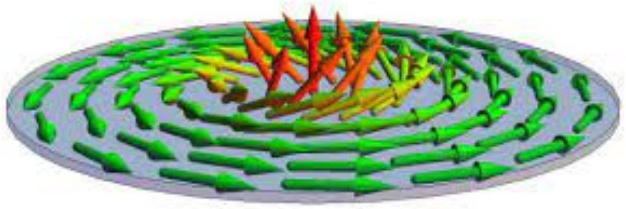
# Energy, geometry, and topology of collective magnetic dynamics



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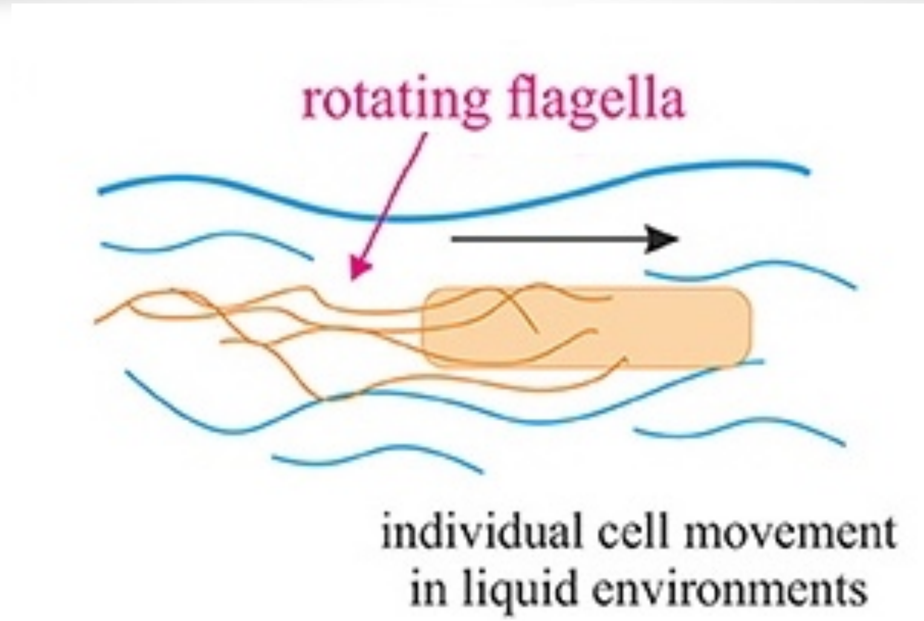
So Takei (*CUNY*), Se Kwon Kim (*KAIST*), Hector Ochoa (*Donostia*),
Ricardo Zarzuela (*Mainz*), Pramey Upadhyaya (*Purdue*), Benedetta Flebus (*Boston*), Ji Zou (*Basel*),
Suzy Zhang (*Max Planck/Dresden*), Chau Dao (*UCLA*), and Eric Kleinherbers (*UCLA*)

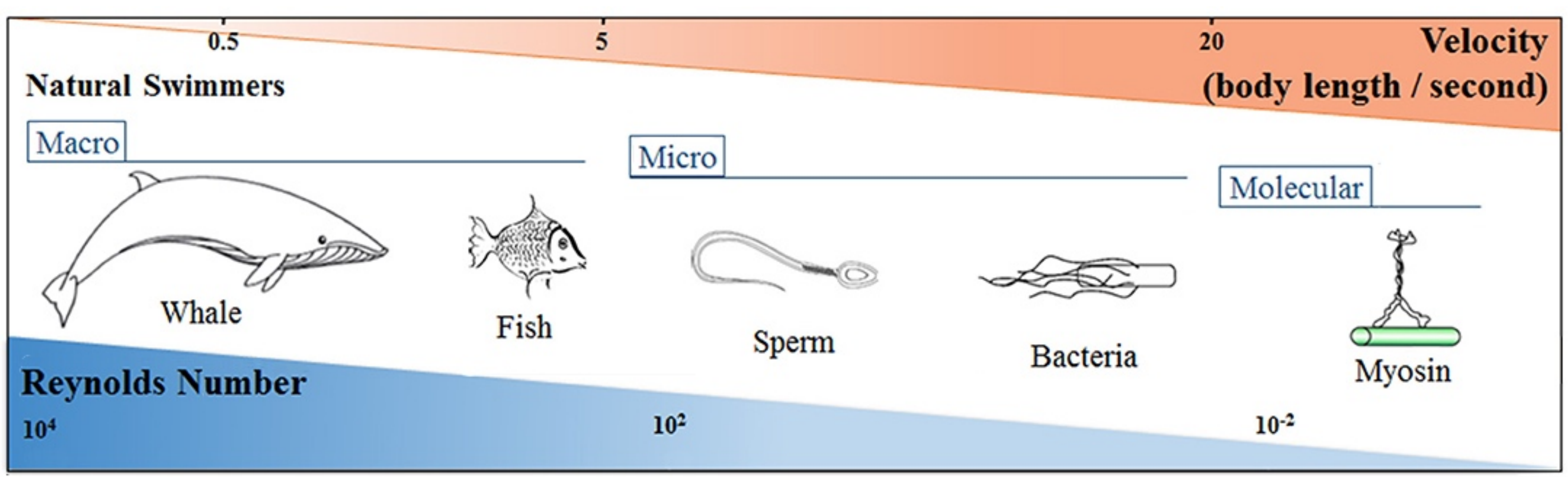
# Bacterial motility




- Bacterial ability to swim using metabolic energy

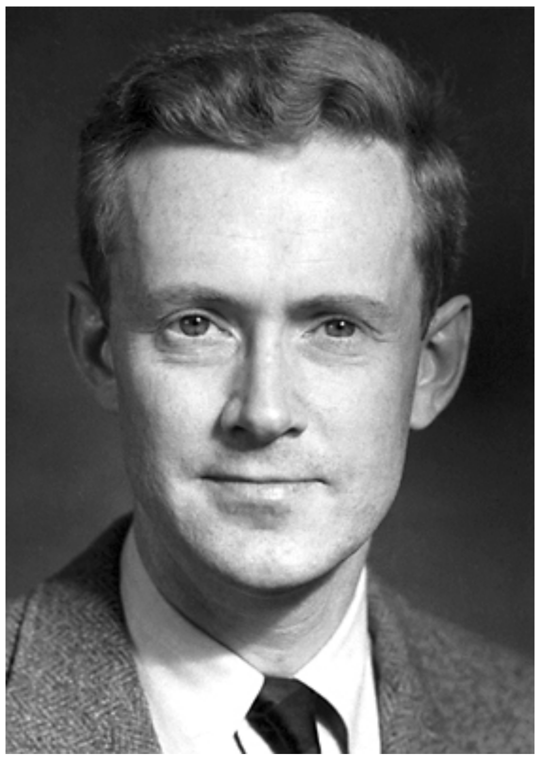
- Each single flagellum (helical appendage) has a rotary motor at the base that can turn clock- or anticlockwise

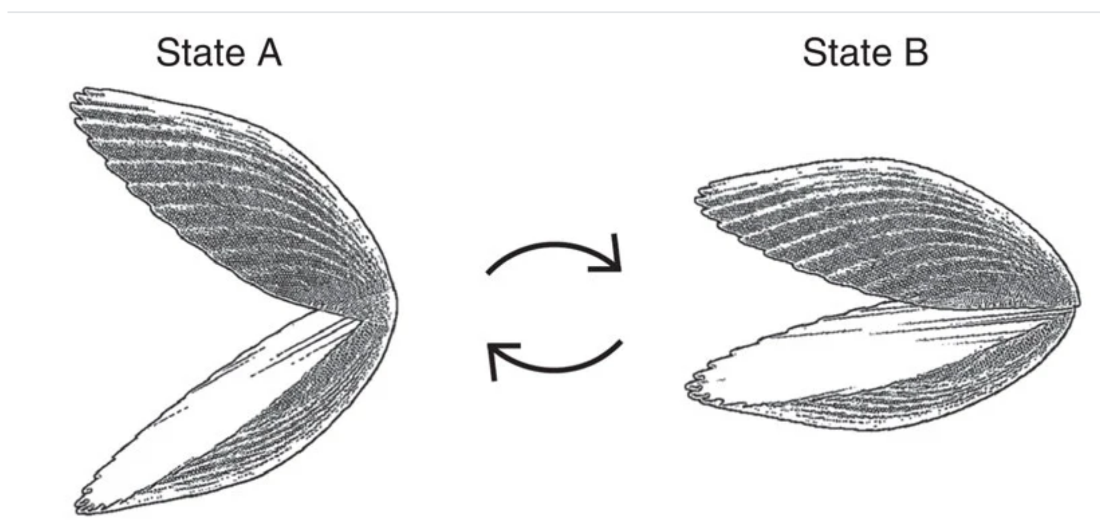
- Small Reynolds number: Viscosity-dominated hydrodynamics; thus, continuous transduction of chemical into mechanical energy



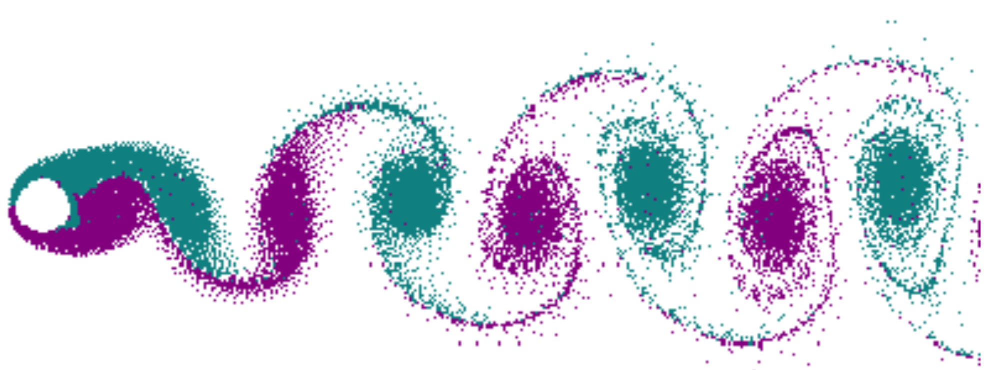
# *Macroscopic mobility strategies*

- Inertia dominated: Typically imparting momentum to the fluid by discrete events, such as vortex shedding, with inertial coasting in between



