGe single crystal



FeGe 180-nm-thick lamella



B20-TYPE CRYSTALS

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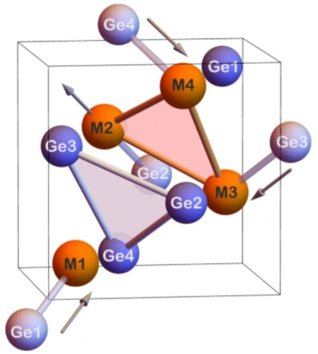

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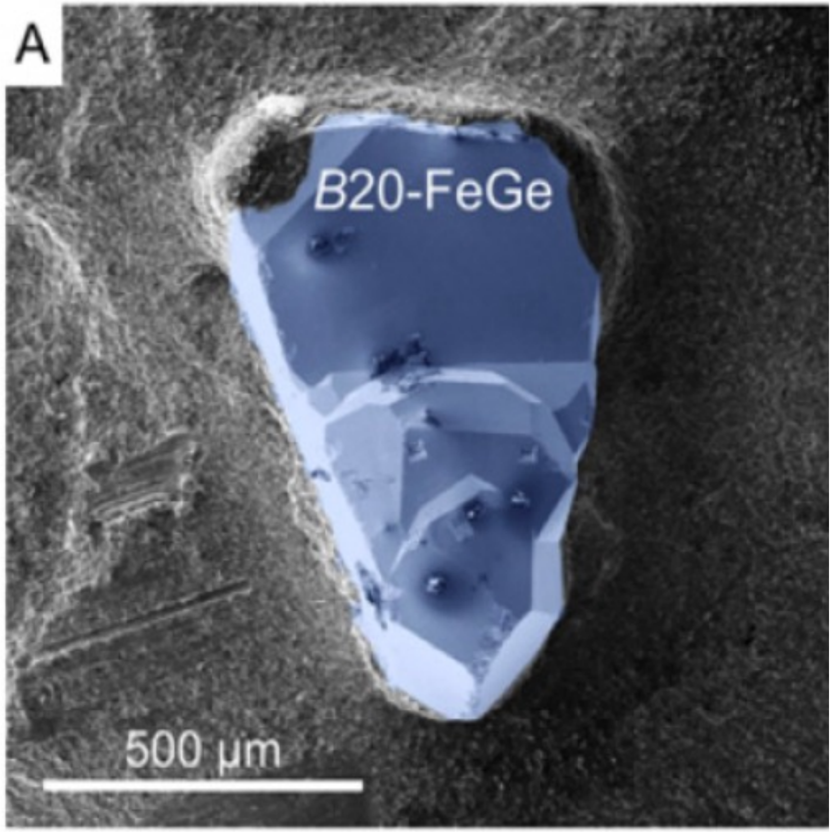


Selected publications on FeGe:

C. Jin, et al, Nat. Commun. **8**, 15569 (2017).

H. Du, et al., Phys. Rev. Lett. **120**, 197203 (2018).

F. Zheng, et al, Nat. Nanotech. **13**, 451 (2018).

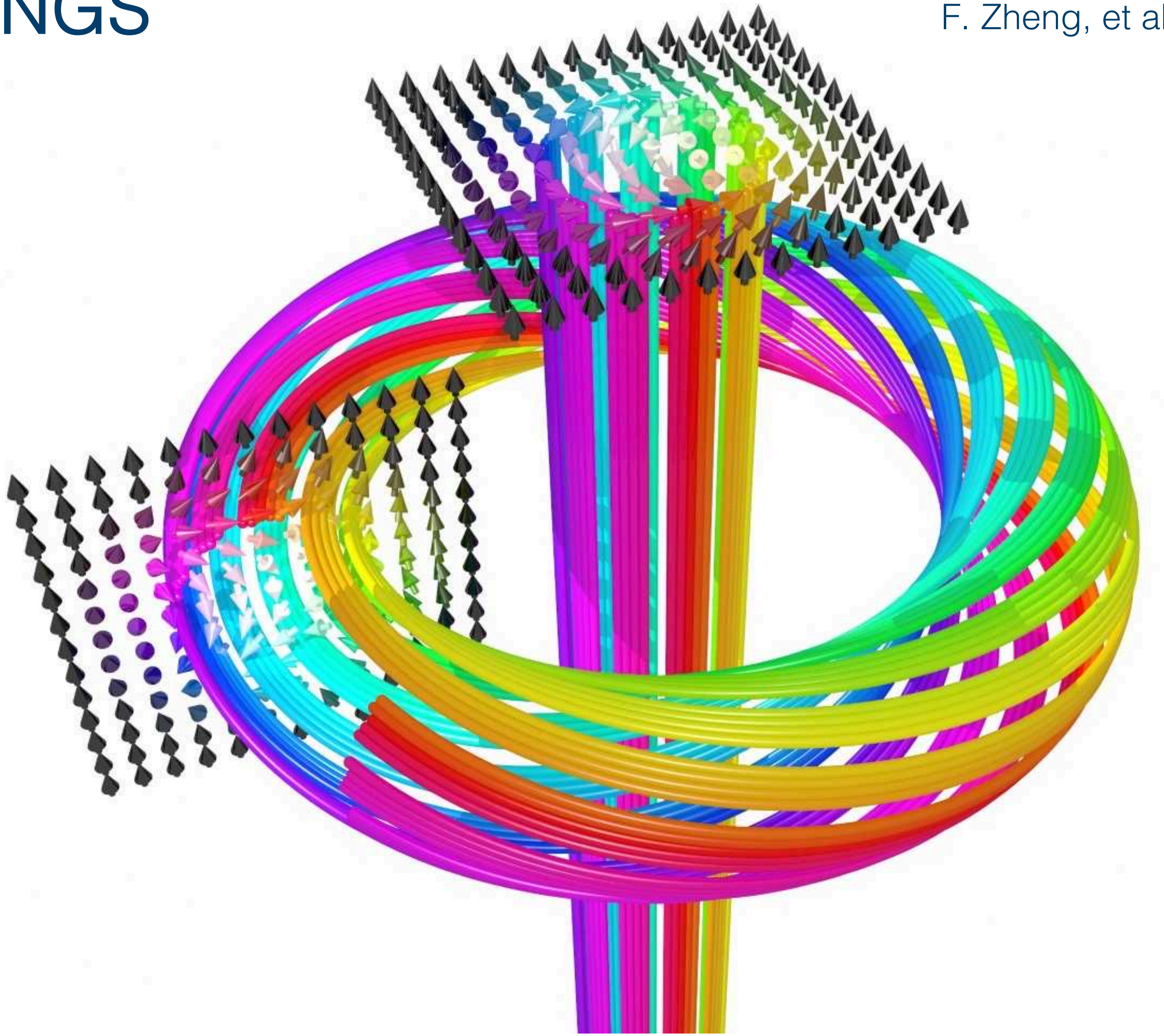
F. Zheng, et al, Nat. Commun. **12**, 5316 (2021).

