Quantization of Heat Flow in Fractional Quantum Hall States

Braun Center for Submicron Research

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WEIZMANN INSTITUTE OF SCIENCE
Wiedemann-Franz Law

\[ \sigma = \frac{I}{V_1 - V_2} \cdot \frac{L}{A} \]

\[ \kappa = \frac{Q}{T_1 - T_2} \cdot \frac{L}{A} \]

\[ \frac{\kappa}{\sigma} = \frac{\pi^2 k_B^2}{3e^2} T \]

Lorentz No.
heat flow in 1-D ballistic channel

J B Pendry
Quantum limits to the flow of information and entropy

thermal energy = temperature \times entropy

together with energy uncertainty

sets an universal upper limit on energy/heat transfer

universality of quantum (upper) limit of heat flow per channel for all non-interacting particles

\( KT \leq \kappa_0 T \)
1D ballistic transport

\[ \frac{dJ_{th}}{dT} = \kappa_0 T \]

\[ \kappa_0 = \frac{\pi^2 k_B^2}{3h} \]

\[ J_{th} = \frac{1}{2} \kappa_0 (T_m^2 - T_0^2) \]

\[ \kappa_0 \approx 9.5 \times 10^{-13} \text{W} / \text{K}^2 \]
Wiedemann - Franz  ballistic 1D channel

for non-interacting electrons

\[ G_{th} = k_0 T \]
\[ G_e = \frac{e^2}{h} \]

\[ \frac{G_{th}}{G_e} = l_{Lorentz} T = \frac{\pi^2 k_B^2}{3e^2} T \]
past experiments... in accord with theory
Non-interacting bosons and fermions both carries the same amount of heat
Interactions....

Pendry’s theory extended for interacting particles

Kane, C. L. & Fisher, M. P. A.
Quantized thermal transport in the fractional quantum Hall effect

Interactions should not affect quantum of thermal conductance !!!

\[ K = \kappa_0 \]

Wiedemann - Franz law breaks down

our 1D interacting system......**FQHE**
Quantum Hall effect: chiral edge modes

Each edge mode carries \( I = \frac{e^2}{h} V \)
1D modes in QHE

bulk of QHE ..........insulating localized quasiparticles

diameter of IQHE ........integer 1D chiral edge modes \( G_H = v e^{2/h} \) \( v = 1, 2, 3, \ldots \)

diameter of FQHE ........fractional 1D chiral edge modes

abelian states .................\( G_H = v e^{2/h} \) \( v = \frac{1}{3}, \frac{2}{5}, \ldots, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots \)

non-abelian states (?!) ...........\( G_H = v e^{2/h} \) \( v = \frac{5}{2}, \frac{12}{5}, \ldots \)
1D modes in FQHE

- **downstream** charge..............................................particle-like

- **downstream** charge + **upstream** neutral ...hole – conjugate & non-abelian
$K$ in lowest $LL$... Kane & Fisher 1997

\[ \nu = \frac{1}{3} \rightarrow K_0 \]

1 composite fermion mode

\[ \nu = \frac{2}{5} \rightarrow 2K_0 \]

2 composite fermion modes

\[ \nu = \frac{2}{3} \rightarrow 0 \]

1 charge down - 1 neutral up

\[ \nu = \frac{3}{5} \rightarrow -K_0 \]

1 charge down - 2 neutral up
hole-like \( v = 2/3 \) non-equilibrated

\[ \nu = 2/3 = 1 - \frac{1}{3} \text{upstream} \]

K=0

MacDonald, A. H.
Edge states in the fractional quantum Hall effect regime
hole-like $\nu = \frac{2}{3}$ equilibrated

$\nu = \frac{2}{3} = \frac{2}{3} - \text{neutral}_{\text{upstream}}$

$K=0$

neutral modes....carrying energy w/o net charge

Kane, C. L., Fisher, M. P. A. & Polchinski, J.  
Randomness at the edge: 
theory of quantum Hall transport at filling 2/3 
equilibration of counter-propagating charge modes

downward arrow

topological neutral modes

* invisible in conductance measurements

* bosonic thermal conductance $\kappa_0$

* associated only with particular FQHE states
Why thermal conductance in FQHE?