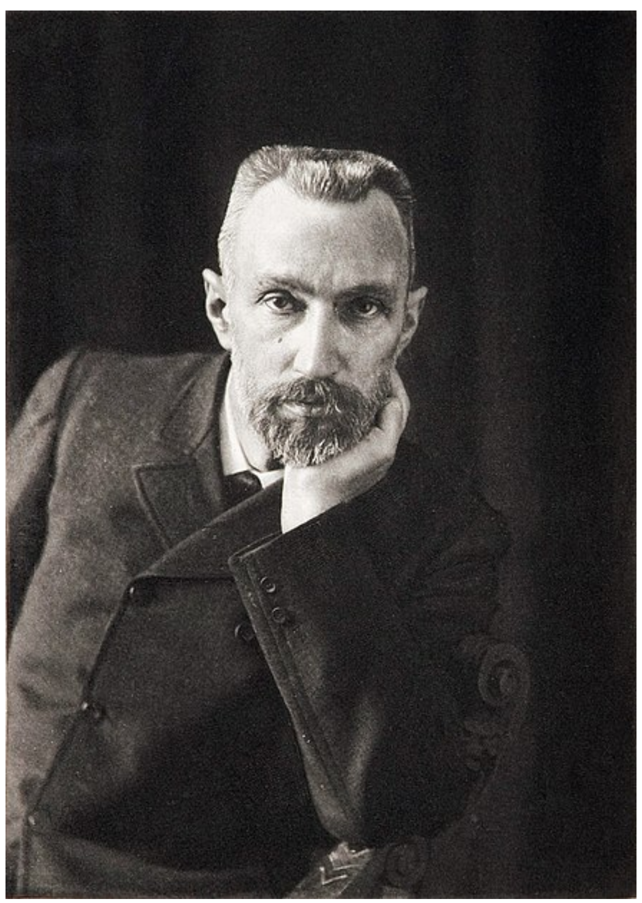



E. M. Purcell: "Fast or slow, it exactly retraces its trajectory and it's back where it started." [without inertia]

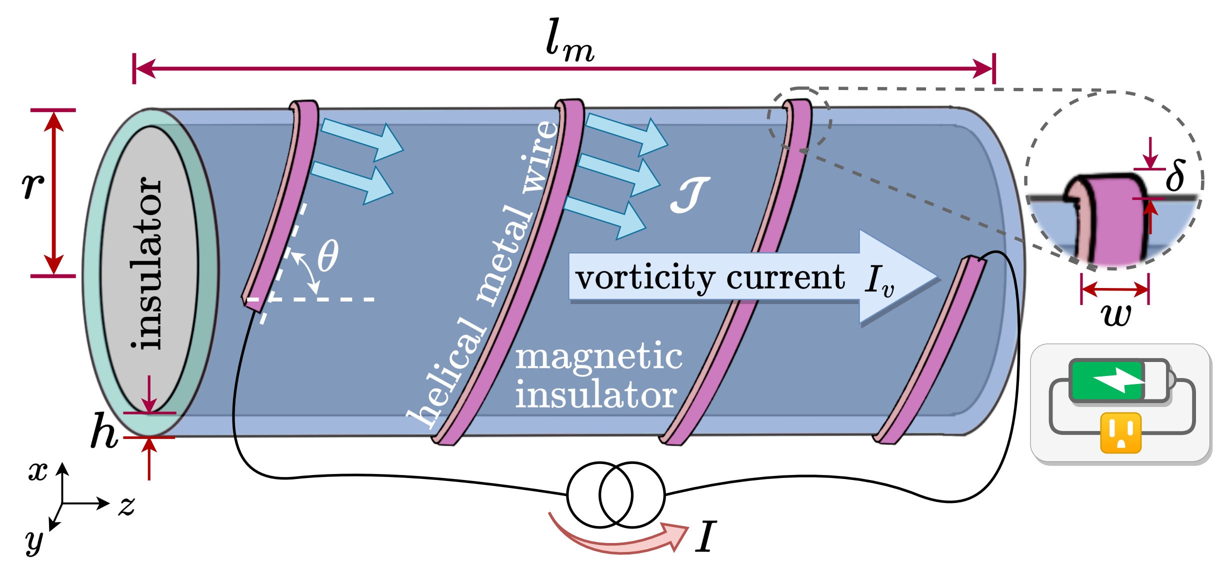


# From evolution to "intelligent" design

- Conversion of featureless electrochemical free energy into dynamic macroscopic configurations
- Geometry is exploited to enable desired process via the reduction of structural symmetries

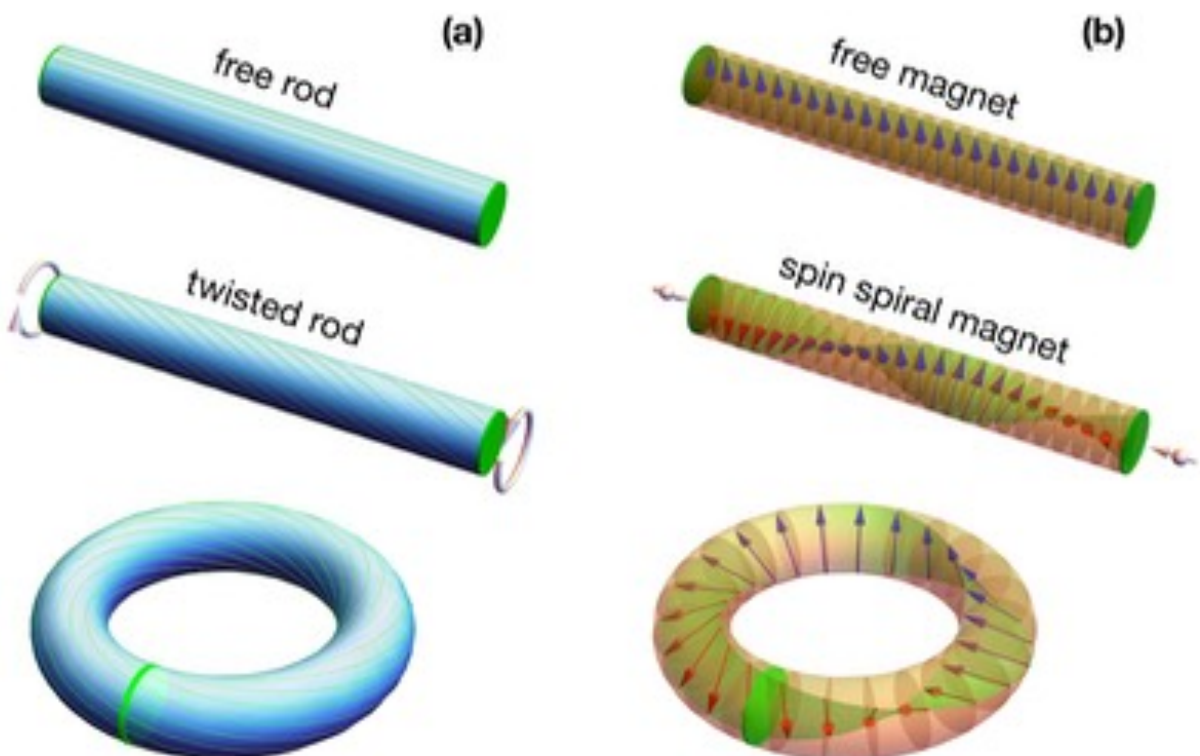


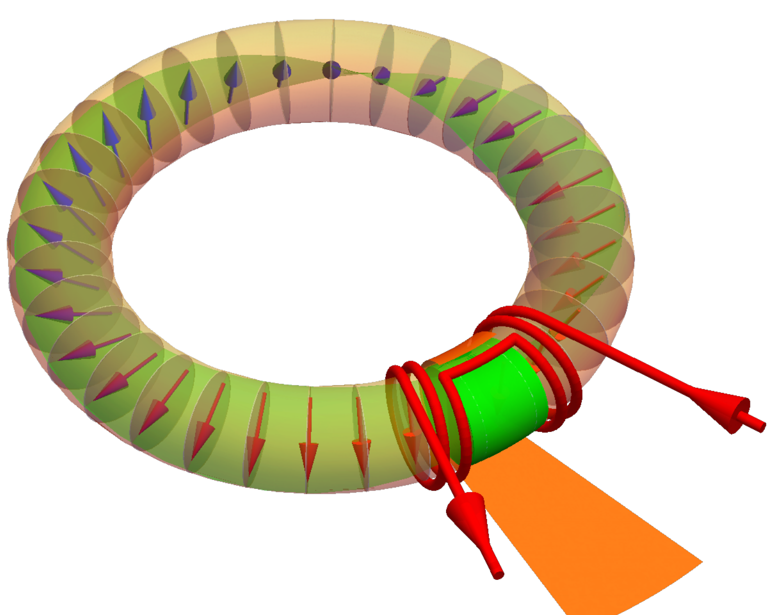
Pierre Curie:
"Asymmetry creates the phenomenon"




- Enable magnetic transport phenomena via geometry
- Utilize topology of collective spin textures to transduce and store free energy
- Interface with (thermo)electric input/output

# The energy-storage concept

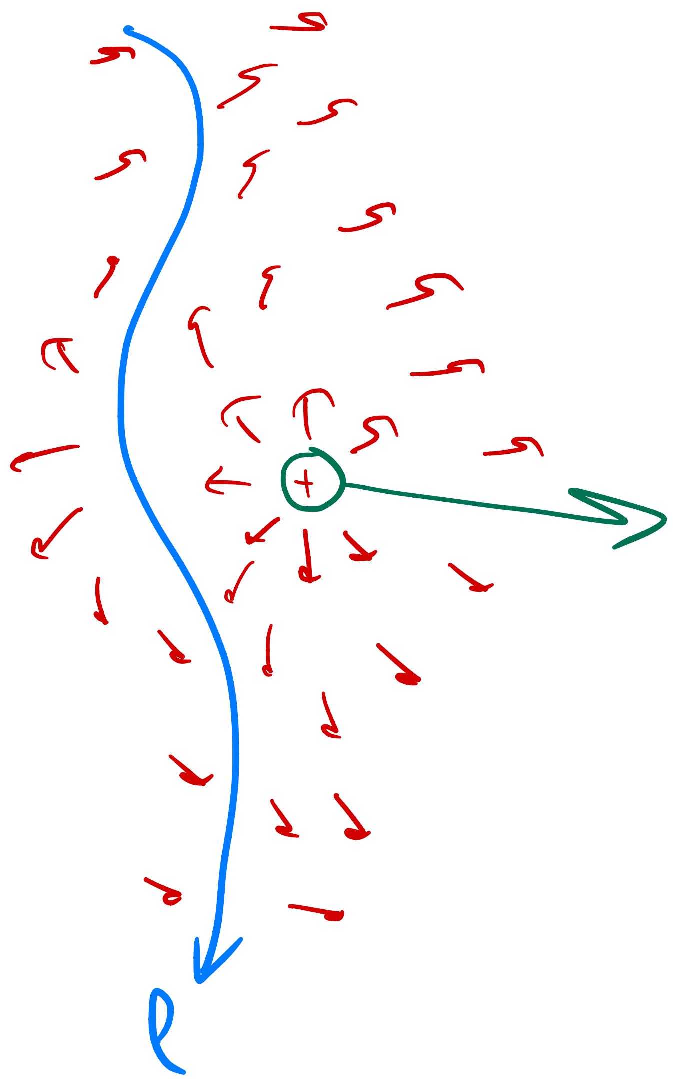


- How to inject/extract such winding in practice?
- Controlling symmetries: Heterostructure design vs curved geometry
- Thermodynamic efficiency of the cross-talk between winding dynamics and electricity?

YT and Xiao, *PRL* (2018)

# An appealing approach



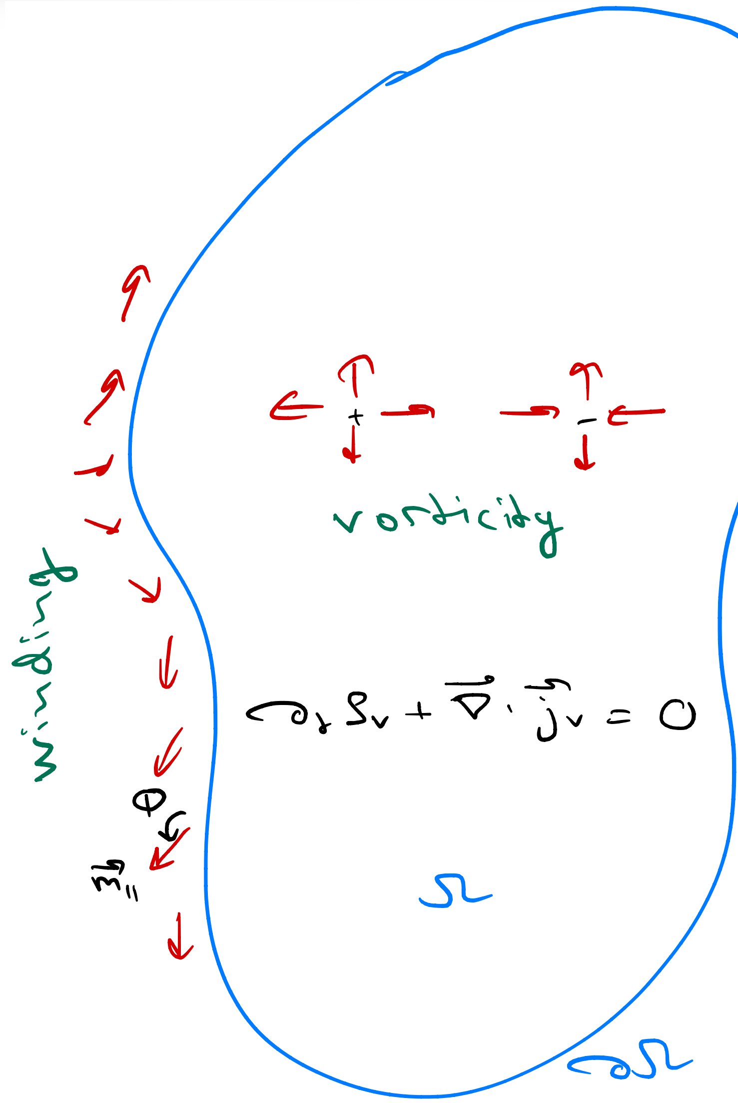

vorticity flow transverse to ℓ

⇕

winding change along ℓ

If we establish practical means to push vorticity within a 2D system, winding would build up in the transverse direction (cf. phase slips in 2D superfluids)