F. Zheng, et al, Nat. Phys. **18**, 863 (2022).

F. Zheng, et al, Nature **623**,718 (2023).

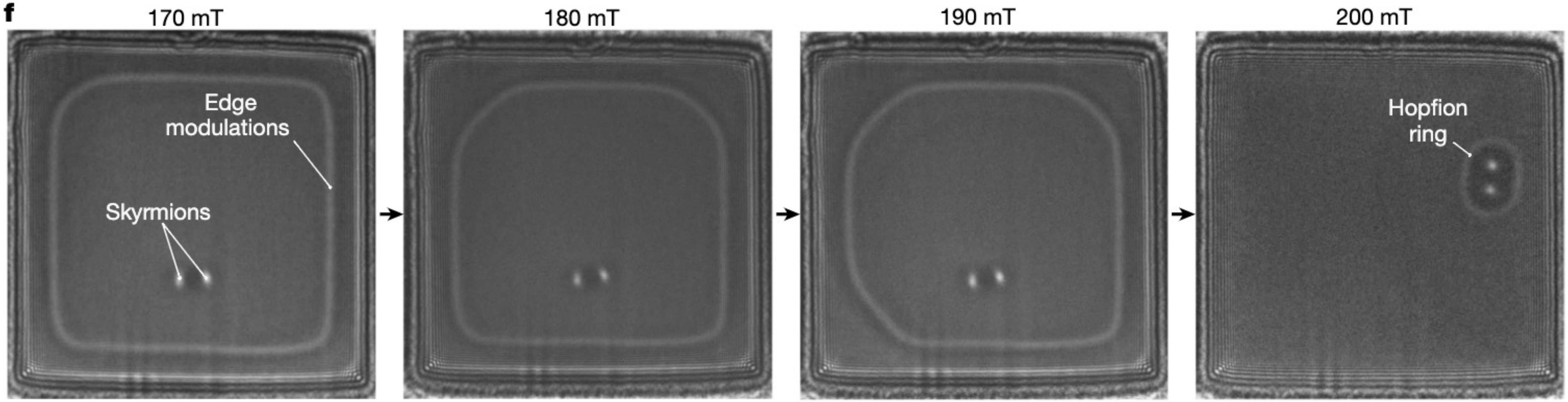
HOPFION RINGS

F. Zheng, et al, Nature **623**,718 (2023).



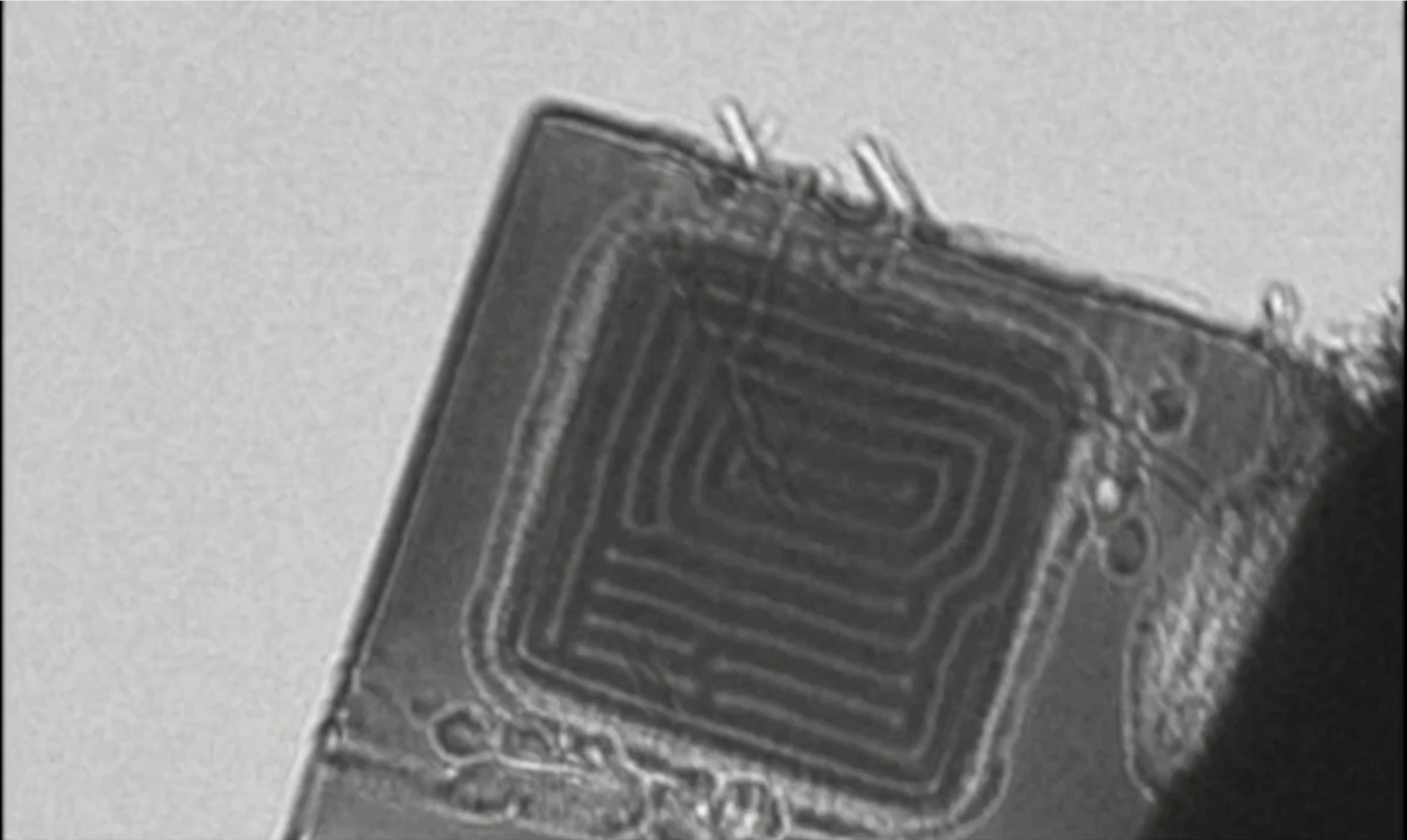
HOPFION RING NUCLEATION

Over-focus Lorentz TEM images

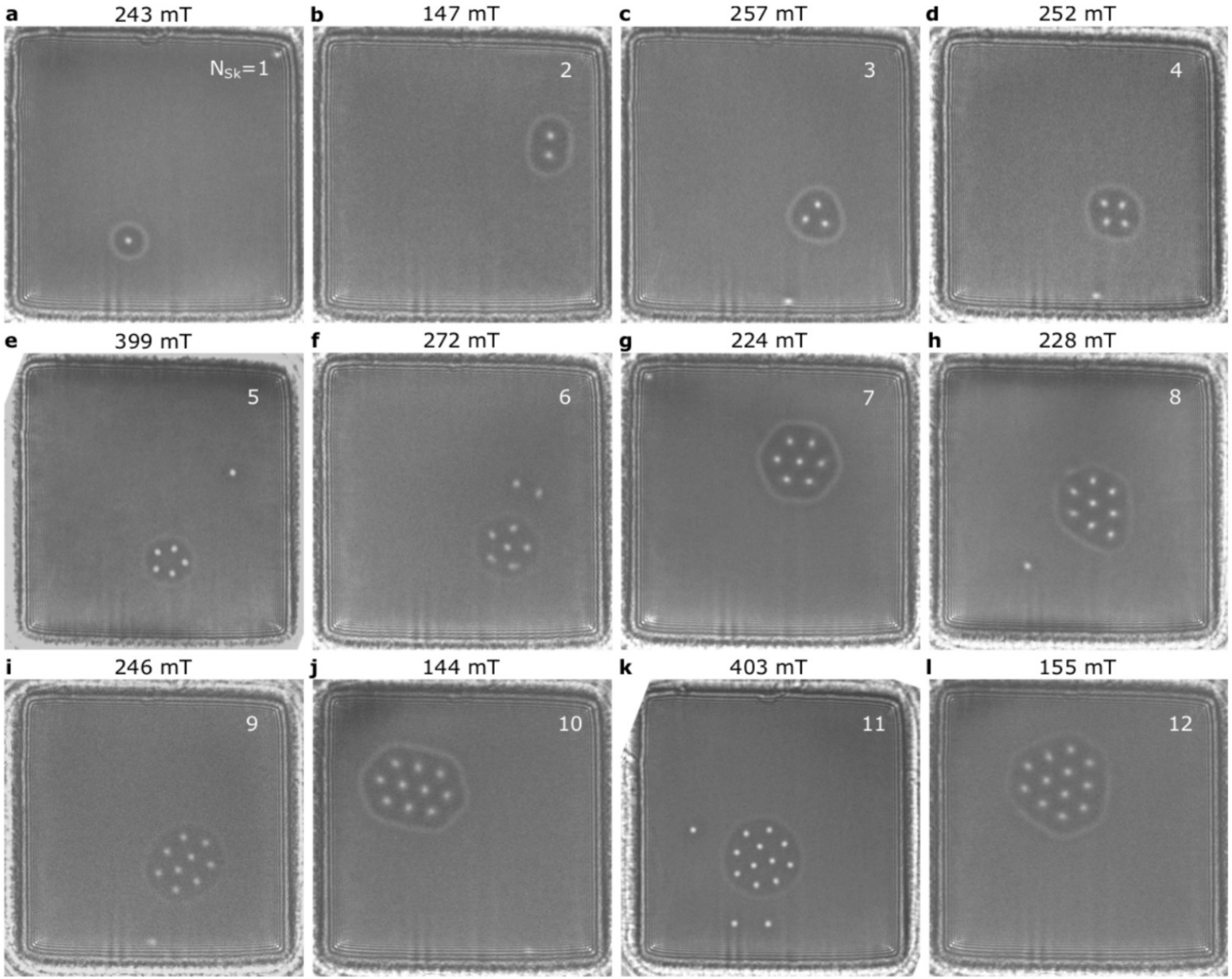


Sample size: 1µm x 1µm x 180 nm

F. Zheng, et al, Nature **623**,718 (2023).



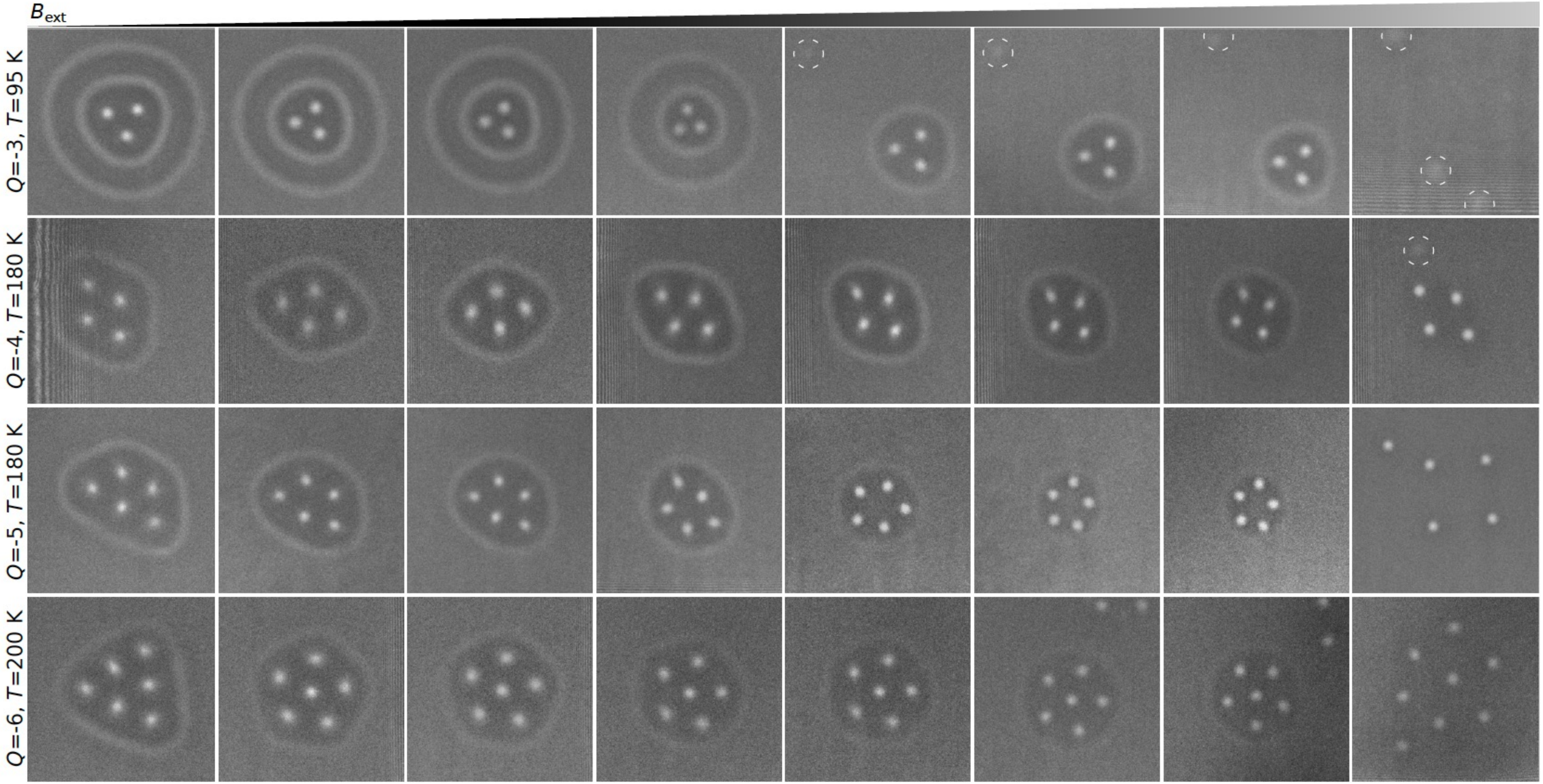
HOPFION RINGS DIVERSITY



F. Zheng, et al, Nature **623**,718 (2023).