* topological constant: determined by bulk wave-function
  * reveals NET chirality of modes (down-up)
  * insensitive to edge reconstruction

these are true for abelian particles

however ....

\[ K_{non-abelian} = \left( n + \frac{1}{2} \right) \kappa_0 \] (Majorana)
The experiment
Working principle:

*flow of dissipated power* ....

\[ J_{tot} = \frac{1}{2} \kappa_0 (T_m^2 - T_0^2) + J_{ph} \]

N - arm device

\[ P_{\text{in}} = \frac{I_{\text{in}}^2}{2G_H} \]

\[ \Delta P = P_{\text{in}} (1 - \frac{1}{N}) \]

\[ P_{\text{out}} = \frac{P_{\text{in}}}{N} \]

\[ N = 4 \]
we measure only temperature...

electron temperature in grounded contacts..... $T_0$

electron temperature in heated reservoir....... $T_m$

$$\Delta P = J_{th}^{total} = 0.5 K (T_m^2 - T_0^2) + \beta (T_m^5 - T_0^5)$$

- small $T$...........phonon term irrelevant
- high $T$............phonon term subtracted
- $K$ determined
measuring temperature

Temperature in grounded contacts..... $T_0$

**shot noise**

Excess temperature in heated reservoir....... $T_m - T_0$

**thermal noise**
measuring $T_0$ ... shot noise

\[ S_i(\omega; 0) = 2e^* I_s t(1-t) \cdot \mathcal{S}(V_s, T) + 4k_B T G \]
measuring $T_m$ ……Johnson-Nyquist noise

dissipated power, $\Delta P$

- modes leave contact with noise $4k_B T_m G$
- even if modes cool down with distance...

low frequency current fluctuations conserved
measuring $T_m$ ....... Johnson-Nyquist noise

excess Johnson - Nyquist noise ... $2k_B G^*(T_m - T_0)$
$T_m$ vs dissipated power

$N = 4$
$v = 2$
actual analysis

\[ J_{tot} = \frac{1}{2} \kappa_0 (T_m^2 - T_0^2) + J_{ph} \]

removing one channel at a time removes phonons contribution
\[ \frac{1}{\Delta N} \delta P_{\Delta N} / 0.5 \kappa_0 (10^{-3} \text{K}^2) \]

\[ \Delta N = 2 \]

slope = 1.00 ± 0.045
realization......N = 4
$N = 3$

QPC pinched
realized structure
**points of consideration** not an easy experiment

- electrons fully equilibrate in the small floating reservoir $T_m$

- outgoing charge channels carry only Johnson-Nyquist noise without shot noise

- no presence of bulk energy modes (may increase the apparent thermal conductance)

- length of arms is limited (~150µm, temperature equilibration between up-down modes)

- equal splitting between arms, amplifier gain determination, contacts’ resistance, ...
weak interaction regime (IQHE) \( \nu = 2, 1 \)

strong interaction regime (FQHE)

- particle-like: \( \nu = \frac{1}{3} \)
- hole-like: \( \nu = \frac{2}{3}, \frac{3}{5}, \frac{4}{7} \)
\( \nu = 2 \quad \nu_{QPC} = 1 \quad N = 2 \)
Results:

\( \nu = 2 \)

\[ \begin{align*}
S_{\text{th}} (10^{29} \text{ A}^2/\text{Hz}) \\
I_S (\text{nA}) \\
T_0 = 30 \text{ mK} \\
N = 4 & \quad + \\
N = 3 & \quad \circ \\
N = 2 & \quad \triangle
\end{align*} \]

\[ \begin{align*}
T_m (\text{mK}) \\
\Delta P (\text{fW}) \\
\nu = 2 \\
N = 4 & \quad + \\
N = 3 & \quad \circ \\
N = 2 & \quad \triangle
\end{align*} \]

\[ \delta P + 2\delta P \]
Results:

\[ \nu = 2 \]

\[ \lambda (10^{-3} \text{ K}^2) \]

\[ \nu = 2 \]

\[ \Delta N = \frac{\Delta P}{\kappa} \]

\[ T_0^2 \]

\[ T_m^2 (10^{-3} \text{ K}^2) \]

\[ \kappa_0 = 0.98 \pm 0.03 \]

\[ \lambda = \Delta P / \kappa \]
particle-like \( \nu = 1/3 \)

