QUANTUM GATING OF CLASSICAL INFORMATION FLOW

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with
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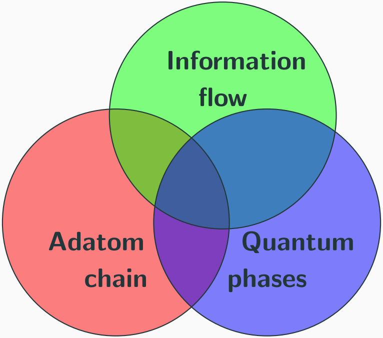
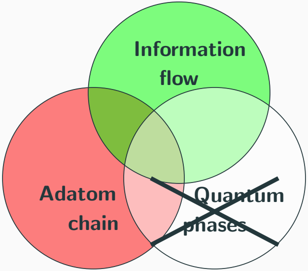
Quantum chains to mediate information flow

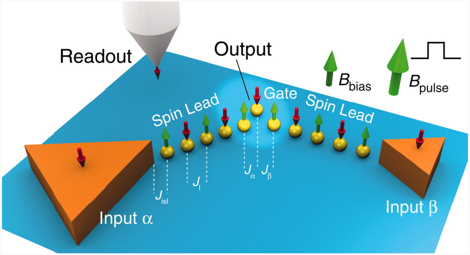
Spin 1:



Introduction

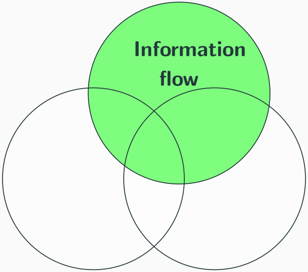
All spin logic in adatom chains

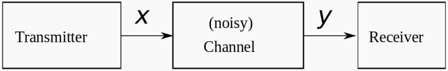




Khajetoorians *et al.*, Science (2011).

Channel capacity





- Transmitter generate data according to a probability distribution p_X.
- Receiver measure data according to the probability distribution p_Y.
- The conditional distribution $p(Y|X)$ describes the quality of the channel.