# Vorticity-winding transmutation




Net winding:

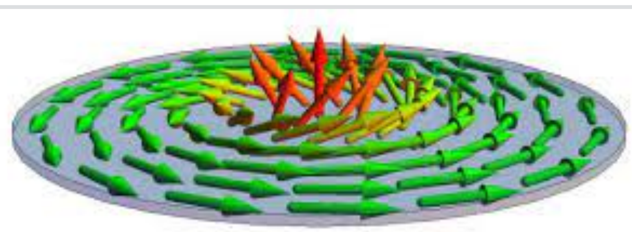
$$Q = \oint_{\partial\Omega} \frac{d\varphi}{2\pi} \quad \vec{m}_{\parallel} \in \mathbb{Z}$$

Stokes $\rightarrow \int_{\Omega} d^2r \; \rho_v$

where $\rho_v \equiv \dfrac{\hat{z}\cdot\partial_x\vec{m}\times\partial_y\vec{m}}{4\pi}$ vorticity density

$$j_v^{(i)} = \frac{\epsilon^{ij}\;\hat{z}\cdot\partial_j\vec{m}\times\partial_t\vec{m}}{4\pi}$$ vorticity flux

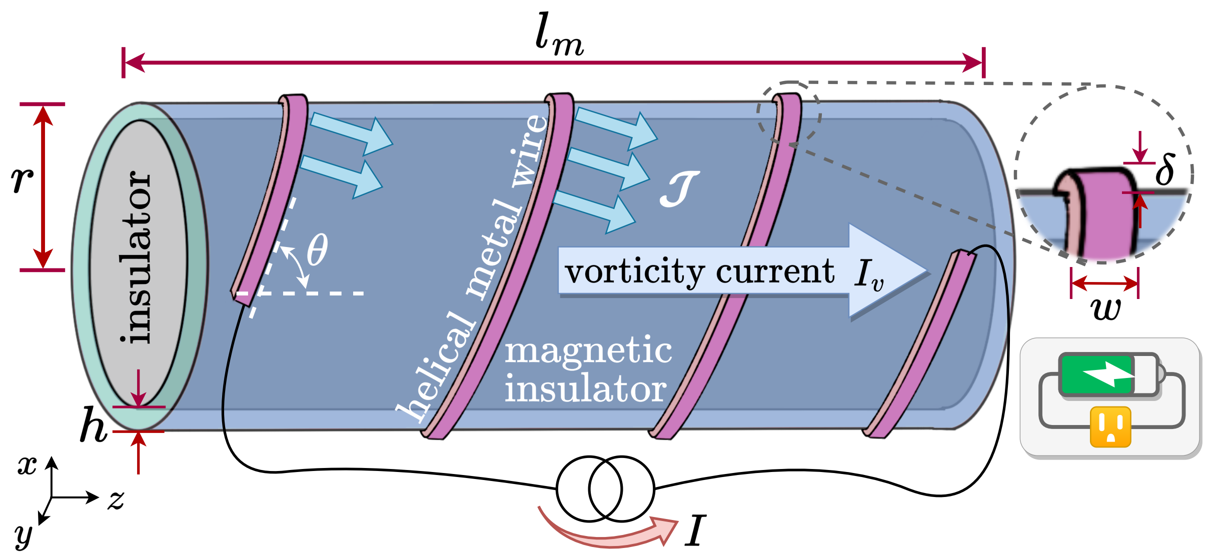
net integrated vorticity is naturally conserved



$$Q = \int d^2r\,\rho_v \rightarrow 1$$

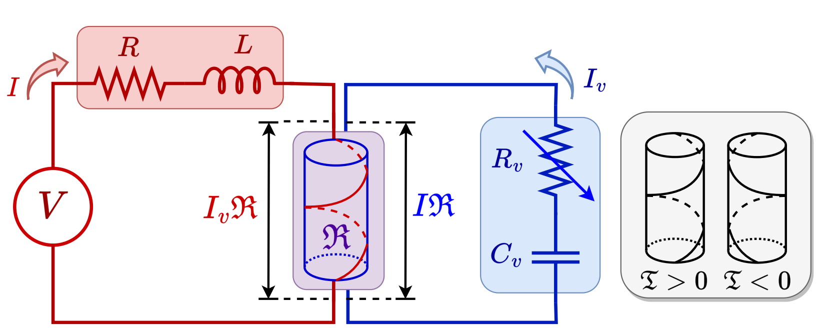
topological charge $Q$ labels winding homotopy classes in the strongly-ordered *XY* limit

# Geometry-enabled topological energy storage




spin-transfer input power: $$\dot{W} = \int dl\, \boldsymbol{\tau} \cdot \mathbf{m} \times \partial_t \mathbf{m} = \mathfrak{R}\, I I_v$$

In this geometry, electric current along the spiral generically drives vorticity flow along the cylinder axis (thus building up uniform transverse winding)



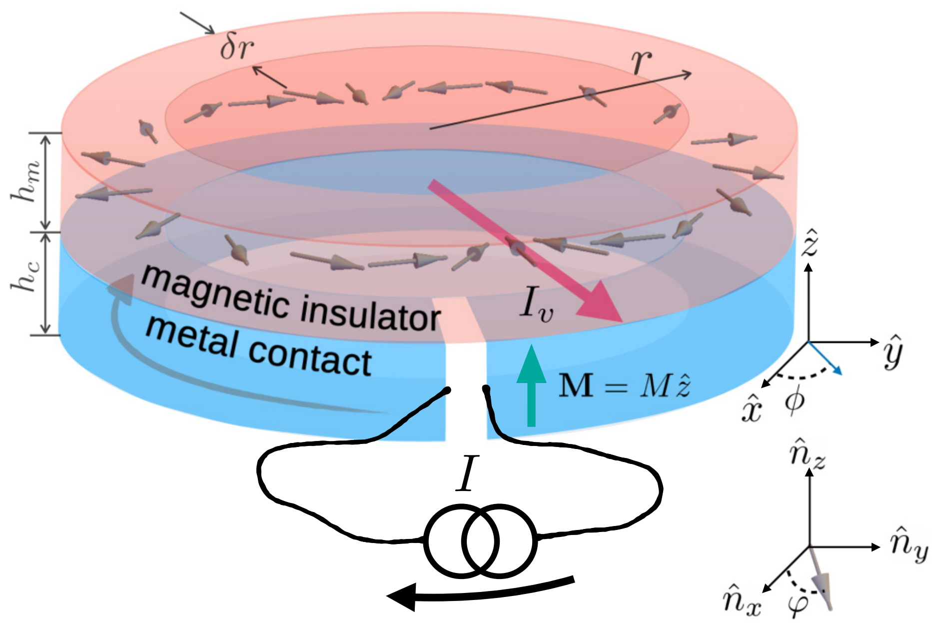

$$\xi = \mathfrak{R}^2 / RR_v$$

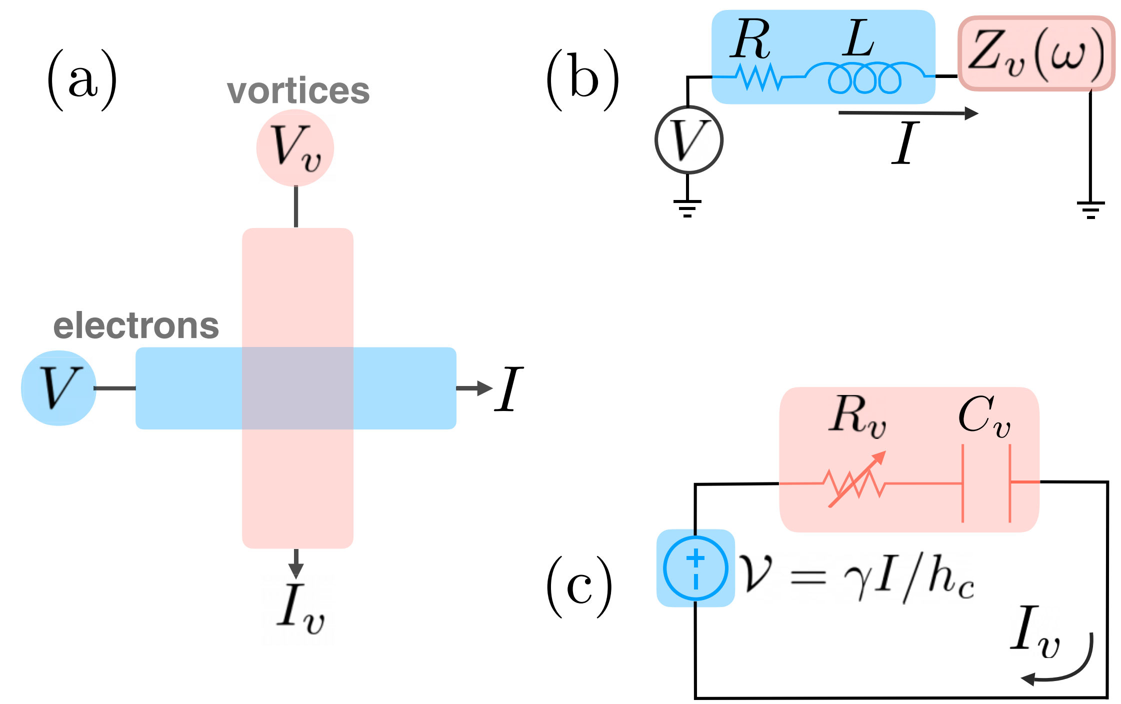
electron-vortex "cooperativity"

the effective dimensionless parameter, which is thermodynamically bounded to [0,1], is formally analogous to the thermoelectric figure of merit called *ZT*

Dao, Zou, Kleinherbers, and YT, *arXiv* (2023)

# *Onsager description of the magnetic energy storage*




- Vortex and electron fluxes cross-couple via Magnus-like friction:

$$\begin{pmatrix} V \\ V_v \end{pmatrix} = \begin{pmatrix} R & \mathfrak{R} \\ -\mathfrak{R} & R_v \end{pmatrix} \begin{pmatrix} I \\ I_v \end{pmatrix}$$