**bulk:** gapped - incompressible liquid

**edge:** single charge mode \( G = G_0/3 \)

\[ K = \kappa_0 \]
$\nu = 1 \ldots 1/3 \rightarrow K_0$

1 electron mode…1 composite fermion mode
\[ \frac{\lambda}{\Delta N} = 2 \]

slope = 1.00 ± 0.045

|  ν | = 1/3

\[ T_m^2 (10^{-3} K^2) \]
interactions do not affect $K$

what about $K$ of neutral modes?
more complex fractions…

fractional hole-conjugate states……\(1/2 < v < 1\)

full Landau level with holes
hence, counter-propagating modes

always with upstream \textit{neutral} modes

\[ v = 2/3, \ 3/5, \ 4/7 \]
$K$ of hole - states... Kane & Fisher 1997

$v = 2/3 \rightarrow K = 0$

1 charge down - 1 neutral up

$v = 3/5 \rightarrow -K_0$

1 charge down - 2 neutral up

$v = 4/7 \rightarrow -2K_0$

1 down charge - 3 neutrals up
\[ \nu = \frac{2}{3} \ldots \text{why } K = 0 ? \]

equal number of down and up modes

full equilibration ONLY at large length.....all emitted heat returns
\[ \frac{\lambda}{\Delta N} \times 10^{-3} K^2 \]

\[ T_m^2 \times 10^{-3} K^2 \]

\( \nu = 2/3 \)

slope = 0.33 \( \pm \) 0.02
Temperature dependence

\[ \nu = \frac{2}{3} \]

\[ \frac{K}{\kappa_0} \text{ vs. } T_0 \text{ (mK)} \]
hole-states with more upstream neutral modes
calculating $T(x)$ & $K$ ...... $v = 2/3$

$J = KT^2$

$0.5n_d \kappa_0 \partial_x T_u^2(x) = -j_t(x)$

$0.5n_d \kappa_0 \partial_x T_d^2(x) = -j_t(x)$

Newton's law of cooling

$$j_t(x) = \frac{K_0}{2\xi_T} \left( T_d^2(x) - T_u^2(x) \right)$$

$n_d = n_u = 1$
\[ \kappa \left( \frac{L}{\xi} \right) / \kappa_0 \]

For \( \nu = 4/7 \),
\[ \frac{\kappa \left( \frac{L}{\xi} \right)}{\kappa_0} = 2 + \frac{4}{3e \frac{2L}{3\xi} - 1} \]

For \( \nu = 3/5 \),
\[ \frac{\kappa \left( \frac{L}{\xi} \right)}{\kappa_0} = 1 + \frac{2}{2e \frac{2L}{2\xi} - 1} \]

For \( \nu = 2/3 \),
\[ \frac{\kappa \left( \frac{L}{\xi} \right)}{\kappa_0} = 0 + \frac{2}{\frac{2L}{\xi} + 1} \]
Summary:

1D electron modes
1D fractional modes
1D neutral modes

quantized thermal conductance
fractional states in first excited Landau level

\[ \nu = 2 + \eta \]

\[ \nu = \frac{7}{3}, \frac{5}{2}, \frac{8}{3} \]
\( \mu \approx 20 \times 10^6 \text{ cm}^2/\text{v-s} \)
at $v = 5/2 \ldots R_{xx} = 0$

BCS of polarized composite fermions w/ odd orbital angular momentum

$B^* = 0$
bulk – edge correspondence

Majorana → half fermion.... $K = \frac{\kappa_0}{2}$
already known for $\nu = 5/2$

- charge $e/4$
- upstream neutral modes
- spin polarized

abelian or non-abelian?
fractional state $\nu = 5/2$

if non-abelian $K = (n \pm 0.5)\kappa_0$

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\[ \Delta N = 4 - 2 \]

\[ T_0 = 11-12 \text{ mK} \]
$v = \frac{7}{3}$  $v = 2 + \frac{1}{3}$  particle-like, downstream

$K = 3\kappa_0$

$K = (2+\varepsilon)\kappa_0$

$\nu = \frac{8}{3}$  $\nu = 2 + \frac{2}{3}$  hole-like, down-up

$\nu = \frac{5}{2}$
measuring $\nu = 1, 2 \at \nu_B = 5/2$
\[ \frac{\left( N = 4 \right) - \left( N = 2 \right)}{2} \]

slope = \( 2.49 \pm 0.03 \)

\( \nu = \frac{5}{2} \)

\( T_0 = 18 \text{ mK} \)
$\Delta N = 4 - 2$

$v_{bulk} = 5/2$

- $T_0 = 18 \text{ mK}$, $K/K_0 = 2.56 \pm 0.02$
- $T_0 = 15 \text{ mK}$, $K/K_0 = 2.64 \pm 0.02$
- $T_0 = 12 \text{ mK}$, $K/K_0 = 2.76 \pm 0.02$
conductance determination with phonons contribution