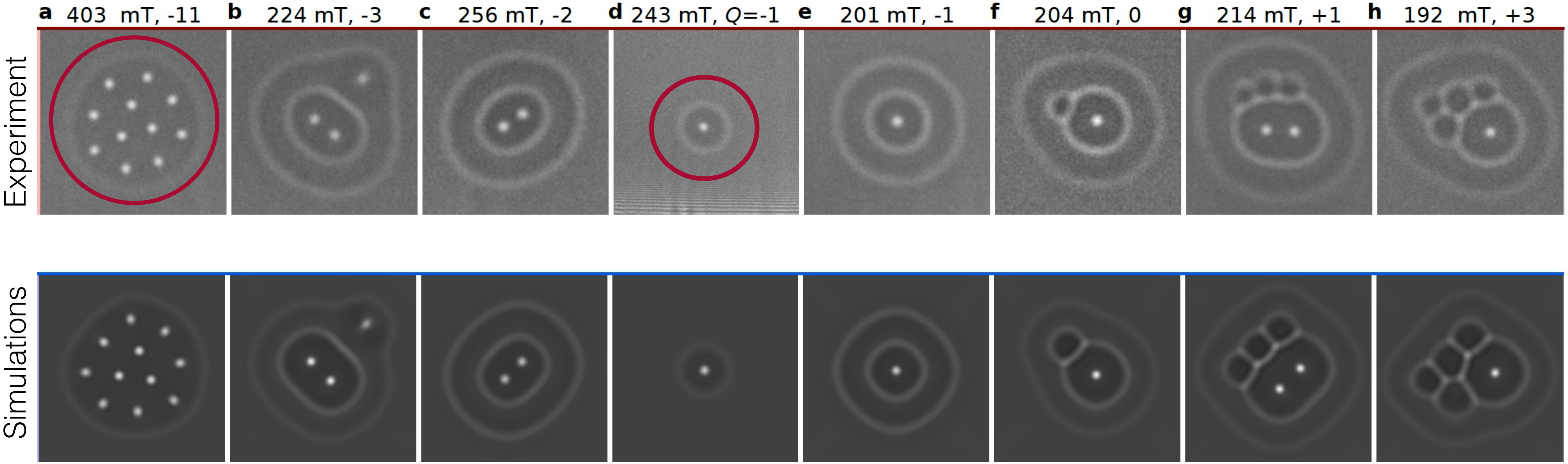
HOPFION RING IN FIELD

Over focus Lorentz TEM images

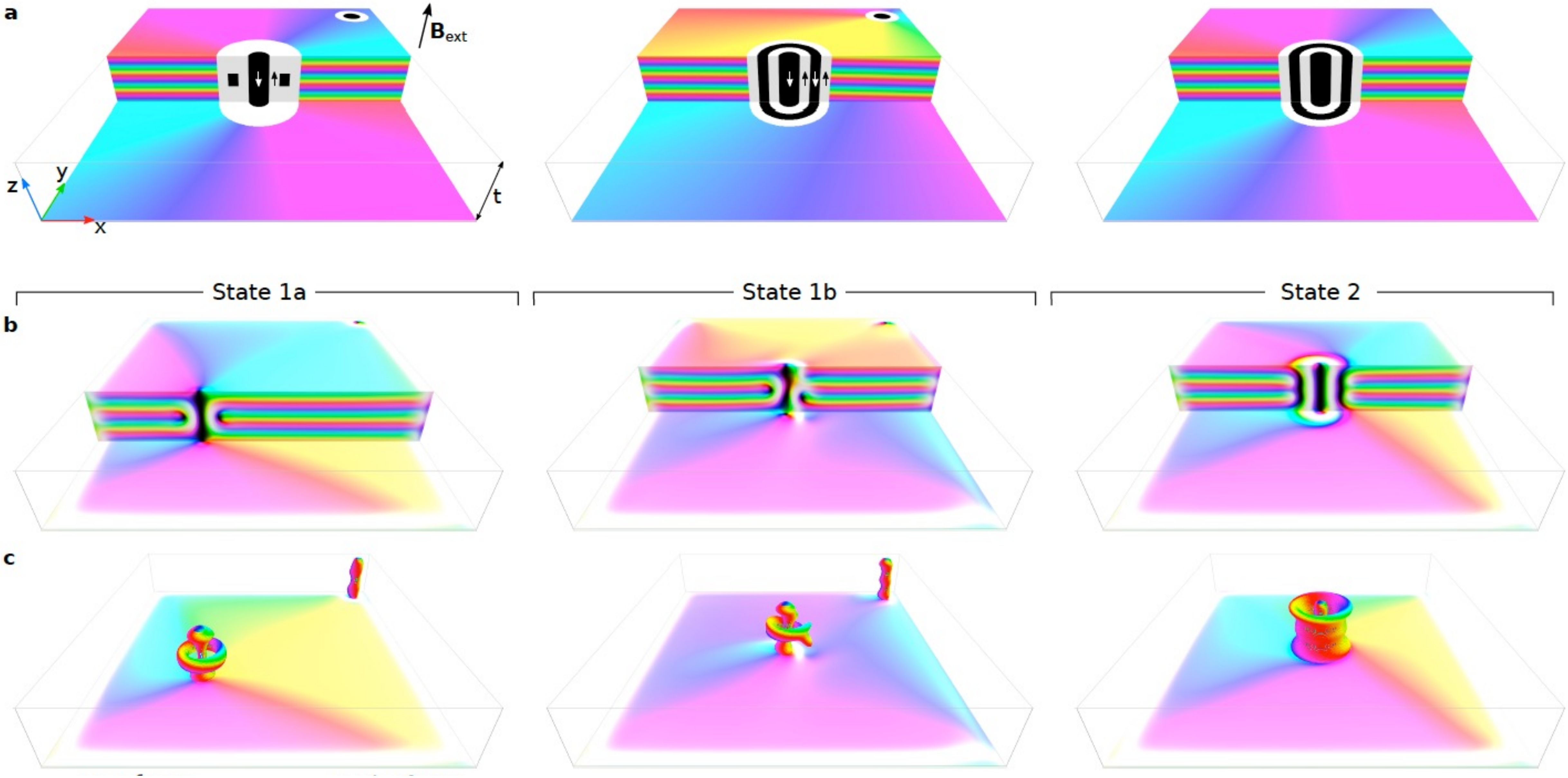


F. Zheng, et al, Nature **623**,718 (2023).

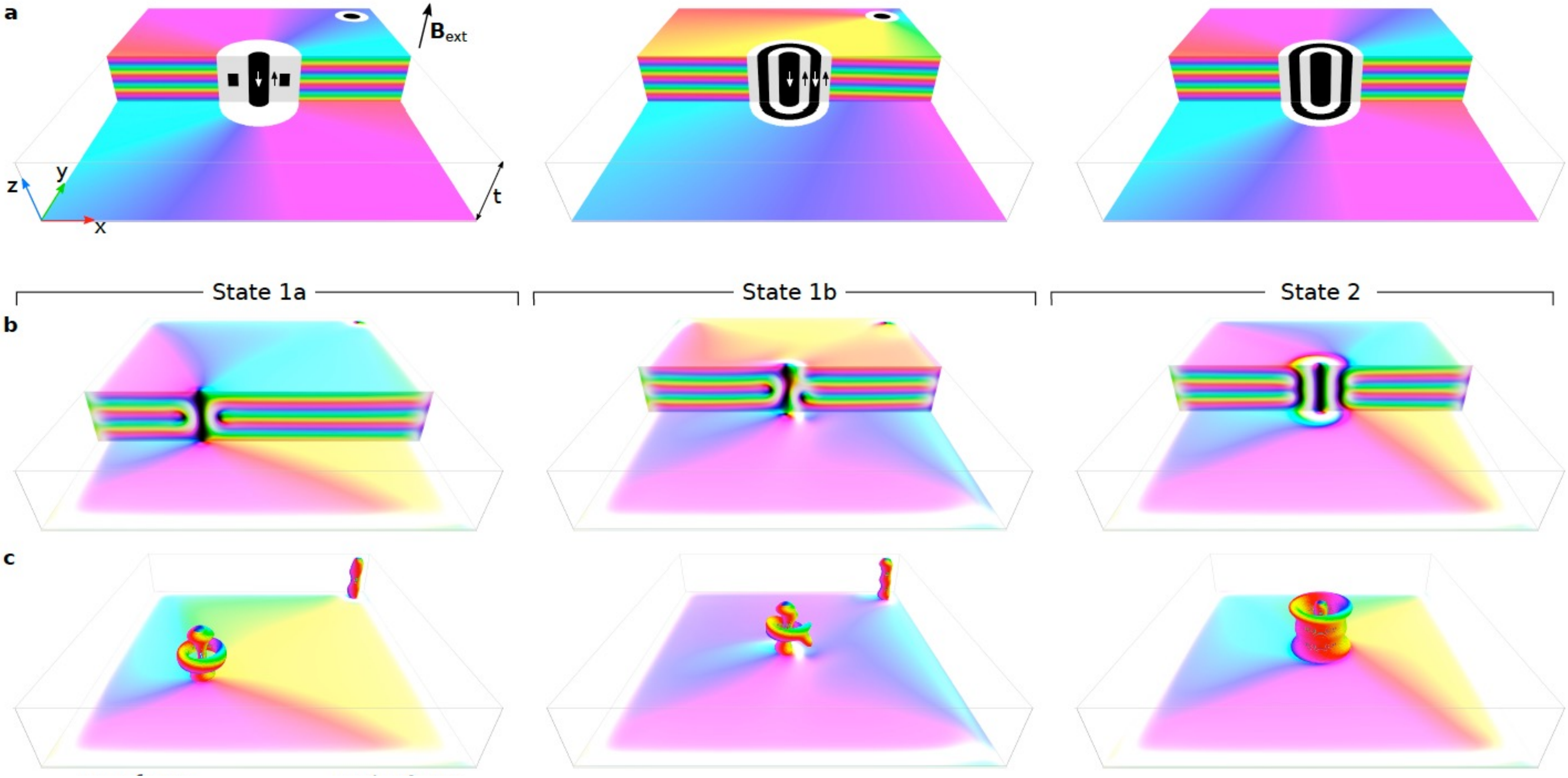
THEORY VS EXPERIMENT



F. Zheng, et al, Nature **623**,718 (2023).



a
B_ext
z
y
x
t
State 1a
State 1b
State 2
b
c


a
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z
y
x
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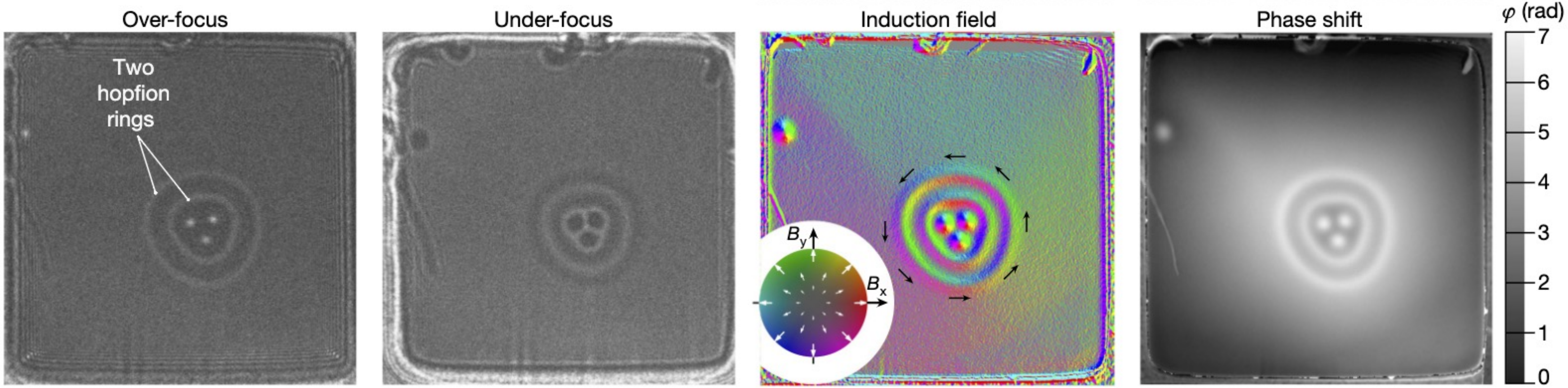
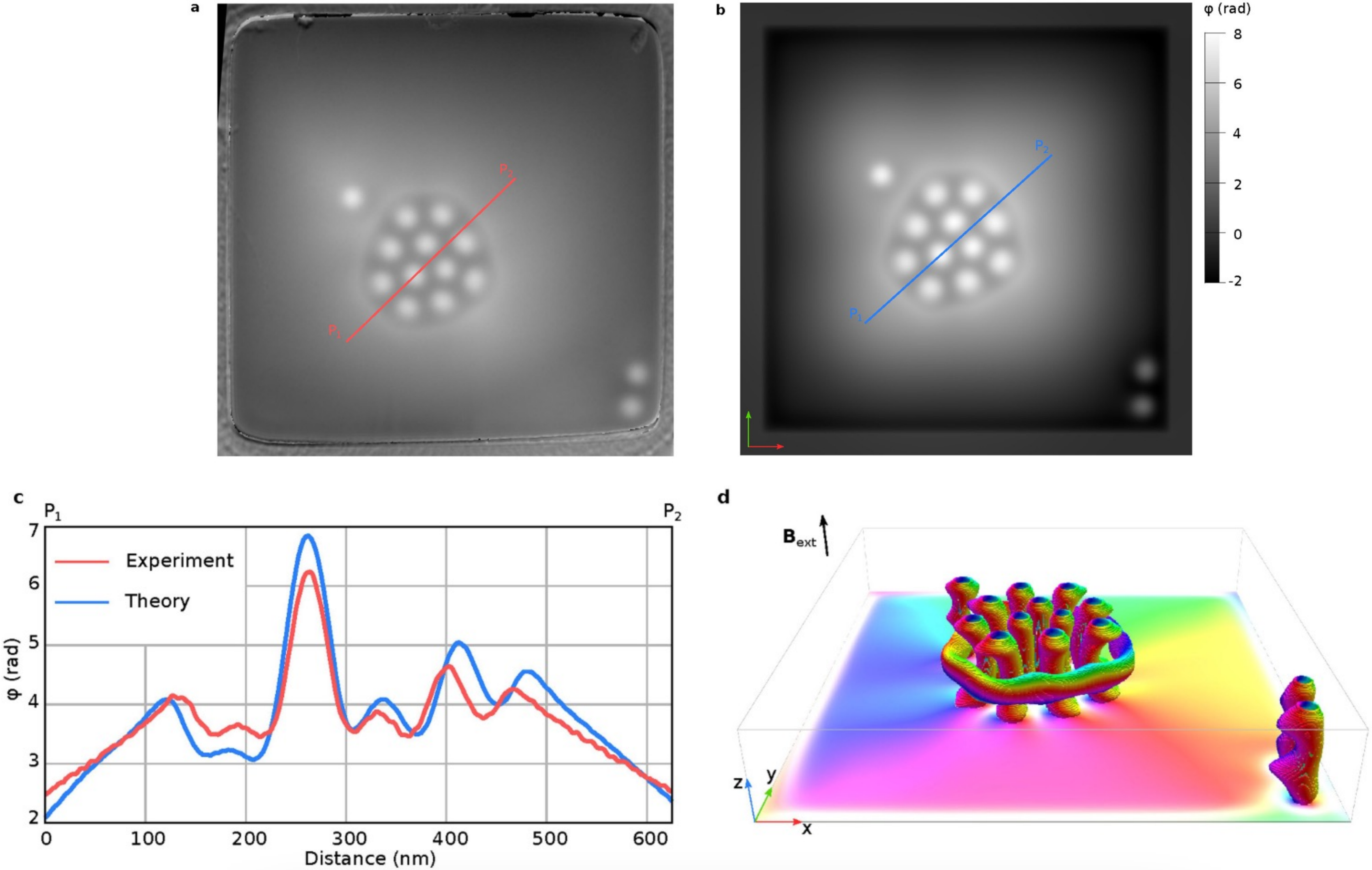
State 1a
State 1b
State 2
b
c