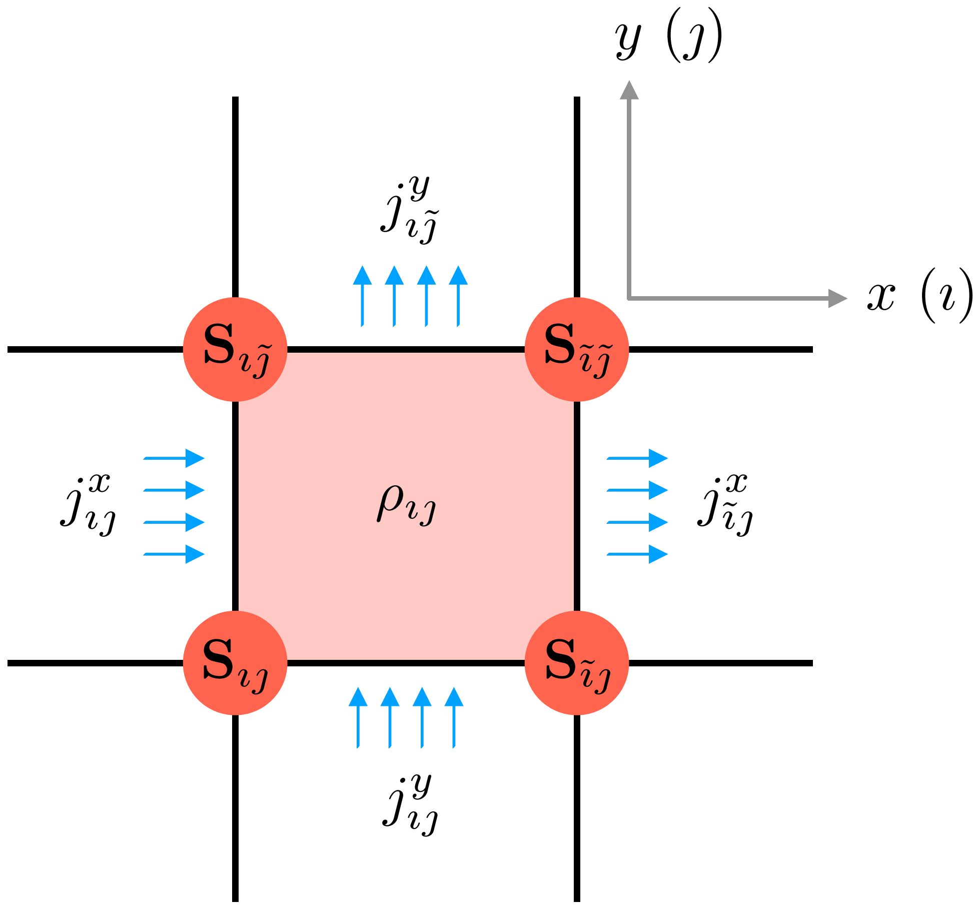
- The magnetic annulus acts as a winding capacitor:

$$E_v = \frac{Q_w^2}{2C_v} \qquad\qquad I_v = \frac{dQ_w}{dt}$$

- It is tempting to think of devices where a tunable vortex conductivity (e.g., associated with their binding and/or defect pinning) is utilized as a switch

Jones, Zou, Zhang, and YT, *PRB* (2020)

# *The vorticity is robust against quantum fluctuations*




$$\partial_t \rho_{\imath\jmath} + \frac{j^x_{\tilde\imath\jmath} - j^x_{\imath\jmath} + j^y_{\imath\tilde\jmath} - j^y_{\imath\jmath}}{a} = 0$$

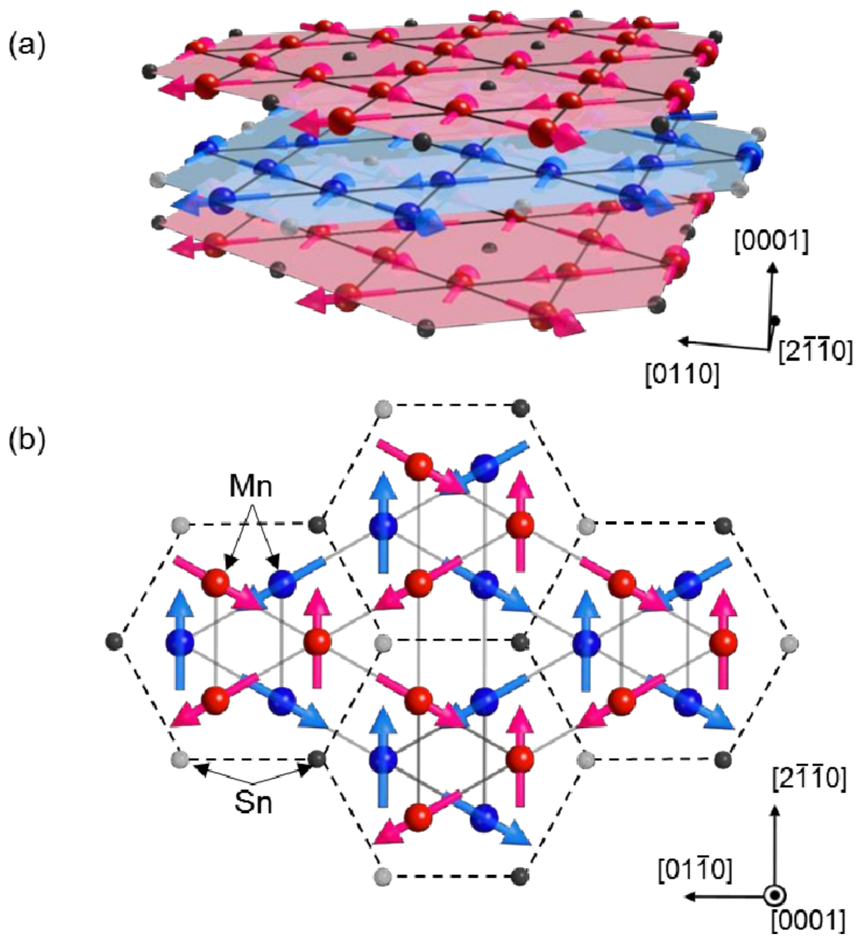
- Vorticity per plaquette is given by the (z component of the) vector chirality: quantum version of the winding around the plaquette

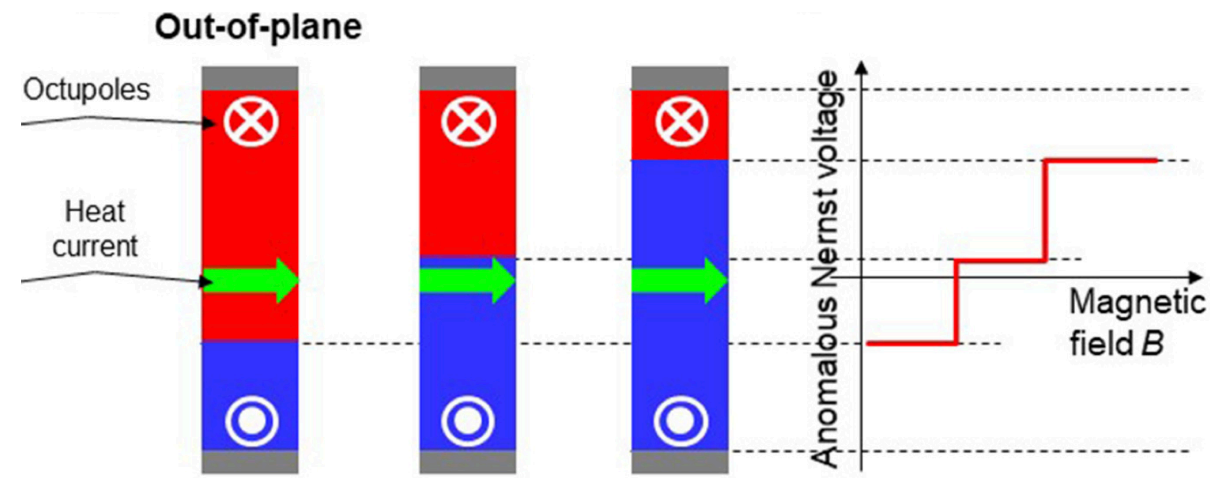
$$\rho_{\imath\jmath} = \frac{\mathbf{z}\cdot\mathbf{c}_{\imath\jmath}}{2\pi a^2} \qquad \text{where} \qquad \mathbf{c}_{\imath\jmath} \equiv \frac{1}{S^2}\sum_l \mathbf{S}_l \times \mathbf{S}_{\tilde l}$$

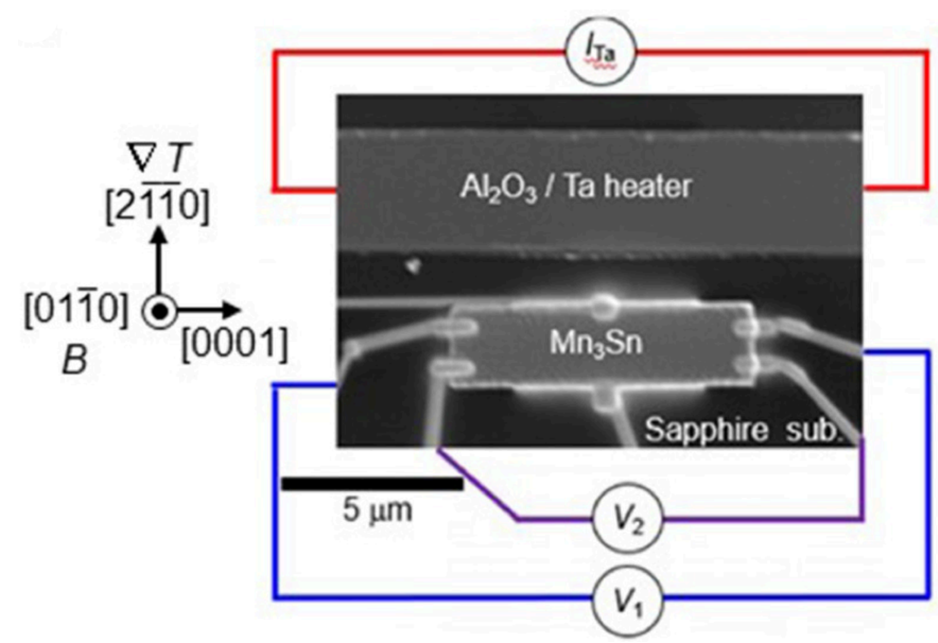
- For smooth classical textures, this reproduces the previous continuum version

YT, Zou, Kim, and Takei, *PRB* (2020)

# A new promising materials platform: $Mn_3Sn(Ge)$




Antiferromagnetic Weyl semimetal with planar (octupolar) spin texture free to rotate within the easy plane

Nernst measurement of a domain-wall motion subjected to a magnetic field

Liu and Balents, *PRL* (2017)

Otani and Higo, *APL* (2021)

# Quantum annealing of the Kosterlitz-Thouless transition

**Observation of topological phenomena in a programmable lattice of 1,800 qubits**

Andrew D. King[1]*, Juan Carrasquilla[2], Jack Raymond[1], Isil Ozfidan[1], Evgeny Andriyash[1], Andrew Berkley[1], Mauricio Reis[1], Trevor Lanting[1], Richard Harris[1], Fabio Altomare[1], Kelly Boothby[1], Paul I. Bunyk[1], Colin Enderud[1], Alexandre Fréchette[1], Emile Hoskinson[1], Nicolas Ladizinsky[1], Travis Oh[1], Gabriel Poulin-Lamarre[1], Christopher Rich[1], Yuki Sato[1], Anatoly Yu. Smirnov[1], Loren J. Swenson[1], Mark H. Volkmann[1], Jed Whittaker[1], Jason Yao[1], Eric Ladizinsky[1], Mark W. Johnson[1], Jeremy Hilton[1] & Mohammad H. Amin[1,3]