$\beta = 5 \times 10^{-9}$ W/K$^5$

$K_{5/2} = 2.54 \kappa_0$

$\nu_{bulk} = 5/2$

$T_0 = 18$ mK
increasing equilibration length at lower temperature

equilibrated

$T_0$ (mK)

$\text{PH-Pfaffian}$

$\text{Pfaffian}$

$\text{SU}(2)_2$

$\text{Anti - Pfaffian}$

$\text{Anti - SU}(2)_2$
Theory of Disorder-Induced Half-Integer Thermal Hall Conductance

David F. Mross, Yuval Oreg, Ady Stern, Gilad Margalit, and Moty Heiblum
Braun Center for Submicron Research, Department of Cond. Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel.

Topological Order from Disorder and the Quantized Hall Thermal Metal: Possible Applications to the $\nu = 5/2$ State

Chong Wang, Ashvin Vishwanath, and Bertrand I. Halperin
Department of Physics, Harvard University, Cambridge MA 02138, USA

On the Interpretation of Thermal Conductance of the $\nu = 5/2$ Edge

Steven II. Simon
Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, Oxford, OX1 3NP, UK
(Dated: January 31, 2018)

Theory of

Disordered $\nu = 5/2$ Quantum Thermal Hall State:
Emergent Symmetry and Phase Diagram

Biao Lian and Javen Wang
Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08544, USA
School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA
\( v = \frac{5}{2} \ldots \) likely non-abelian

measuring thermal conductance reveals hidden information

ARTICLE

Observation of half-integer thermal Hall conductance

Mitali Banerjee\textsuperscript{1}, Moty Heiblum\textsuperscript{1*}, Vladimir Umansky\textsuperscript{1}, Dima F. Feldman\textsuperscript{2}, Yuval Oreg\textsuperscript{1} & Ady Stern\textsuperscript{1}

https://doi.org/10.1038/s41586-018-0184-1

Thank you !!!
Starting with K

• Measuring K at $v=5/2$ at short distances – aided with noise measurements to check down and up neutral modes

• Doing the same in graphene (and bi-layer graphene) – as flakes are small

• Measuring K at $v=5/2$ at different $B$’s and different $n$’s – with aid of a back gate – testing if K is universal in this state or depends on parameters

• Studying K at $v=12/5$. Not easy as accuracy has to be better than $0.5k_0$.

• Studying $v=7/2$ – also predicted to be non-abelian (but our quick measurement didn’t see it...)

Interference

• Can we prevent neutral modes in GaAs 2DEG?

• If we can, looking for interference

• Looking for neutral modes in graphene (and others monolayer materials...)

• If no neutral modes, look for interference in graphene – first integer and fractions

Thank you !!!
temperature profile, $v = \frac{2}{3}$

$$L/\xi = 10, \quad T_m = 2T_0$$

$$\frac{K}{\kappa_0} = \frac{2}{1 + \frac{L}{\xi}}$$

length dependence diffusive heat transport

thermal conductivity

$$x/\xi$$
heating the reservoir

\[ P_{in} = \frac{I_{in}^2}{2G_H} \]
calculating \( T(x) \) & \( K \) ...... \( v = 3/5 \)

\[ \frac{\partial}{\partial t} \left( J_T \right) = \kappa \frac{\partial}{\partial x} T^2 \]

\[ 0.5n_u \kappa_0 \frac{\partial}{\partial x} T^2_u(x) = -j_t(x) \]

\[ 0.5n_d \kappa_0 \frac{\partial}{\partial x} T^2_d(x) = -j_t(x) \]

\( n_d = 1 \), \( n_u = 2 \)

\( J = KT^2 \)
temperature profile, $v = 3/5$

Temperature independent

$L/\xi = 10, T_m = 2T_0$

\[
\frac{T^2}{T_0^2} \approx \frac{T_m^2}{T_0^2} e^{-\frac{x}{\xi}}
\]
difficulties due to structure:

➢ ‘bulk heat conductance’…..free electrons in the donor layers

➢ poor contact of the floating reservoir – hence, reflections

➢ instability of QPC’s
$\nu = \frac{7}{3}$ \hspace{1cm} $\nu = 2 + \frac{1}{3}$ \hspace{1cm} particle like, downstream

$\nu = \frac{8}{3}$ \hspace{1cm} $\nu = 2 + \frac{2}{3}$ \hspace{1cm} hole-like, down + up

$\nu = \frac{5}{2}$ \hspace{1cm} $\nu = 2 + \frac{1}{2}$

- measured
  - $K = 3\kappa_0$
  - $K = (2+\varepsilon)\kappa_0$

- what do we know?