ELECTTRON HOLOGRAPHY EXPERIMENT



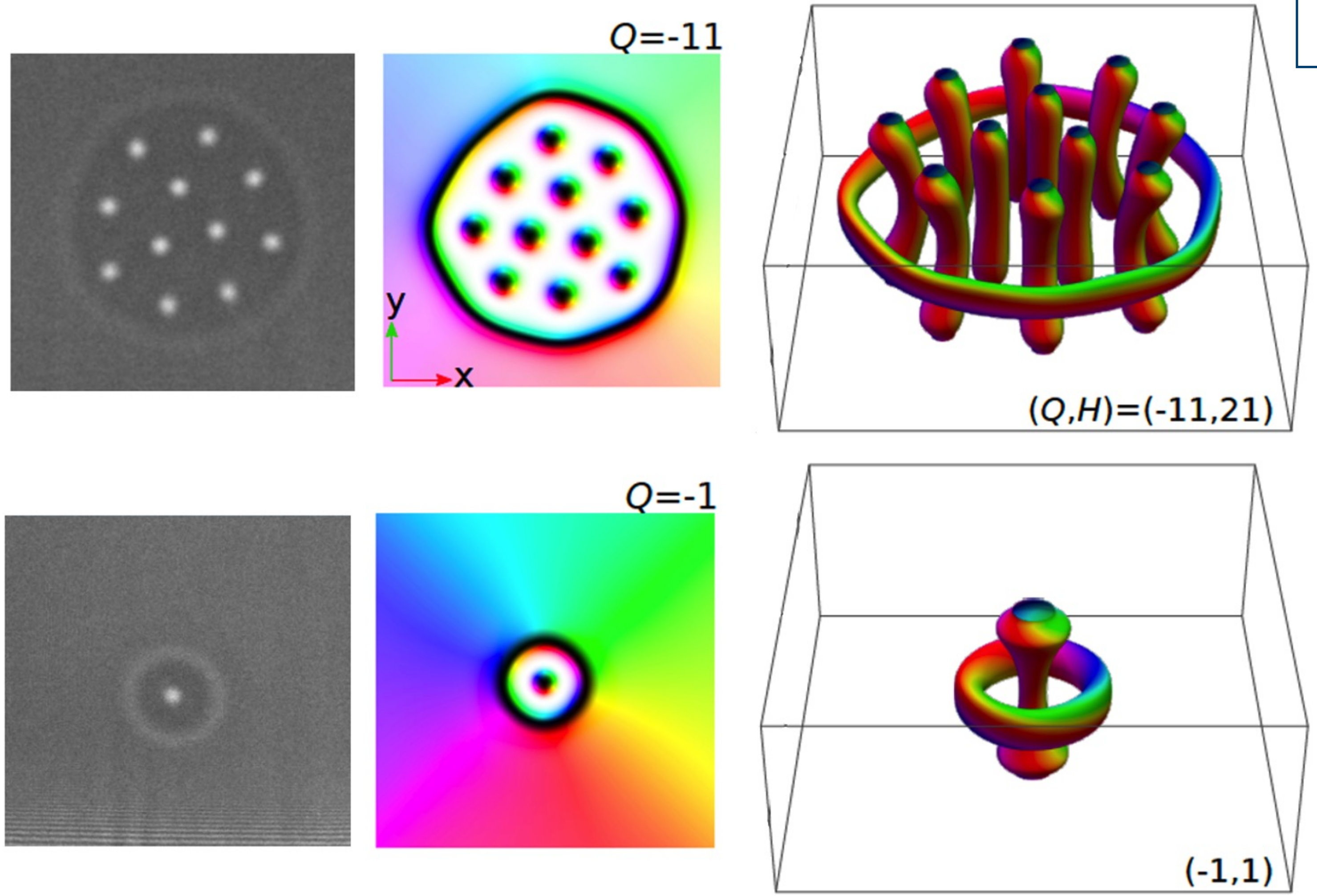
F. Zheng, et al, Nature **623**,718 (2023).

ELECTTRON HOLOGRAPHY EXPERIMENT



JÜLICH Forschungszentrum

HOMOTOPY GROUP ANALYSIS



Second homotopy group of S^2

Third homotopy group of S^2

$$G = \pi_2(\mathbb{S}^2, \mathbf{m}_0) \times \pi_3(\mathbb{S}^2, \mathbf{m}_0) = \mathbb{Z} \times \mathbb{Z},$$

$$Q = \frac{1}{4\pi} \int_\Omega \mathrm{d}r_1 \mathrm{d}r_2 \; \mathbf{F} \cdot \hat{\mathbf{e}}_{r_3},$$

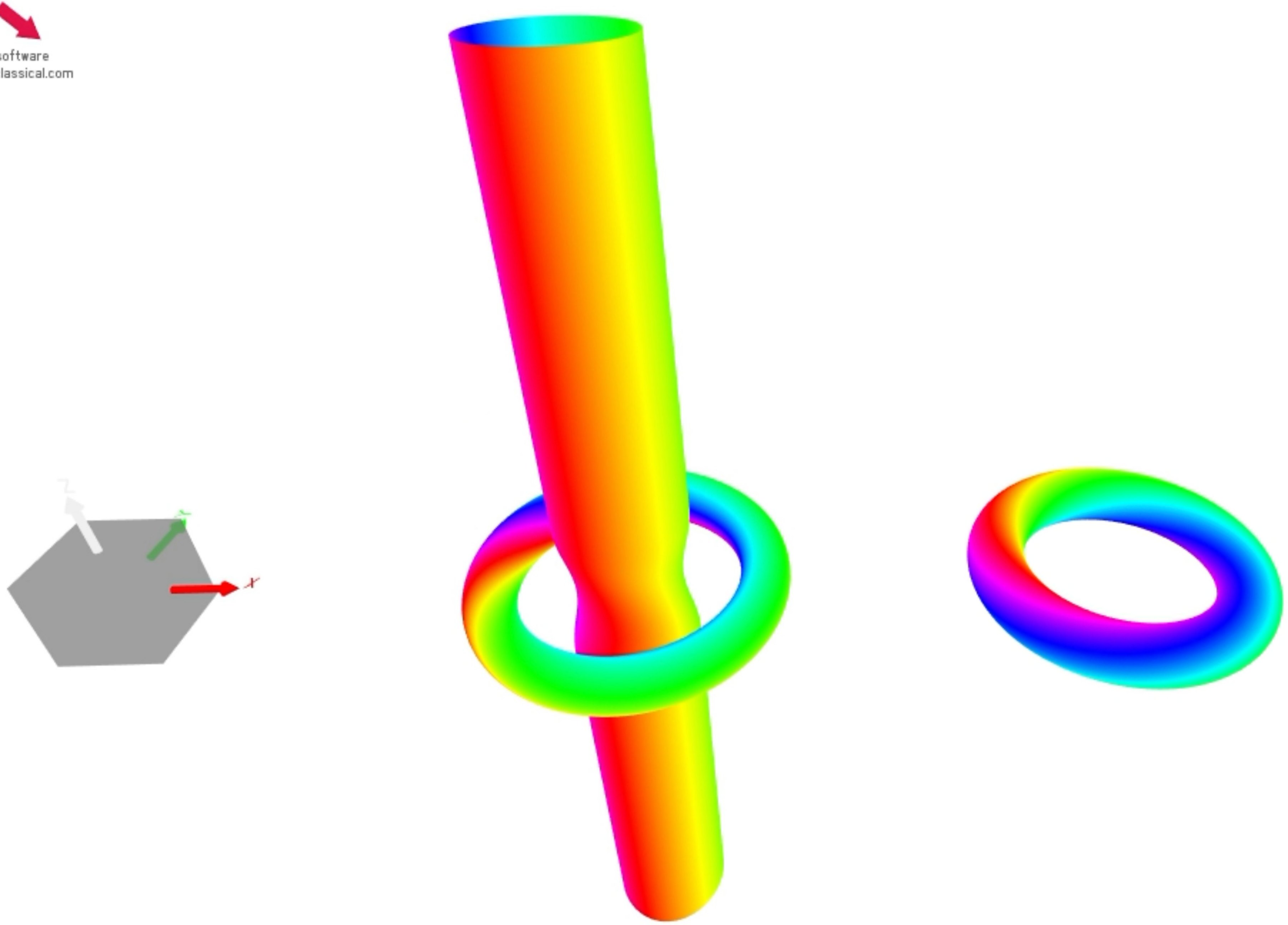
$$H = -\frac{1}{16\pi^2} \int_\Omega \mathrm{d}r_1 \mathrm{d}r_2 \mathrm{d}r_3 \; \mathbf{F} \cdot [(\nabla\times)^{-1}\mathbf{F}].$$

where

$$\mathbf{F} = \begin{pmatrix} \mathbf{m} \cdot [\partial_{r_2}\mathbf{m} \times \partial_{r_3}\mathbf{m}] \\ \mathbf{m} \cdot [\partial_{r_3}\mathbf{m} \times \partial_{r_1}\mathbf{m}] \\ \mathbf{m} \cdot [\partial_{r_1}\mathbf{m} \times \partial_{r_2}\mathbf{m}] \end{pmatrix}$$

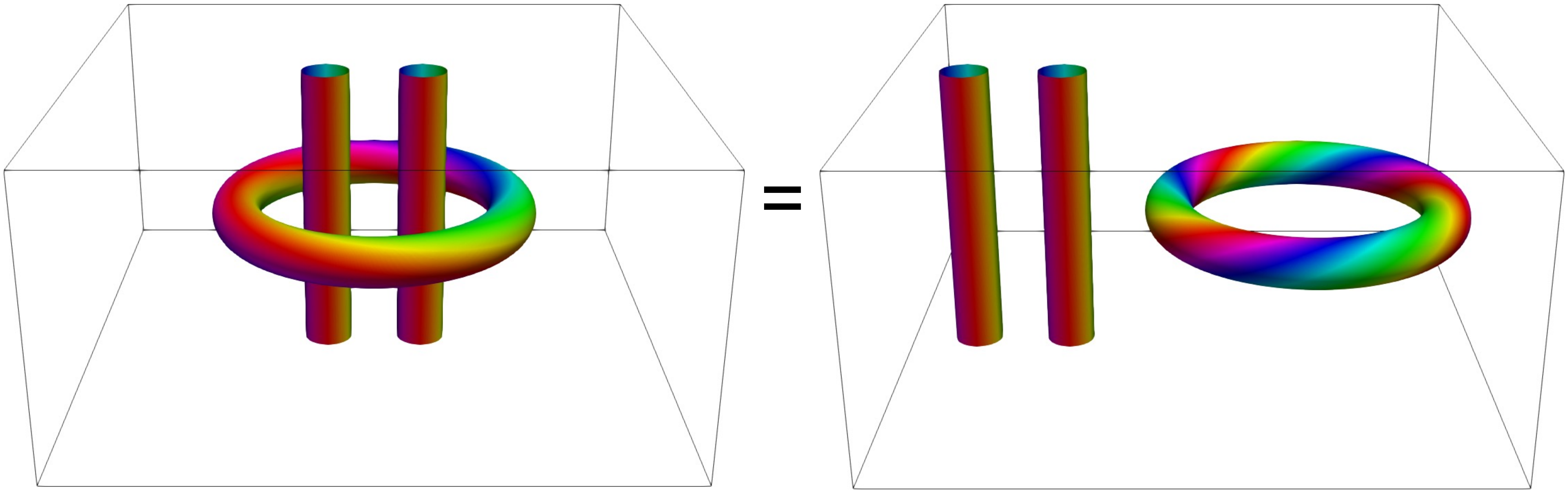
is the vector of curvature and r_1, r_2, r_3 are local right-handed Cartesian coordinates