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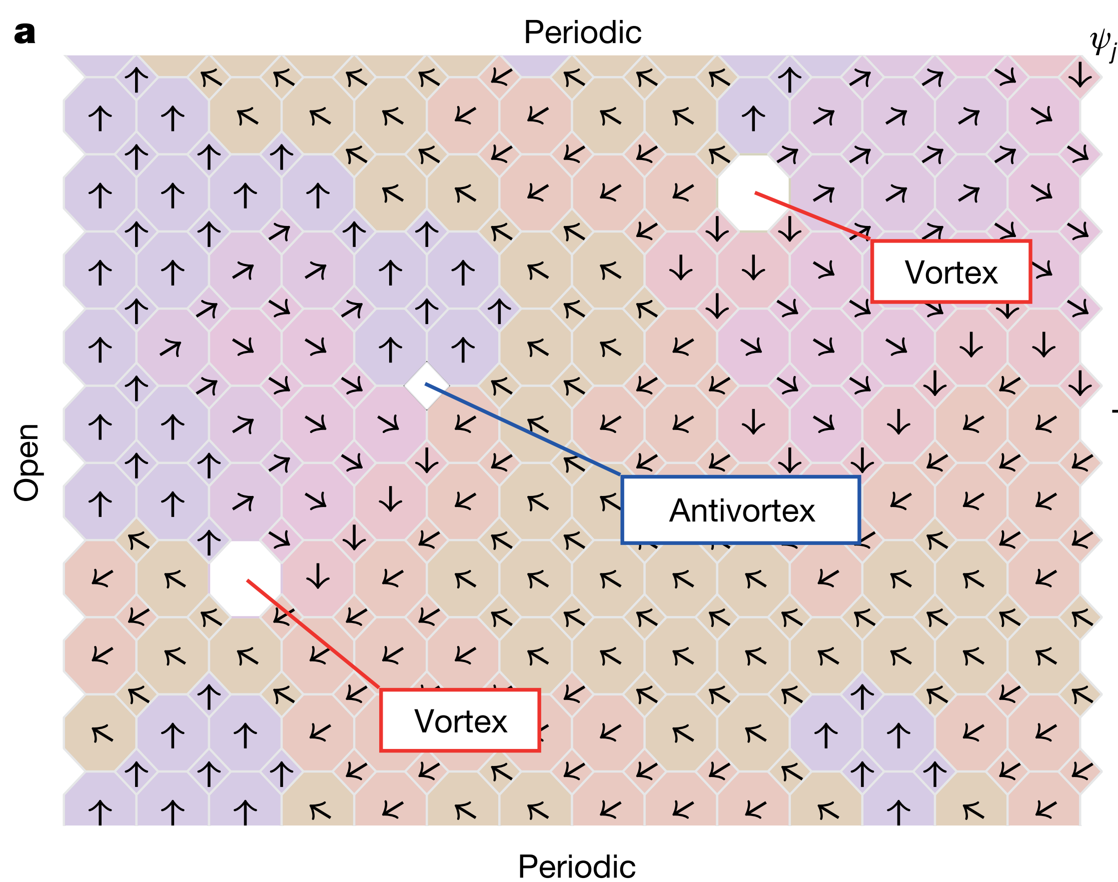
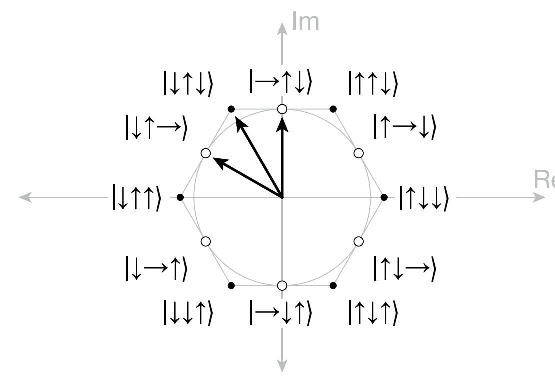
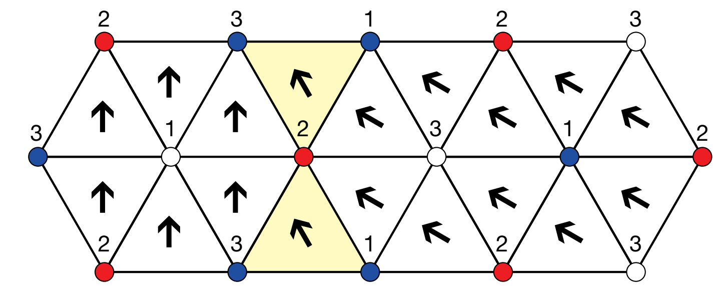
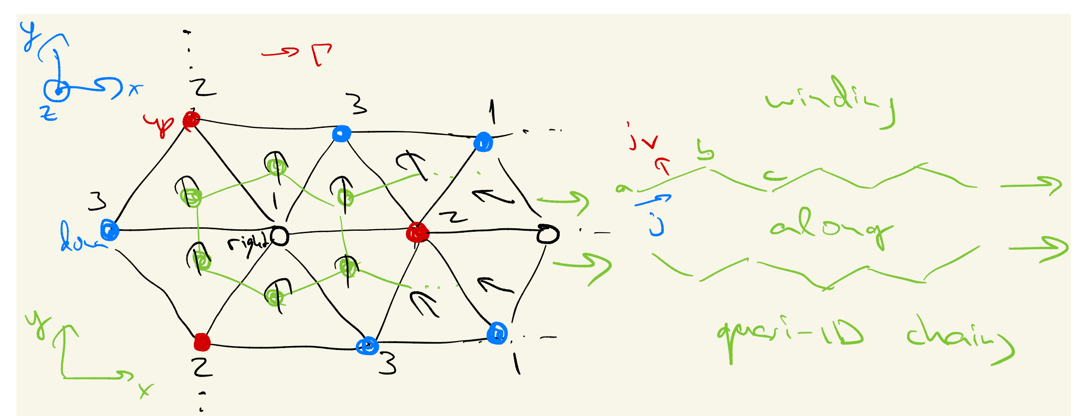


$$\psi = \hat{\sigma}_1^z + \hat{\sigma}_2^z e^{2\pi i/3} + \hat{\sigma}_3^z e^{-2\pi i/3}$$




$T < T_{KT}$

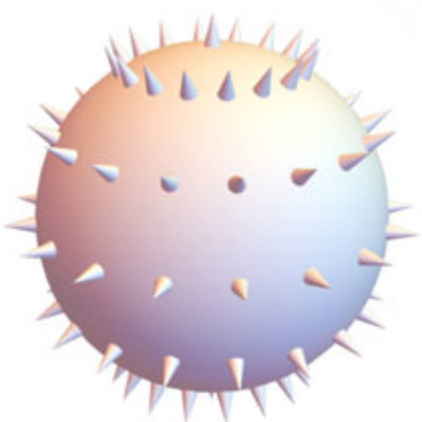
vortices bind with antivortices:
vorticity insulator

$T > T_{KT}$

(anti)vortices unbind and proliferate:
vorticity metal (two-component plasma)

# 3D version of vortices: Hedgehogs

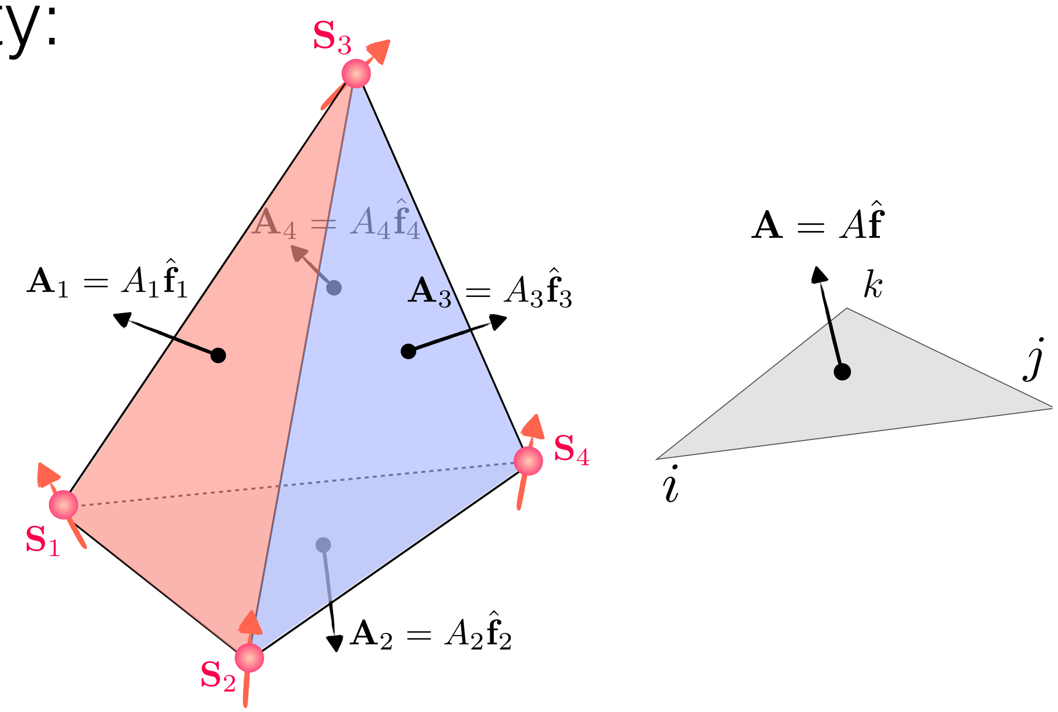
- The continuum hedgehog hydrodynamics



$$j^\mu = \epsilon^{\mu\alpha\beta\gamma}\partial_\alpha \mathbf{n}\cdot(\partial_\beta \mathbf{n}\times\partial_\gamma \mathbf{n})/8\pi\,, \quad \partial_\mu j^\mu = 0$$

$$\mathbf{n}(\mathbf{r},t): \quad \mathbf{n},\mathbf{r}\in\mathbb{R}^3$$

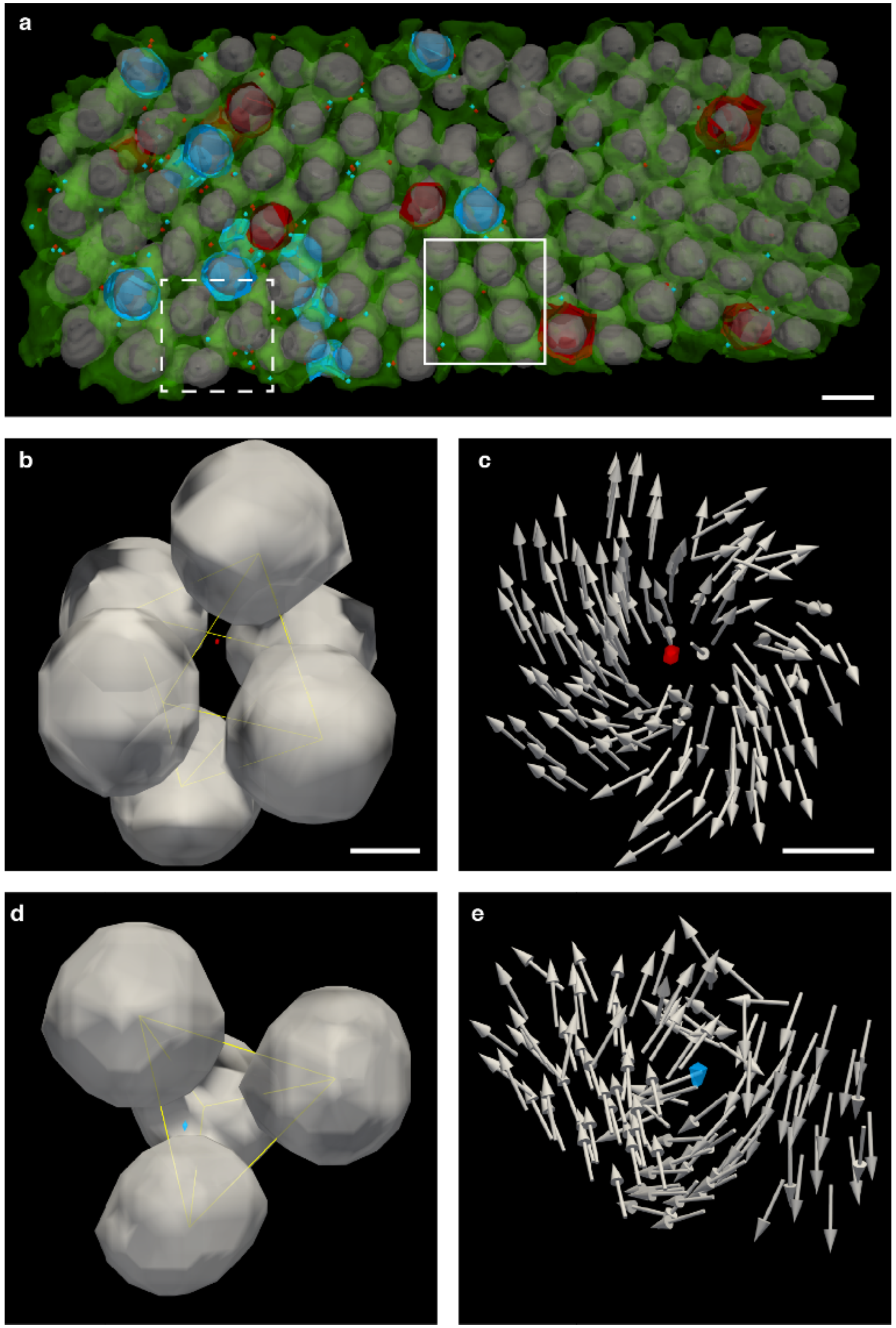
has a natural quantum underpinning on an arbitrary lattice, in terms of scalar spin chirality:

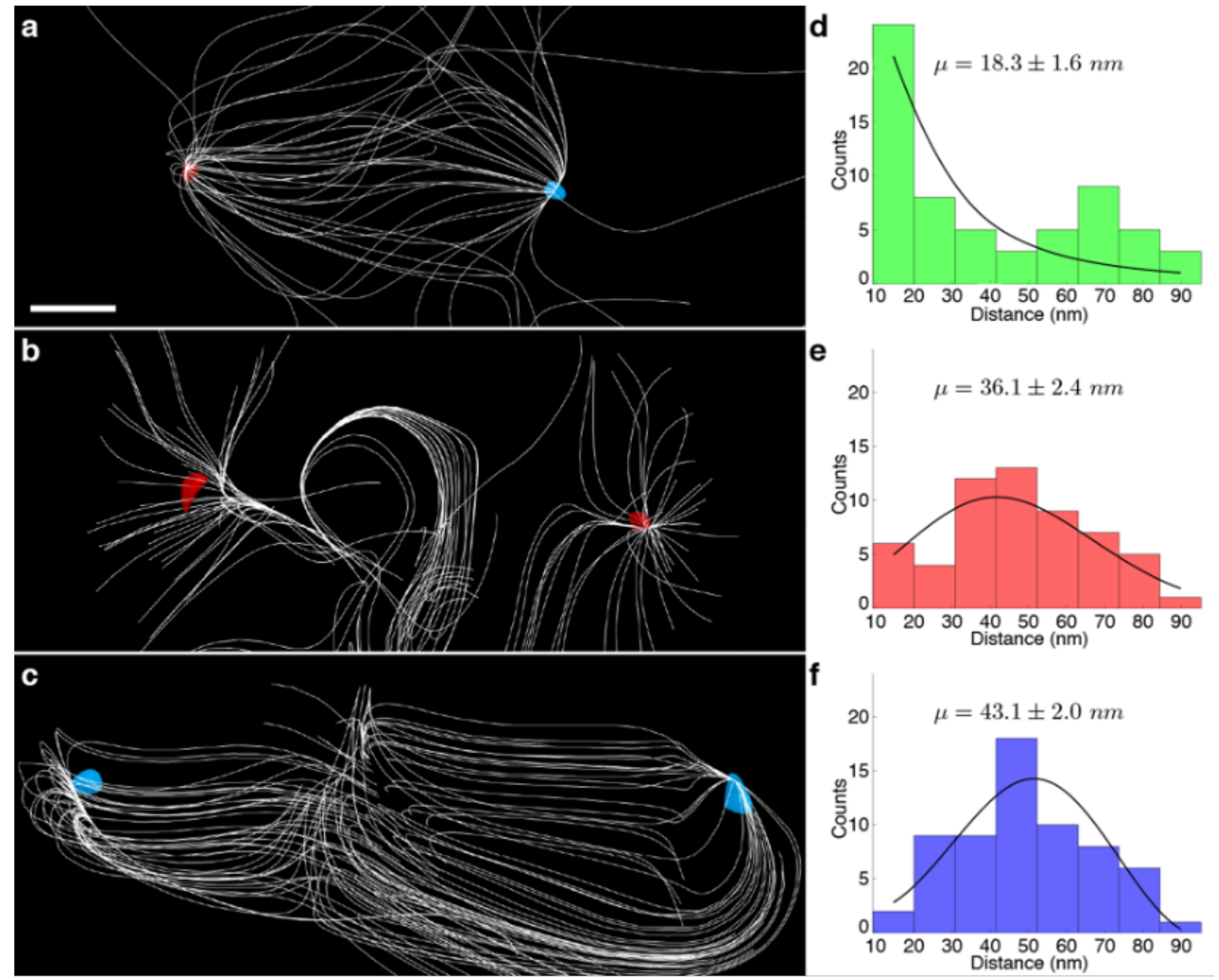



$$\rho = \frac{c_{123}+c_{142}+c_{243}+c_{134}}{8\pi V} \quad \text{where} \quad c_{ijk} = \frac{\mathbf{S}_i\cdot\mathbf{S}_j\times\mathbf{S}_k}{S^3}$$

Zou, Zhang, and YT, *PRL* (2020)

# *Engineering and imaging hedgehog metamaterials*

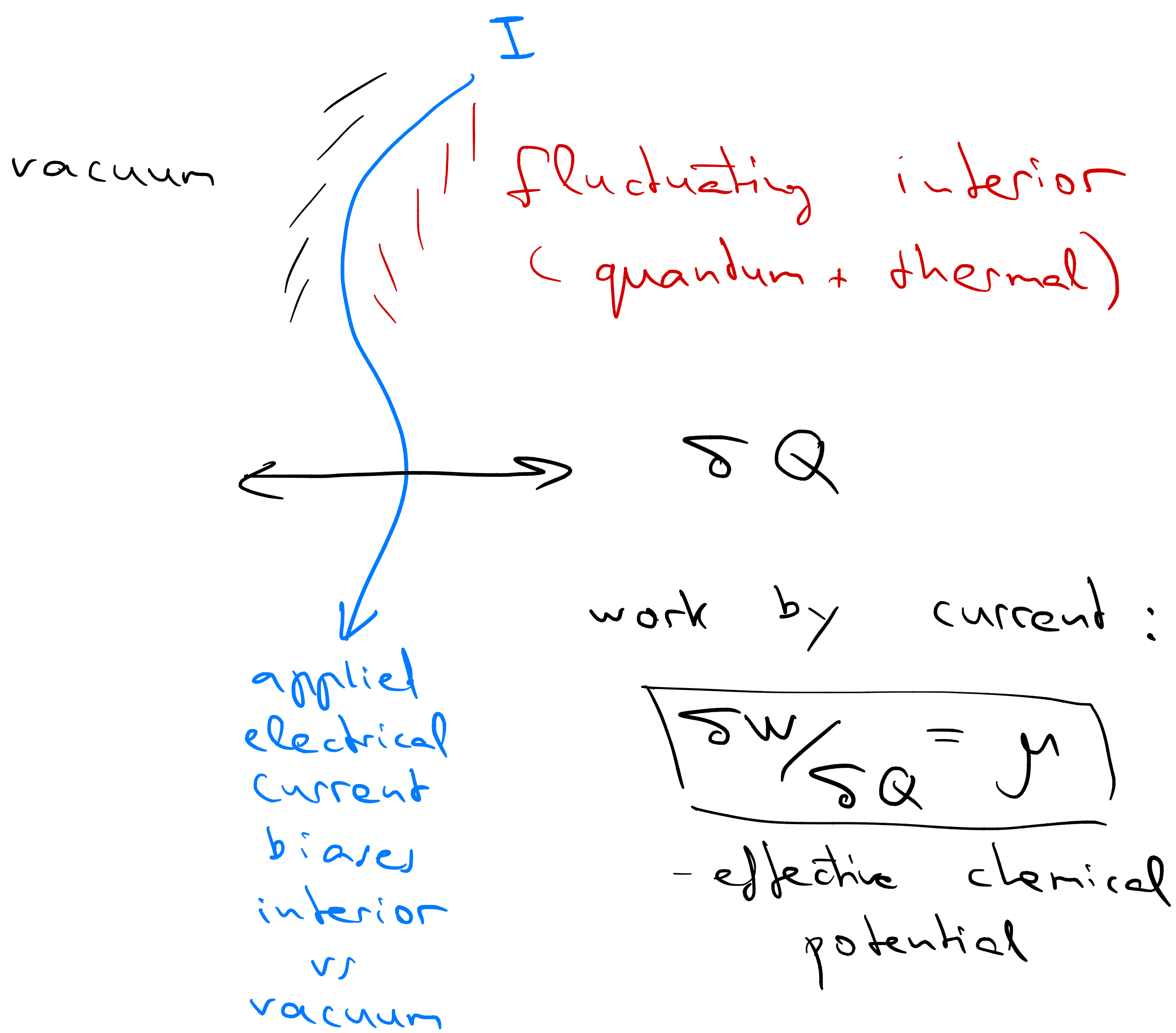



*Creation and observation of topological magnetic monopoles and their interactions in a ferromagnetic metalattice*

Soft X-ray imaging by Miao's group at UCLA [Rana *et al.*, *Nature Nano.* (2023)]

# Electrical injection (detection) of topological charge $\delta Q$




- A nonequilibrium flow of Q must be consistent with crystalline and structural symmetries, Onsager reciprocity, and (1st/2nd) laws of thermodynamics
- In principle, we generally expect such injection to happen for a generic drive (electrical/ microwave/optical etc.), as long as it is not disallowed by symmetries

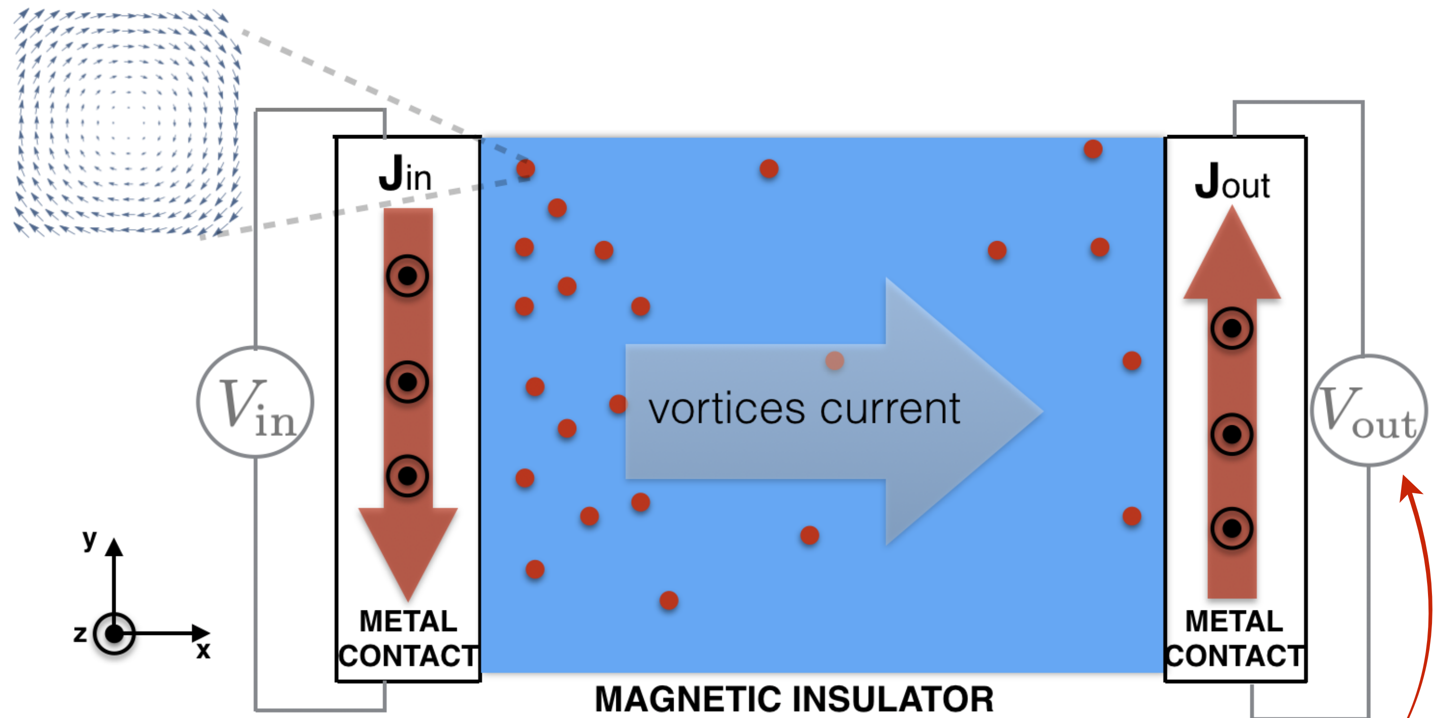
Pierre Curie: Asymmetry creates the phenomenon