  - quasiparticle charge $e^* = e/4$
  - upstream energy modes
  - spin polarized
Points of consideration:

* noiseless source current (DC I_{in} \to in most cases)
* electrons fully equilibrate in the floating contact (with T_m)
* outgoing currents only carry J-N noise (low contact resistance)
* measurements at low temperature (J_{e-ph} \ll J_e)
* no bulk energy modes exist (may increase apparent conductance)
* required length of arms (allowing temperature equilibration)
\( \nu = \frac{5}{2} \) state ....... if non-abelian \( \frac{K}{\kappa_0} = n + \frac{1}{2} \)

from electrons ....... to non-abelian quasiparticles

* half - filled LL on top of two filled LL’s........... \( 2 \frac{1}{2} = 2 + \frac{1}{2} \)

* flux attachment......spin polarized CFs 
  at zero average magnetic field

* CFs pair into Cooper pairs 
  p-wave superconductor

* vortices are charged......\( e^* = e/4 + \text{Majorana} \)

* chiral edge modes: charged + Majorana

* ground state degeneracy (braiding is non-abelian)
shallow DX centers over doping

delta doping in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ ($x=23\div25\%$)

$\delta$-doping

$E_F$

- NO illumination
- NO parallel conductance
- NO bulk thermal conductance
- stable gates (QPC replaced by continuous gate)
- low resistance floating reservoir
\( R_{xx} (\Omega) \)

- 7/3
- 5/2
- 8/3

\( K/K_0 \)

- 1.96
- 2.9
- ?
- ?
- 3.1
thermal conductance of $\nu = 8/3$ $R_{xx} > 0$

$K_{8/3} = 2.16 \, K_0$

$\nu = 8/3$

$T_0 = 18 \, \text{mK}$
\[ \frac{[(N = 4) - (N=2)]}{2} \]

slope = 2.31 ± 0.02

\[ \nu = \frac{8}{3} \]

\[ T_0 = 18 \text{ mK} \]
$R_{xx}(\Omega)$

$\nu$

$K$

$\gamma = \frac{8}{3}$

$\nu$

$K/K_0$

2 of 8/3.....2.07
thermal conductance of $\nu = 5/2$

$K_{5/2} = 2.54 \ K_0$

$\nu = 5/2$

$T_0 = 18 \ mK$
thermal conductance of \( \nu = 5/2 \)

\[
\nu = 5/2 \\
T_0 = 18 \text{ mK}
\]
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integer, \( e, \kappa = 1 \)

fraction, \( e/4, \kappa = 1 \)

neutral, \( 0, \kappa = 1 \)

Majorana, \( 0, \kappa = 0.5 \)

**upstream**
### Integer, $e$, $\kappa = 1$

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### Fraction, $e/4$, $\kappa = 1$

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### Neutral, $0$, $\kappa = 1$

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### Majorana, $0$, $\kappa = 0.5$

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### PH-Pfaffian

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Son 2015, Feldman 2016

Majorana likely found!!!
Heat conductance w/length

$L/\xi = 10, \ T_m = 2T_0$

$v = \frac{3}{5}$

\[
\frac{\kappa(L/\xi)}{\kappa_0} = 1 + \frac{2}{2e^{2L/\xi} - 1}
\]
\[ \nu = \frac{2}{3} \]

\[ J_e \approx 0.33 \cdot 0.5\kappa_0 (T_m^2 - T_0^2) \quad T_0 = 10 \text{mK} \]

\[ \kappa > 0 \] ....symmetric up and down of arms, hence.... actual \[ \kappa / 2 \]
$\nu = \frac{3}{5}$

$$J_e \approx 1.04 \cdot 0.5 \kappa_0 (T_m^2 - T_0^2) \quad T_0 = 10 \text{mK}$$
\[ v = \frac{2}{3} \]

\[
\frac{K}{\kappa_0} = \frac{2}{1 + \frac{L}{\xi_T}}
\]

\[ L \approx 150 \mu m \]

\[ J_e \approx 0.33 \cdot 0.5 \kappa_0 (T_m^2 - T_0^2) \quad T_m^{\text{ava}} = 20 mK \quad \Rightarrow \xi_T = 30 \mu m \]

\[ J_e \approx 0.25 \cdot 0.5 \kappa_0 (T_m^2 - T_0^2) \quad T_m^{\text{ava}} = 45 mK \quad \Rightarrow \xi_T = 20 \mu m \]
example......\( v = \frac{2}{3} \)......why \( K = 0 \) ?