F. Zheng, et al, Nature **623**,718 (2023).



HOMOTOPY GROUP ANALYSIS

Q=-2, H=3



HOMOTOPY GROUP ANALYSIS

Nanomagnetism in 3D

Workshop, April 30th - May 2nd 2024

Combination of hopfion with skyrmion

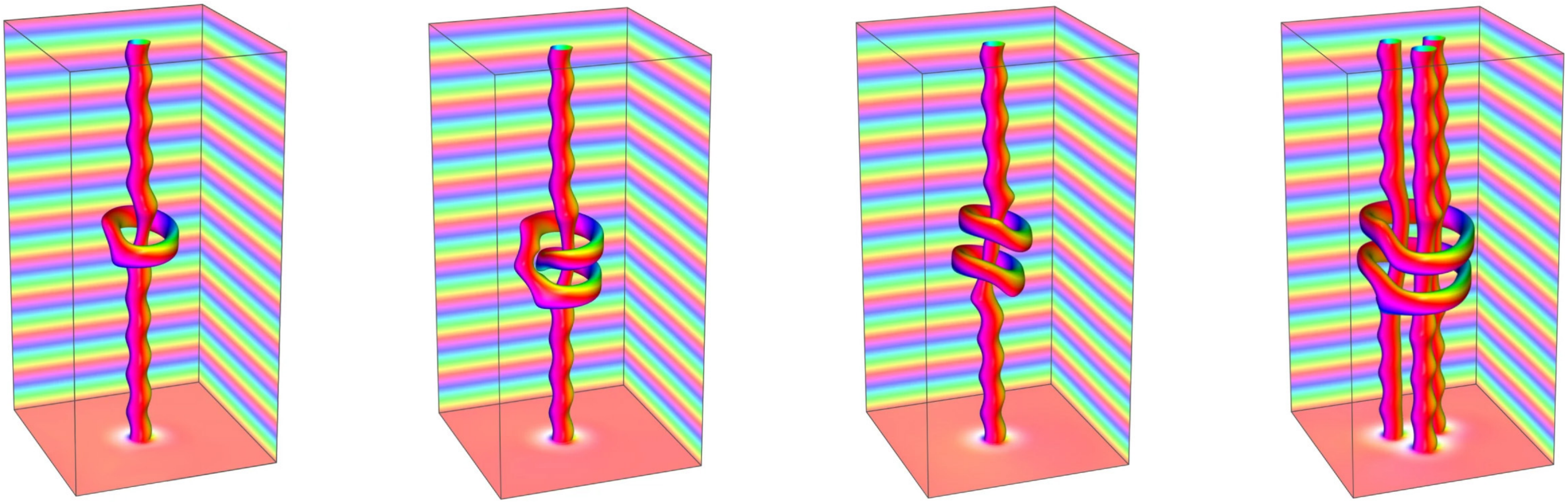
SPICE Workshop on Nanomagnetism in 3D, April 30th - May 2nd 2024

Phillip Rybakov

Magnetic skyrmions and hopfions are essentially particles made of spins. The topological nature of these states is two- and three-dimensional, respectively. Recent experiments have confirmed the existence of hopfions in magnetic crystals [1]. As it turned out, hopfions naturally combine with skyrmions and form stable bound states. We will discuss in detail the combinations of such topologically different particles. We will also consider the homotopy group, which simultaneously classifies skyrmions, hopfions, and their combinations.

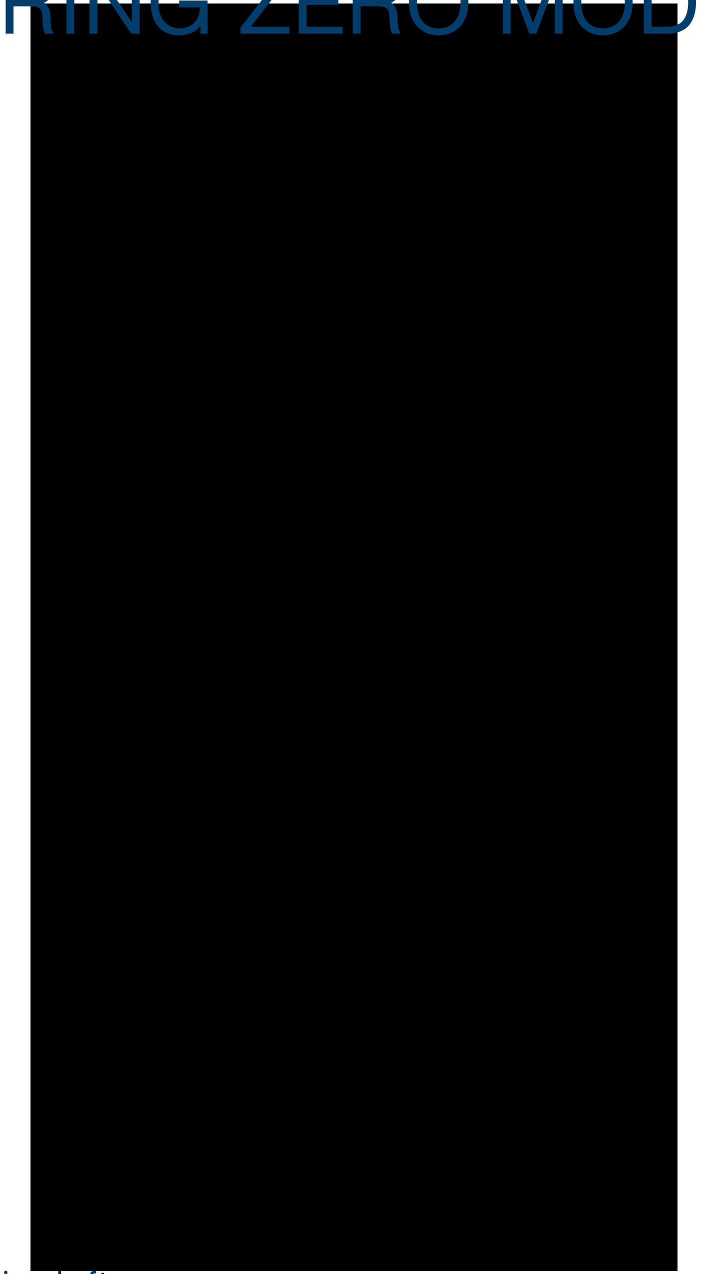
[1] F. Zheng, et al. Hopfion rings in a cubic chiral magnet. Nature 623, 718 (2023)