# Electrical injection (detection) of vorticity

## Energetics (thermodynamics) and symmetries:

spin torque by the input current at the left interface:

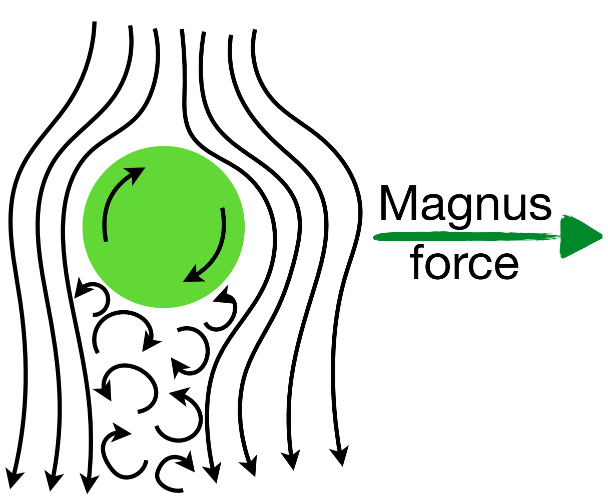
$$\boldsymbol{\tau} = \eta \mathbf{M} \cdot \mathbf{m} (\mathbf{J} \cdot \boldsymbol{\nabla}) \mathbf{m}$$




$\mathbf{M} \propto \mathbf{z}$ - magnetization of the metal contact

$\mathbf{m}(x, y, t)$ - magnetic texture of the insulator

Onsager-reciprocal motive force




$$\dot{W} = \int dy \, \boldsymbol{\tau} \cdot (\mathbf{m} \times \partial_t \mathbf{m}) \to \eta \, \mathbf{z} \cdot \mathbf{J} \times \mathbf{j}$$

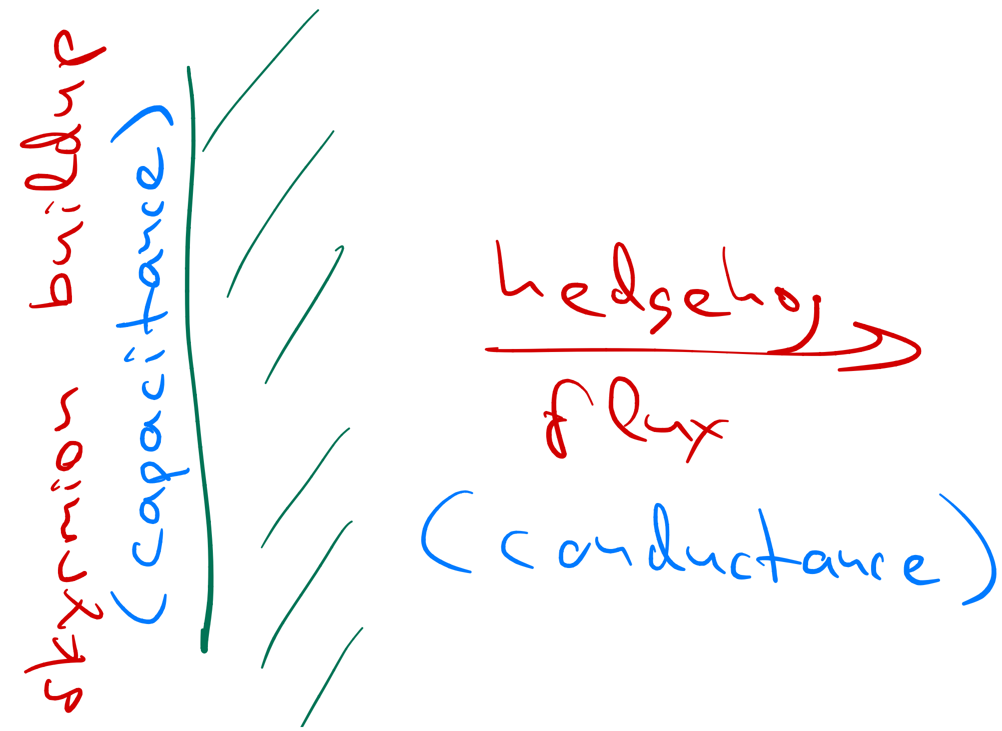
charge current

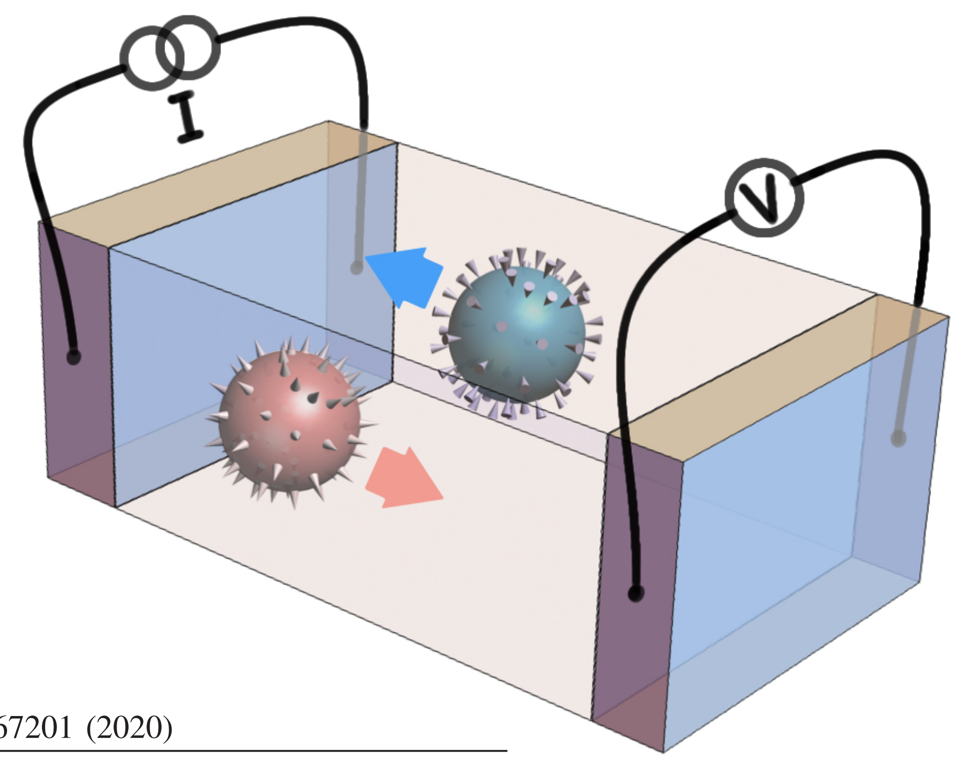
vortex flux

Zou, Kim, and YT, *PRB* (2019)

# Extending to 3D bulk: Hedgehog flow

The 3D hedgehog flow with skyrmionic bulk/boundary correspondence is closely analogous to the 2D vorticity flow with winding bulk/boundary correspondence




PHYSICAL REVIEW LETTERS **125,** 267201 (2020)

Editors' Suggestion

**Topological Transport of Deconfined Hedgehogs in Magnets**

Ji Zou, Shu Zhang,* and Yaroslav Tserkovnyak
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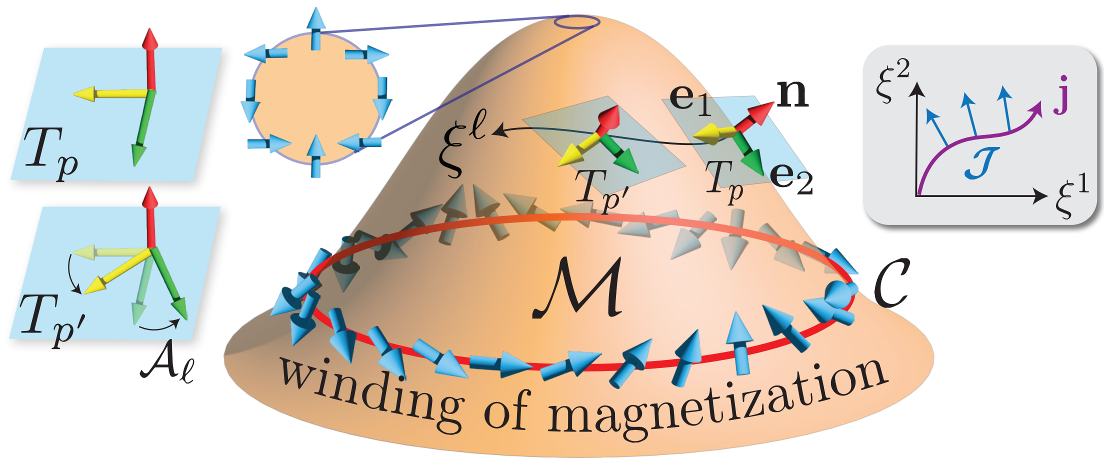
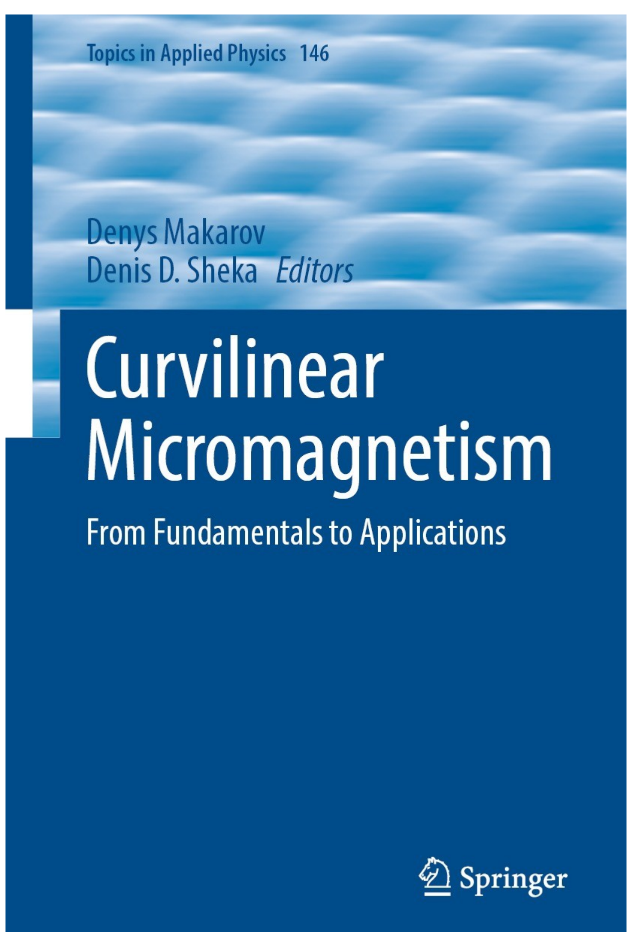
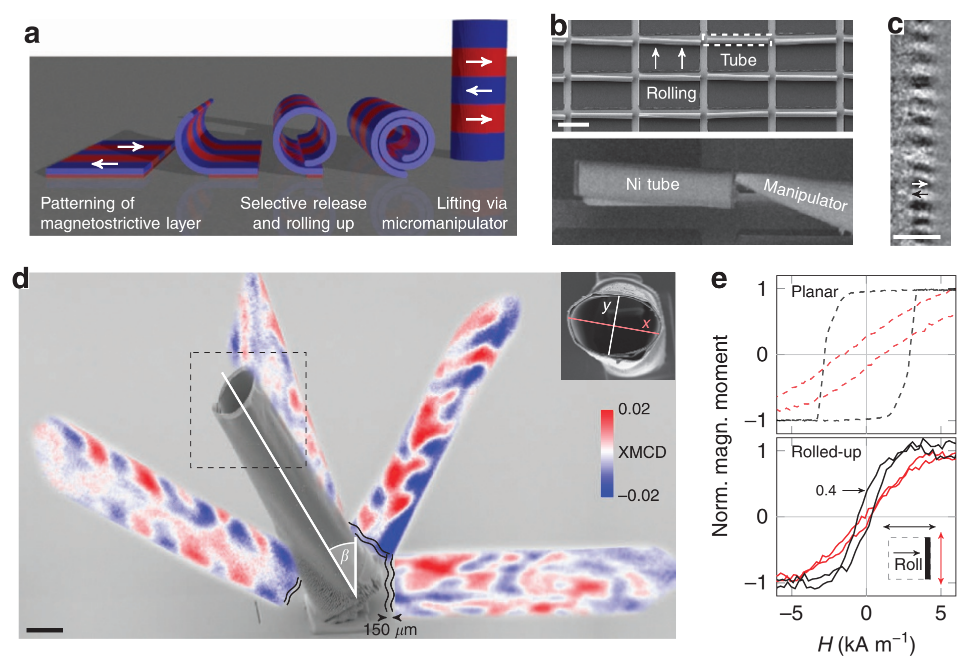
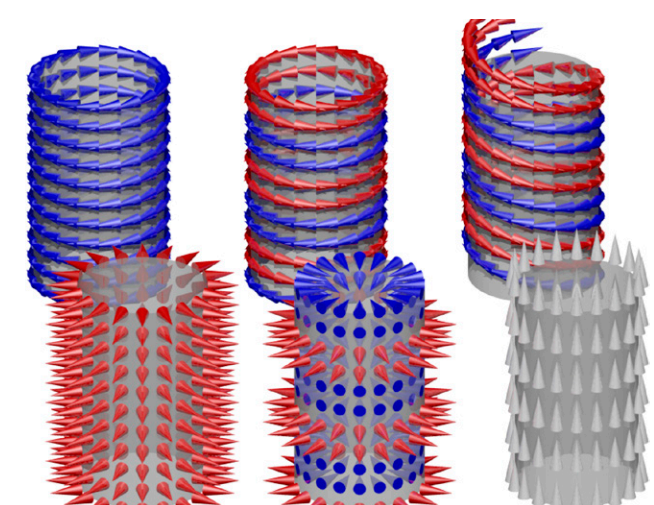
# Transport of vorticity on curved surfaces