heat diffuses
thermal conductivity vs thermal conductance
length dependence thermal conductance
observation of upstream neutral edge modes

shot noise
\( \nu = \frac{2}{3}, \frac{3}{5}, 1+\frac{2}{3}, 2+\frac{2}{3} \) & \( \frac{5}{2} \)

QD thermometry
\( \nu = \frac{2}{3} \) edge at \( \nu = 1 + \) bulk heat transport
Venkatachalam, Nature Physics (2012)

QD thermoelectric current
\( \nu = \frac{2}{3} \)

QD thermometry
\( \nu = 1+\frac{1}{3} + \) bulk heat transport
Altimiras, PRL (2012)
two-arm device
Results:

\( \nu = 1/3 \rightarrow K_0 \)

1 electron mode... 1 composite fermion mode

\[ S_{th}(10^{-29} \text{A}^2 \text{Hz}^{-1}) \]

\[ T_0 = 10 \text{ mK} \]

\[ \nu = 1/3 \]

\[ I_s (nA) \]

\[ + \quad N = 4 \]

\[ \circ \quad N = 2 \]
Results:

\[ N = 4 \quad \Delta P = 4\kappa_0 T_m^2 \quad T_0 = 10 \text{ mK} \]

\[ N = 2 \quad \Delta P = 2\kappa_0 T_m^2 \]

\[ \Delta P(fW) = 1/3 \]

\[ T_2^m(10^{-3} \text{ K}^2) \]

\[ \Delta N = 2 \quad \nu = 1/3 \]

\[ \lambda/\Delta N(10^{-3} \text{ K}^2) \]

\[ \text{slope} = 1.00 \pm 0.045 \]
What sets the limit on heat flow

\( E.t \approx \hbar \) sets a lower bound to the energy flow

In steady state, \( \dot{E} = \frac{\pi k_B^2}{12\hbar} T^2 \) \& \( \dot{S} = \frac{\pi k_B^2}{6\hbar} T \)

Expression relating single channel entropy and energy flow is

\[ \dot{S}^2 \leq \frac{\pi k_B^2}{3\hbar} \dot{E} \]

using, \( \dot{Q} = \dot{E} \) \& \( \frac{\dot{Q}}{T} \leq \dot{S} \)

\[ \dot{Q} \leq \frac{\pi k_B^2}{3\hbar} T^2 \] or \( J \leq \frac{\pi^2 k_B^2}{6\hbar} T^2 \)

hole-like states:

\[ \nu = \frac{2}{3}, \quad \kappa = 0 \]

\[ \nu = \frac{3}{5}, \quad \kappa = -\kappa_0 \]

\[ \nu = \frac{4}{7}, \quad \kappa = -2\kappa_0 \]
hole-like states + neutral modes..... $v = \frac{2}{3}$

expected ...... $K = 0$ all electrical heat returns

distance~150µm, $T_0$~10mK
thermal noise - spectral density

\[ S_{th}(nA) \]

\[ \nu = 2/3 \]

\[ T_0 = 10 \, mK \]

\[ N = 4 \]

\[ N = 2 \]
Generalization....

G.C Rego and G Kirczenow

Fractional exclusion statistics and the universal thermal conductance: A unifying approach


\[
J_q = \frac{q}{h} \int d\varepsilon (\eta_R - \eta_L) \ldots \ldots \text{electric current}
\]

\[
J_{th} = \frac{1}{h} \int d\varepsilon \varepsilon (\eta_R - \eta_L) \ldots \ldots \text{heat current}
\]

\[
\eta_g = \frac{1}{Z(x, g) + g}
\]

\[
x = \frac{\varepsilon - \mu}{k_B T}
\]

\[
g = 0 \quad \text{bosonic}
\]

\[
g = 1 \quad \text{fermionic}
\]

\[
g = 3 \quad \nu = 1/3
\]

\[
G_q = \frac{1}{g} \cdot \frac{e}{h} \cdot e \ldots \ldots \text{g dependent}
\]

\[
G_{th} = 1 \cdot \frac{\pi^2 k_B^2}{3h} \cdot T \ldots \ldots \text{g independent}
\]