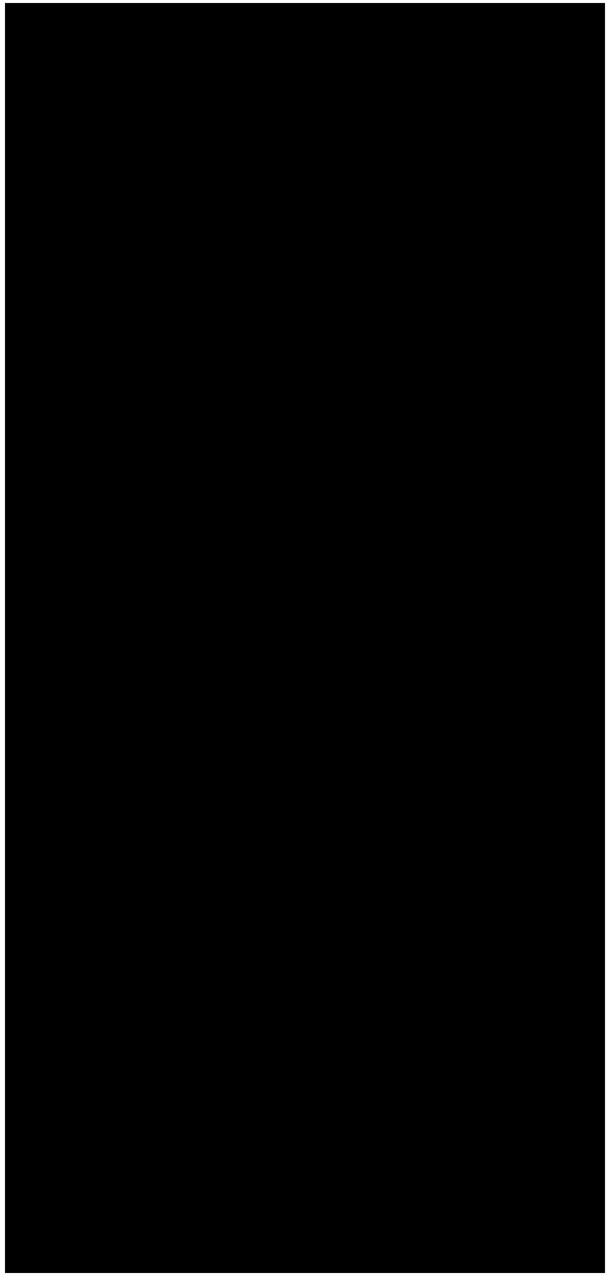
HOPFION RINGS IN BULK



F. Zheng, et al, Nature **623**,718 (2023).

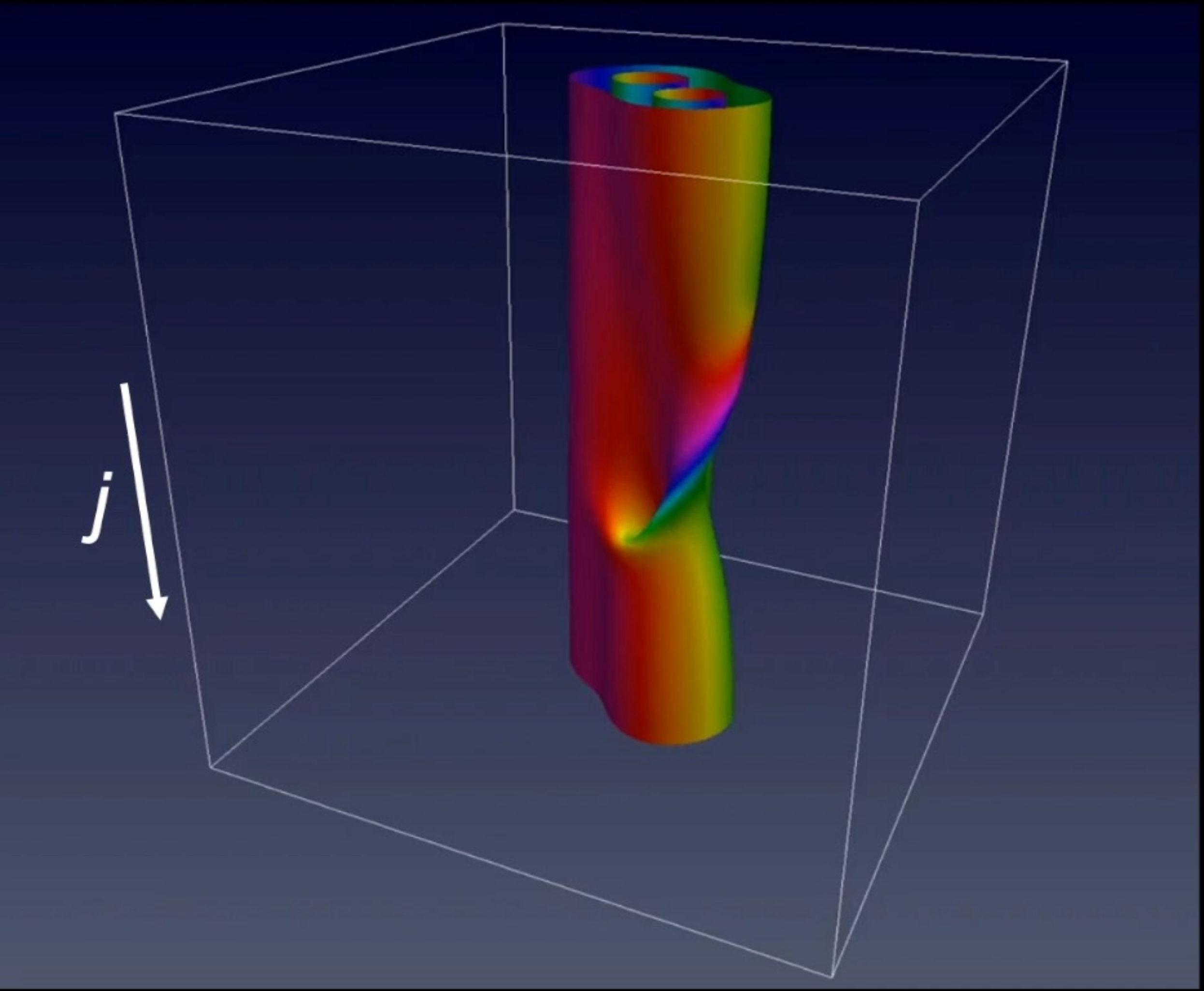
HOPFION RING ZERO MODE





HOPFION CHARGE OF HYBRID SKYRMIONS TUBE

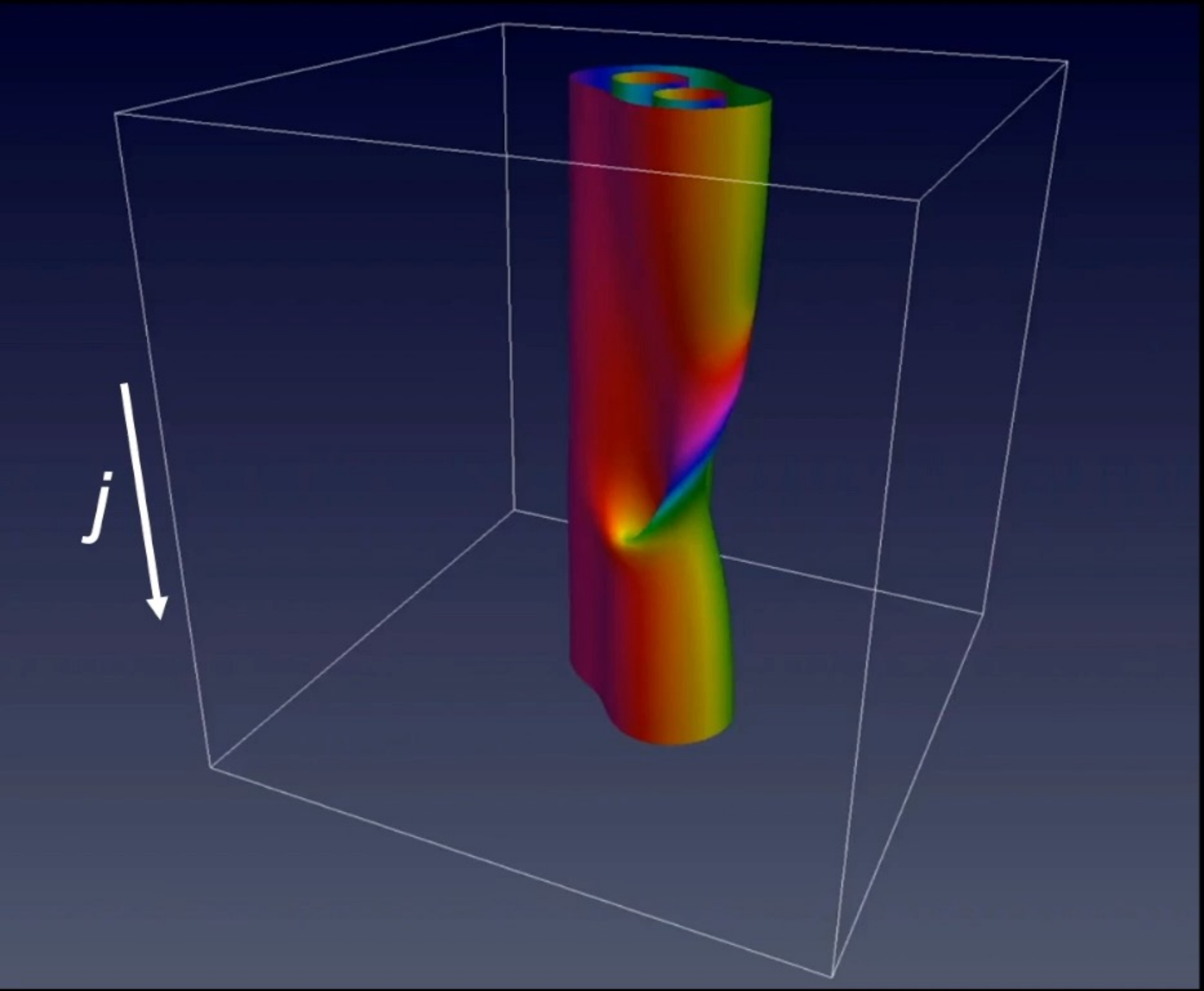
(Q,H)=(1,-1)

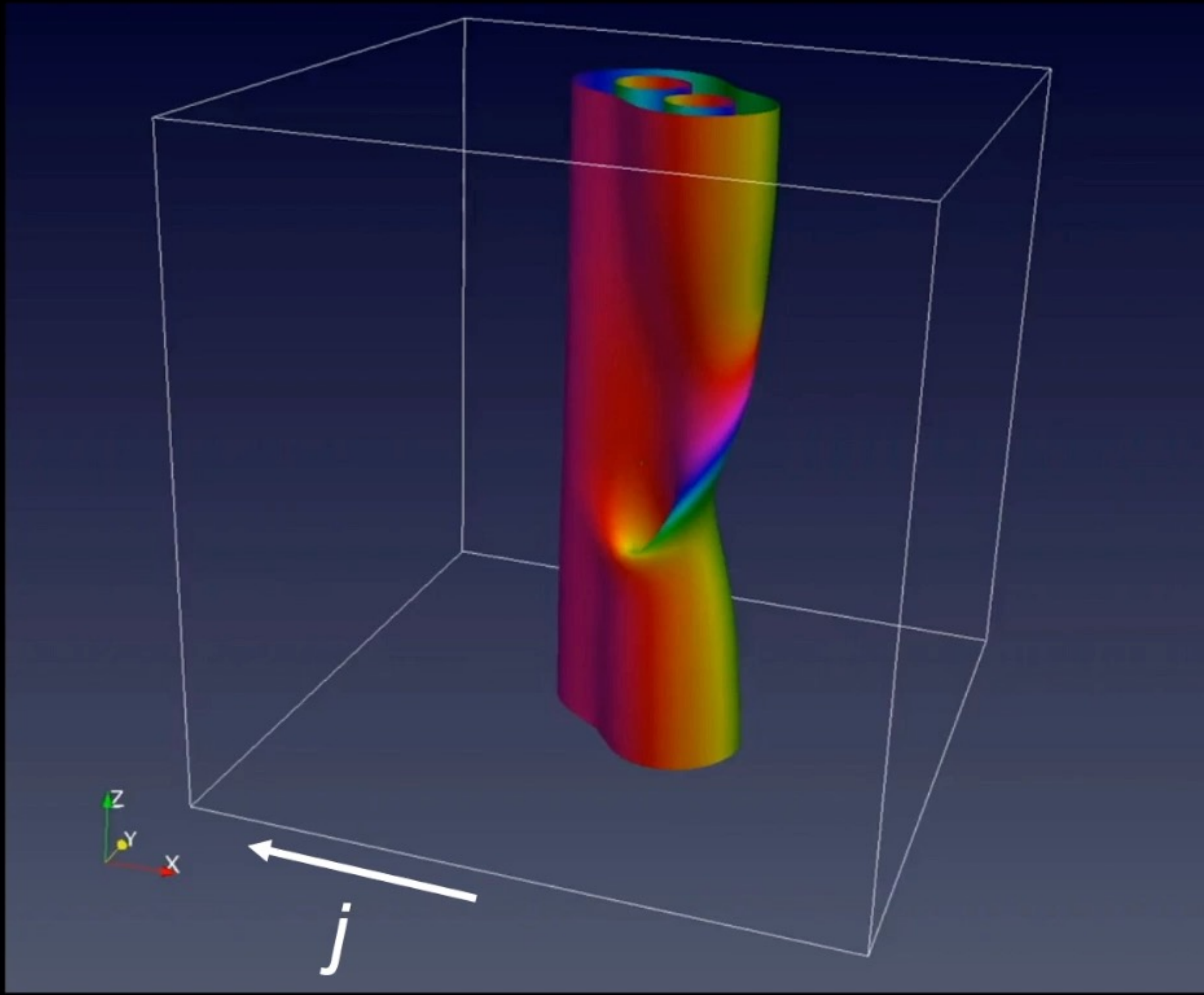


V.M. Kuchkin & NSK, APL Mater. 10, 071102 (2022).