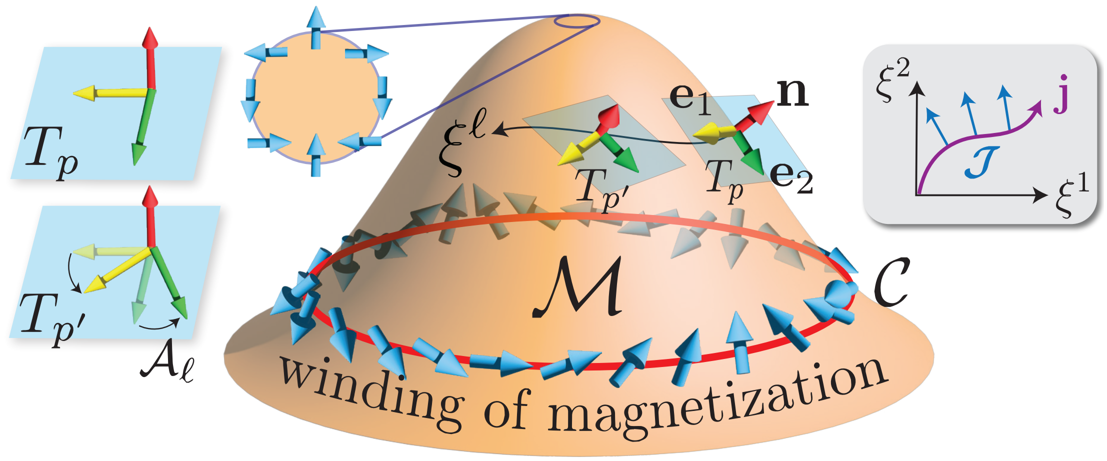

- Can nontrivial geometry reduce the symmetries enough to allow to drive topological hydrodynamics even in a single magnetic layer?



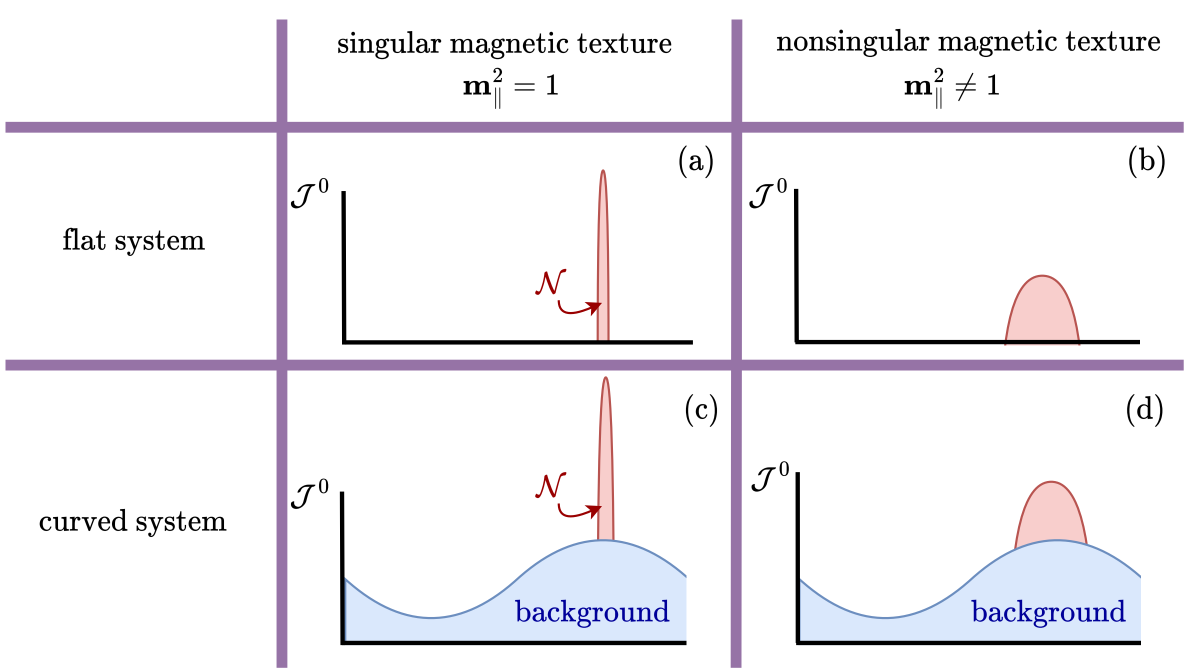



# Hydrodynamics on curved surfaces




- Net topological charge is defined in terms of a winding 1-form (the density is constructed via exterior derivative, utilizing generalized Stokes' theorem):

$$\mathcal{Q} = \int_{\partial\mathcal{S}} \rho_w = \frac{1}{2\pi}\int_{\partial\mathcal{S}} d\xi^i \mathbf{m}_\parallel^2 D_i\varphi \qquad \text{where} \qquad D_i\varphi \equiv \partial_i\varphi - \mathbf{e}_1 \cdot \partial_i \mathbf{e}_2$$



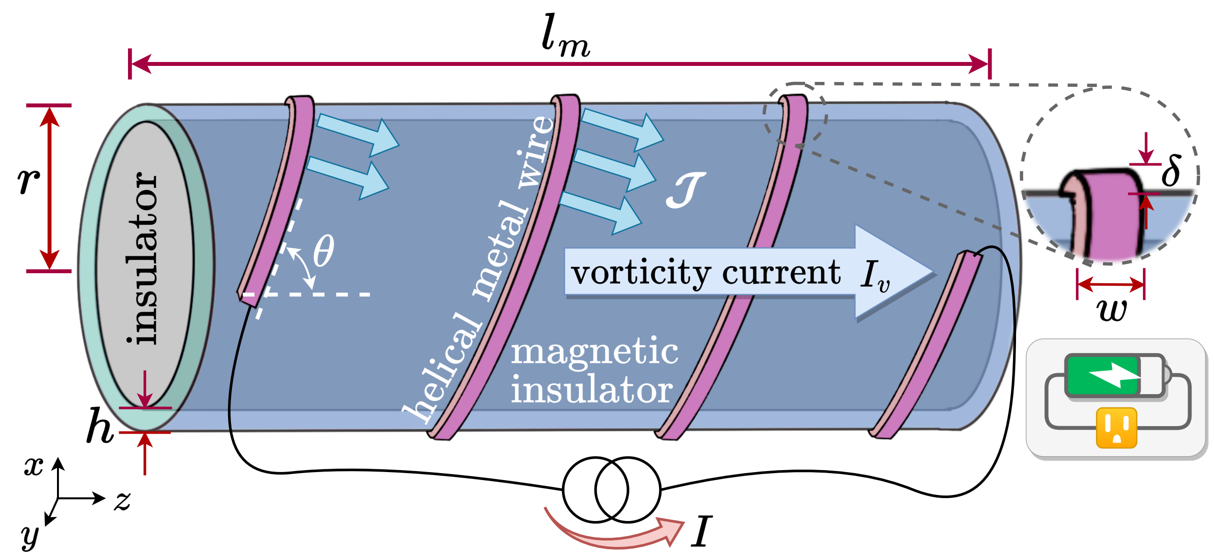

in the strongly easy-plane limit:

$$\mathcal{Q} = \mathcal{N} - \frac{1}{2\pi}\int_{\mathcal{S}} d\xi^1 d\xi^2 \sqrt{g}\mathcal{K}$$

(Mermin-Ho)

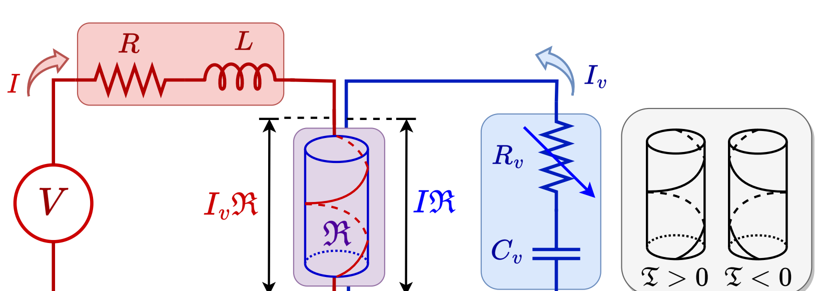
Dao, Zou, Kleinherbers, and YT, *arXiv* (2023)

# "Simplified" topological energy storage




spin-transfer input power: $$\dot{W} = \int dl\, \boldsymbol{\tau} \cdot \mathbf{m} \times \partial_t \mathbf{m} \propto \mathfrak{T}\, I I_v$$

$\mathfrak{T}$ - (pseudoscalar) "torsion of a curve" (electrical wire), which can effectively (from the symmetry point of view) convert the local geometric normal to the surface into an out-of-plane magnetization




$$\xi \propto \mathfrak{T}^2 / R R_v$$

the effective dimensionless parameter, which is thermodynamically bounded to [0,1], is formally analogous to the thermoelectric figure of merit called *ZT*

# Outlook

- Dynamics of collective order-parameter textures can have robust low-energy behavior rooted in topological conservation laws and responsive to geometric controls
- Spin-based systems are abundant, versatile, and amenable to the wealth of spintronic tools
- This can lead to new strategies for probing materials as well as applications, such as energy storage and nontraditional computing (both classical and quantum)
- Myriad connections across different fields of physics, from astrophysics to turbulence
- On the quantum front, intriguing outlooks concern direct transport probes of condensed-matter dualities (e.g., vortex condensation at the superfluid-insulator transition), interplay between real-space and momentum-space topologies, and integration with optically-active quantum impurities for sensing and generation of quantum entanglement

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