HOPFION CHARGE OF HYBRID SKYRMIONS TUBE

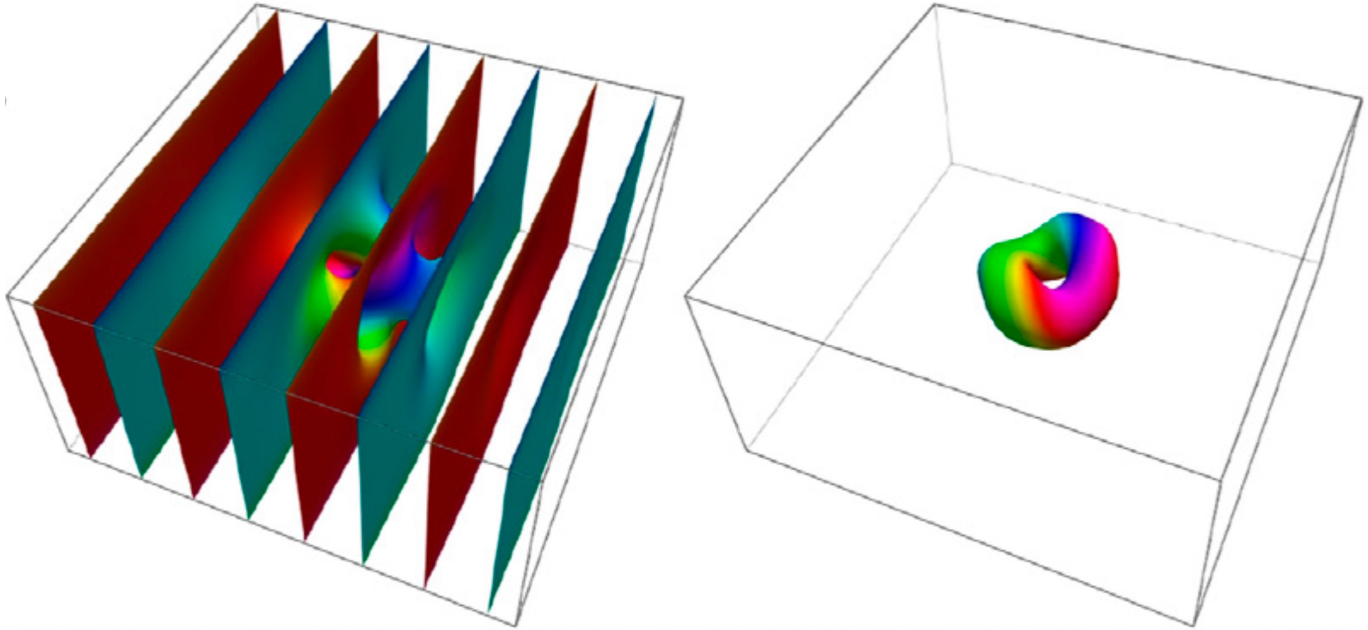
$(Q,H)=(1,-1)$



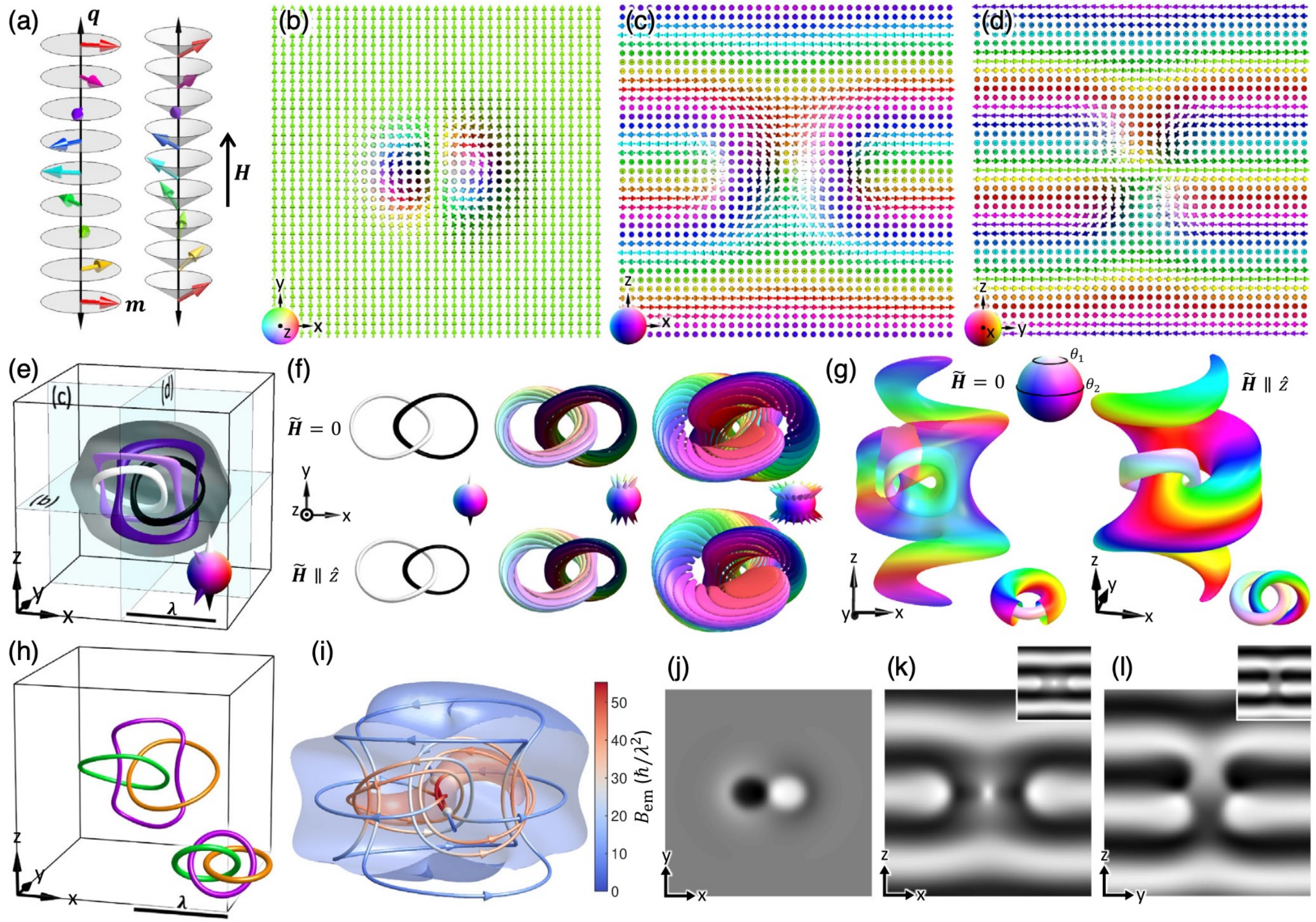


V.M. Kuchkin & NSK, APL Mater. 10, 071102 (2022).

HELIKNOTON

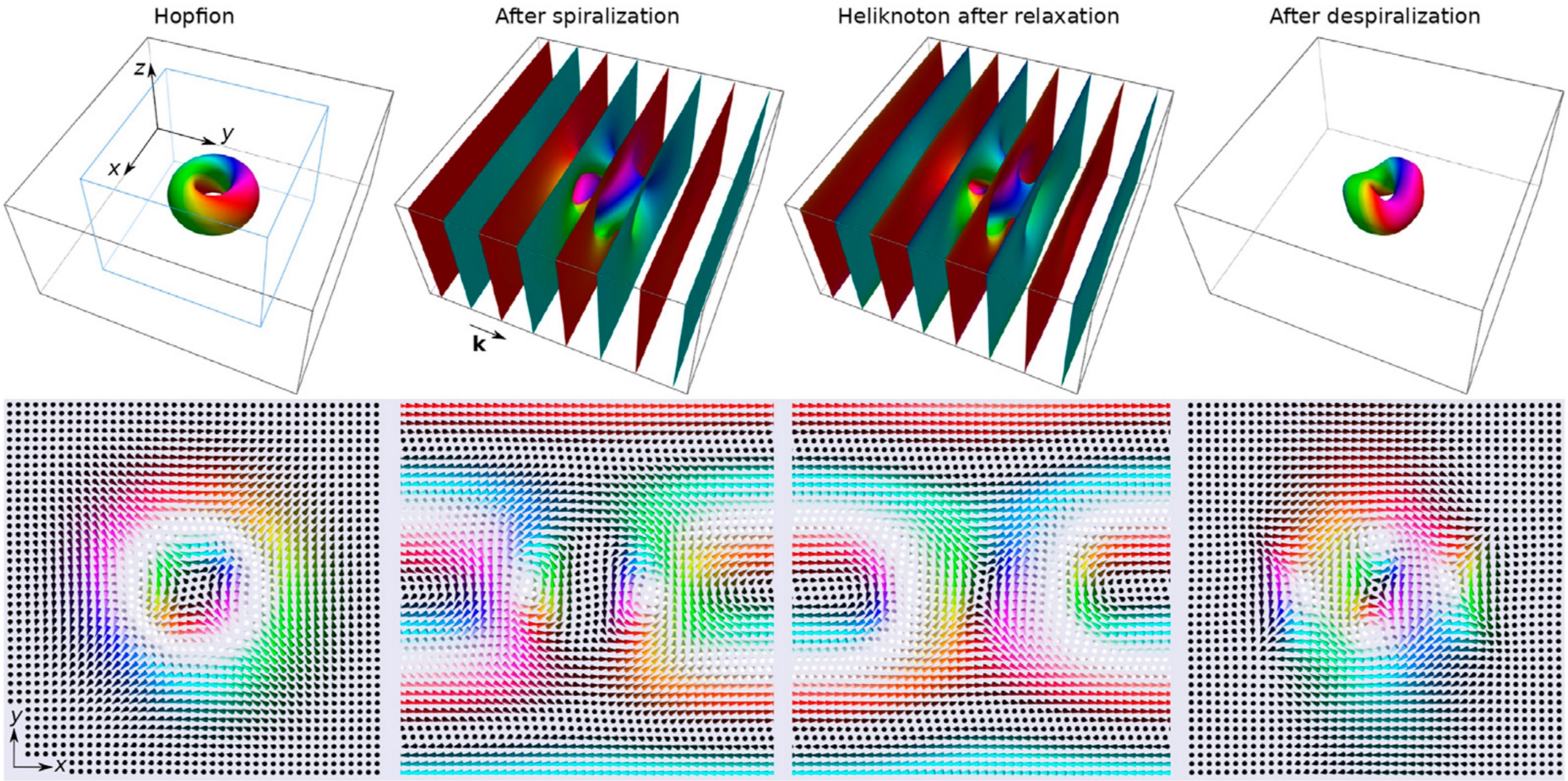


HELIKNOTON IN A CHIRAL MAGNET



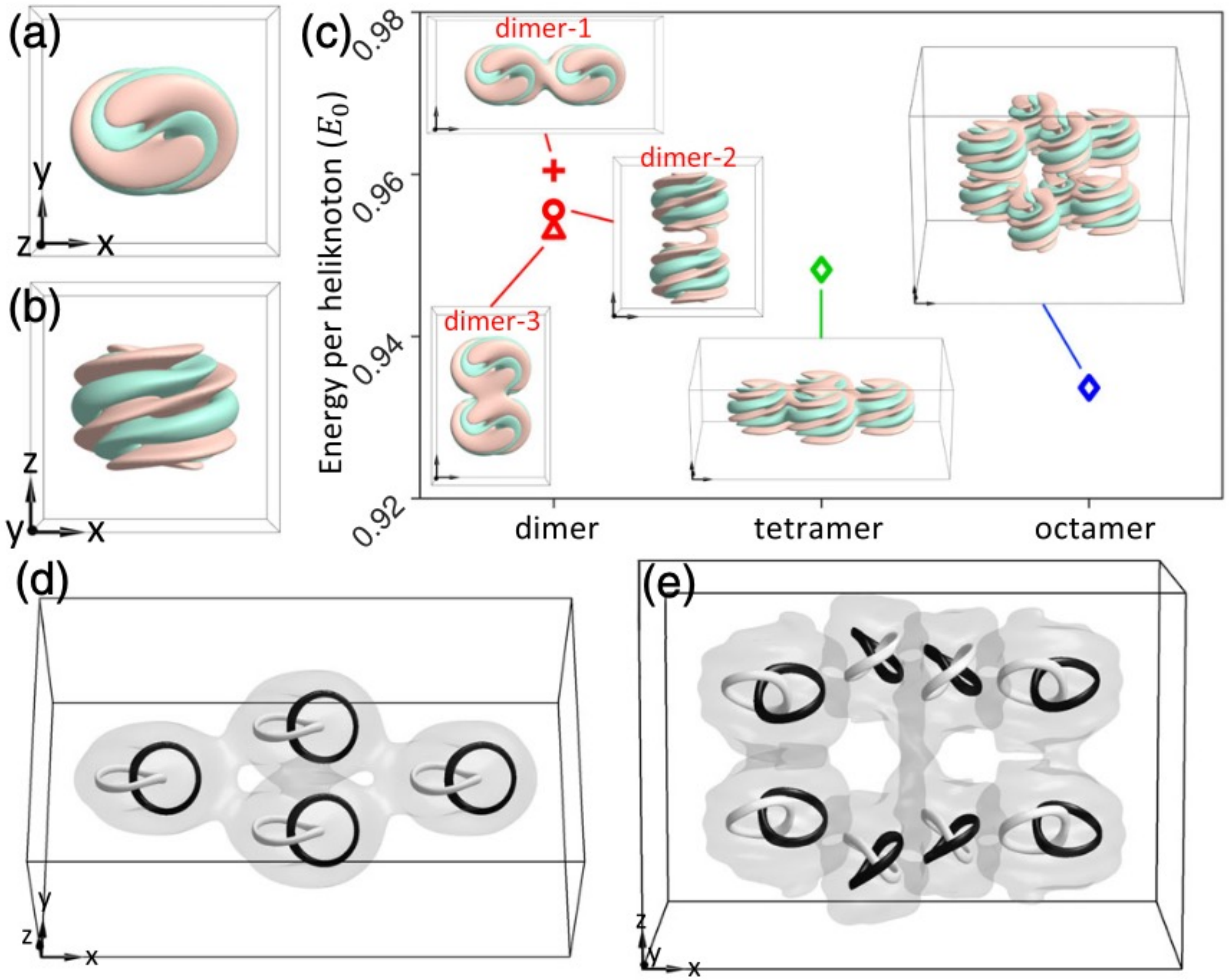
R. Voinescu, J-S. B. Tai, I. I. Smalyukh, Phys. Rev. Lett. 125, 057201 (2020)
J-S. B. Tai, I.I. Smalyukh, Science 365:1449–53 (2019).

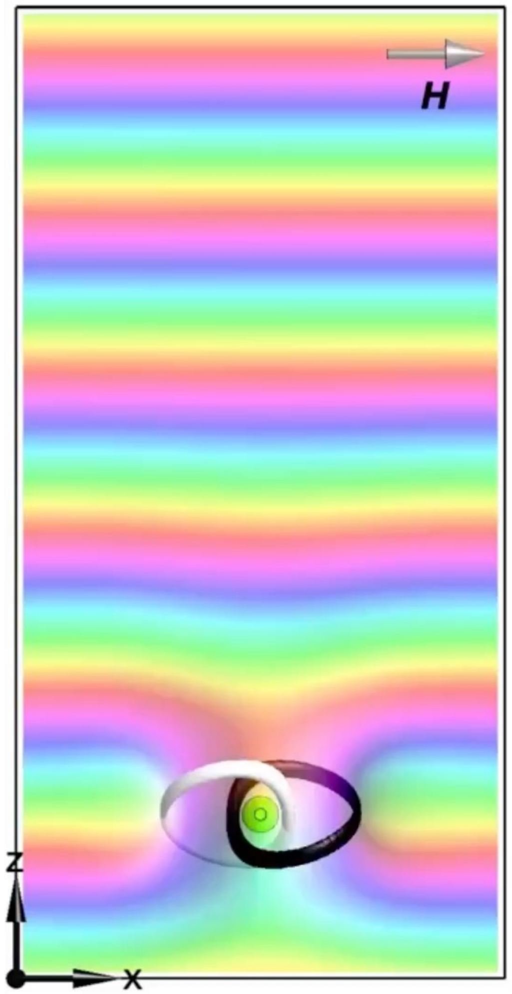
HELIKNOTON IN A CHIRAL MAGNET



V.M. Kuchkin, et al., Front. Phys. 11:1201018 (2023).

HELIKNOTON IN A CHIRAL MAGNET

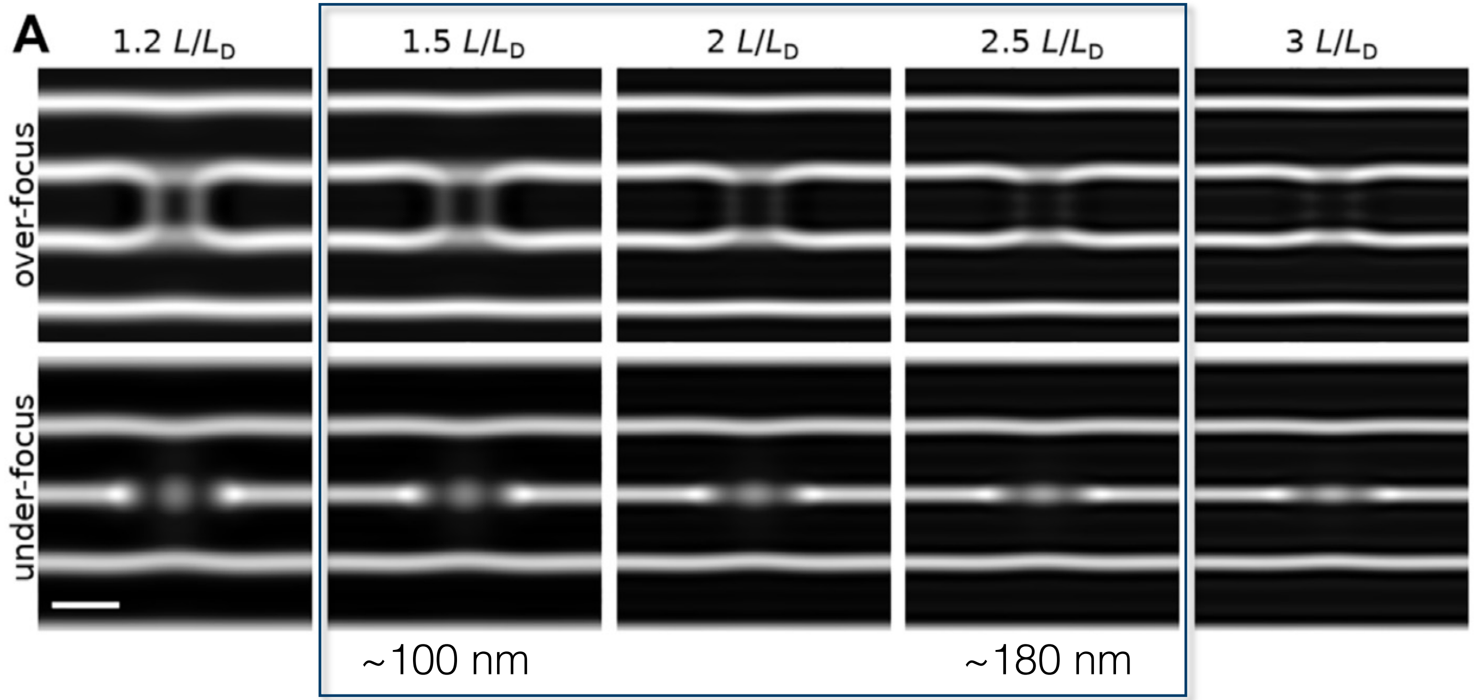


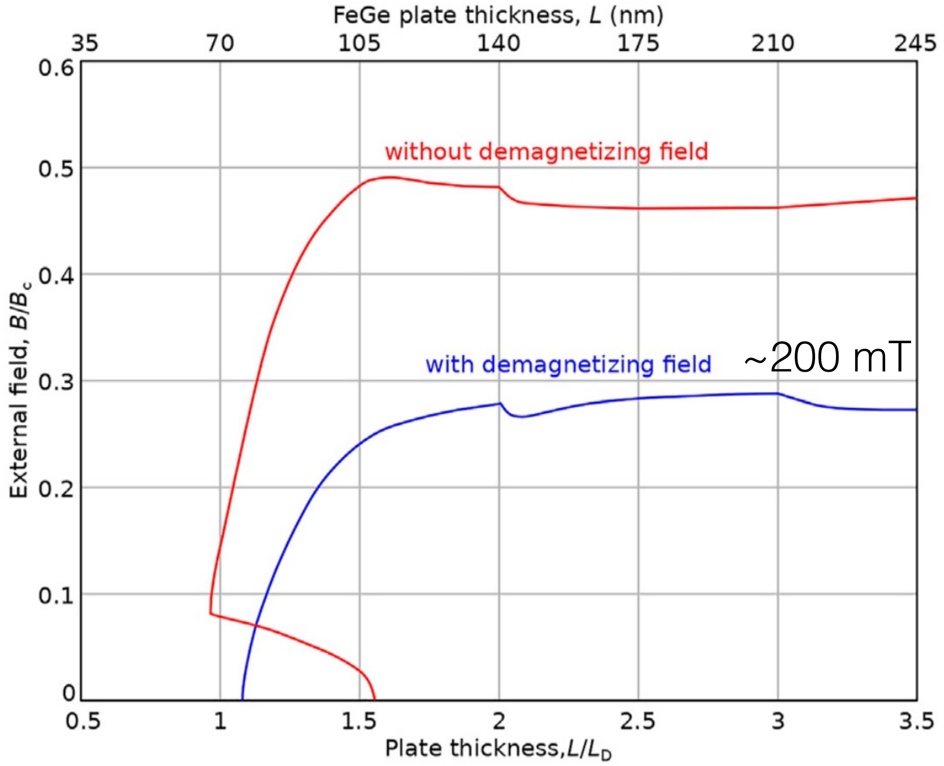


R. Voinescu, J-S. B. Tai, I. I. Smalyukh, Phys. Rev. Lett. 125, 057201 (2020)
J-S. B. Tai, I.I. Smalyukh, Science 365:1449–53 (2019).

HELIKNOTON IN A FILM OF FEGE

Lorentz TEM





V.M. Kuchkin, et al., Front. Phys. 11:1201018 (2023).

CONCLUSIONS

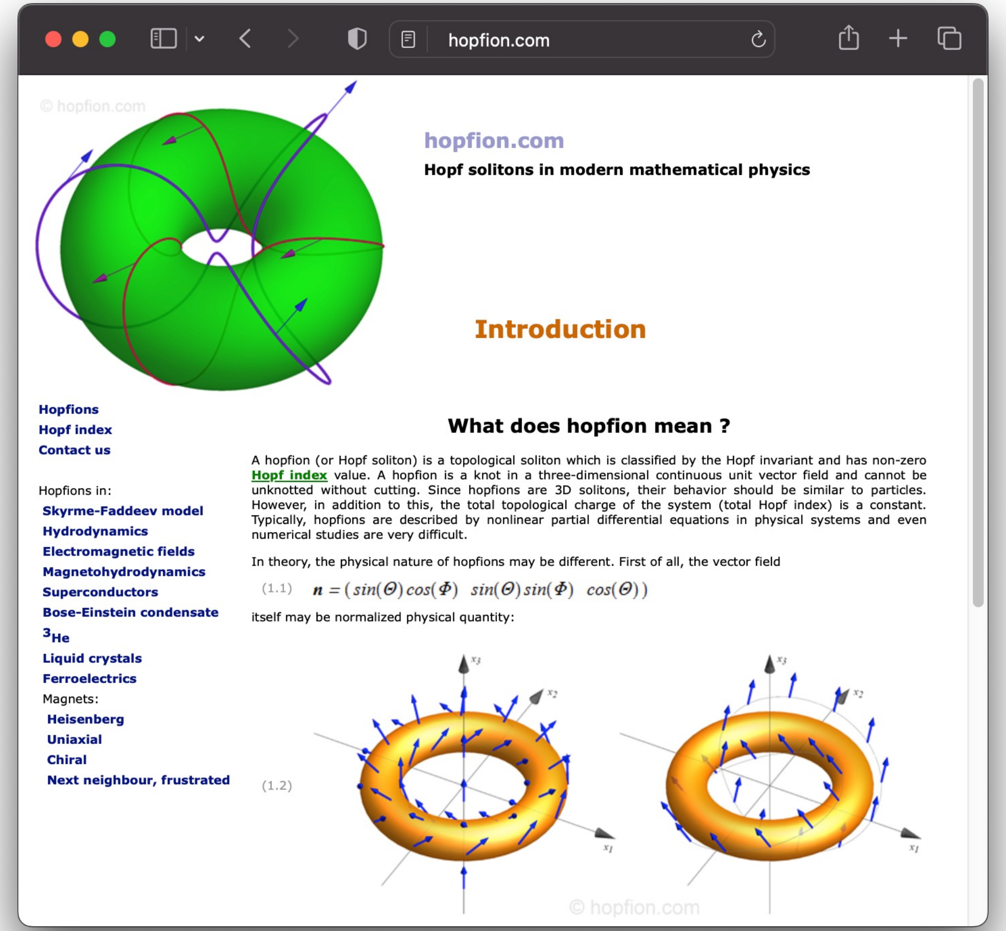
- There are only a few realistic models for magnetic hopfions:
  - frustrated magnets,
  - chiral magnets.

- Isotropic chiral magnets can host an ecxotic typy of hopfions coupled to skyrmion strings.

- The topological charge of hopfion rings is definned by a pair of integers: Q and H.

- Hopfion rings are expected to be very mobile, which might be useful in applications.

- Heliknoton is another promising type of 3D topological soliton in a cubic chiral magnet.



www.hopfion.com

Thank you for your attention!