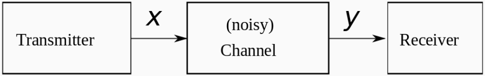
Capacity of a memoryless channel

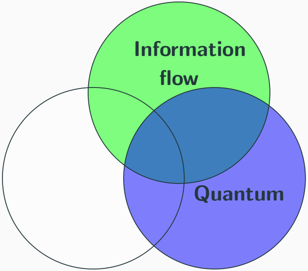
$$C = \max_{p_X}[H(Y) - H(Y|X)]$$

Shannon: transmission errors can be corrected if the transmission rate $R < C$

The Shannon entropy H measures the uncertainty of a random variable

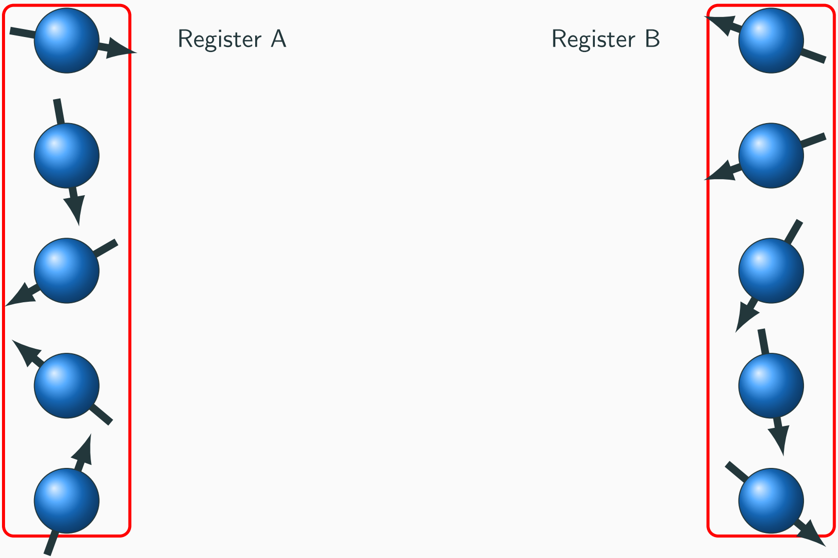
Quantum channel capacity





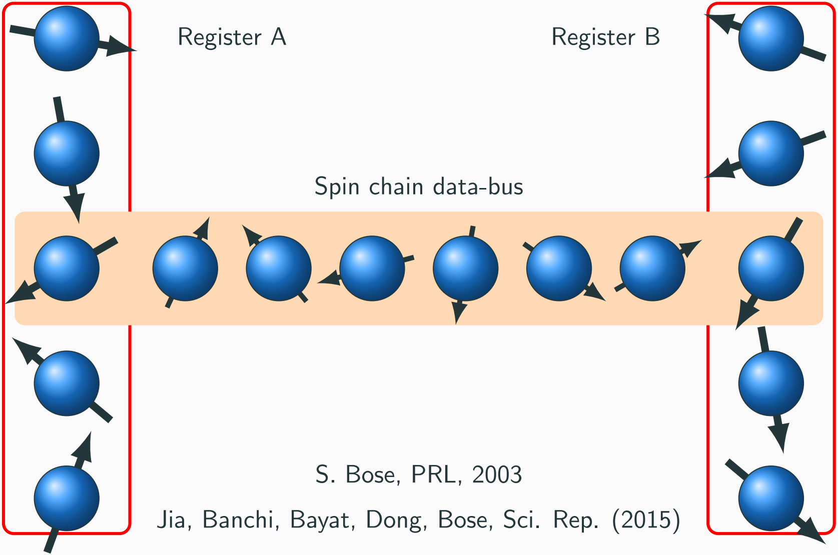
- Transmitter encodes the information onto quantum states.
- Receiver performs quantum measurements to retrieve the classical data.

Quantum data-bus



Quantum data-bus

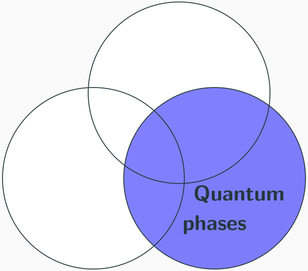
Quantum data-bus



S. Bose, PRL, 2003

Jia, Banchi, Bayat, Dong, Bose, Sci. Rep. (2015)

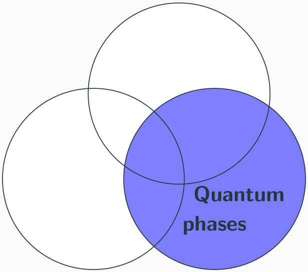
Quantum phase transitions (QPT)



$$\mathcal{H} = -J \sum_j S_j^z S_{j+1}^z - \frac{B}{2} \sum_j S_j^x$$

- $J = B$: critical point, gapless
- $J \neq B$: gapped

Quantum phase transitions (QPT)



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$|B| > |J|$:

- Unique ground state. E.g. ($J = 0$)

$$|\rightarrow\rangle^{\otimes L}$$

- $\langle O_i O_j \rangle \approx e^{-|i-j|/\xi}$

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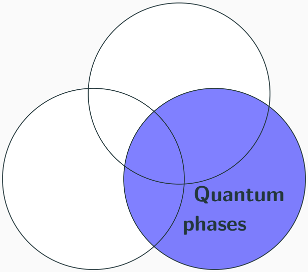

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$$|\rightarrow\rangle^{\otimes L}$$

- $\langle O_i O_j \rangle \approx e^{-|i-j|/\xi}$

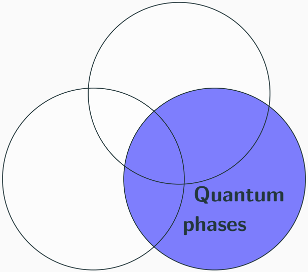
$|B| < |J|$:

- Degenerate ground state. E.g. ($B = 0$)

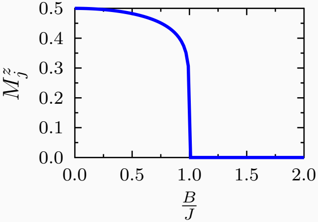
$$|\downarrow\rangle^{\otimes L} \quad \text{or} \quad |\uparrow\rangle^{\otimes L}$$

- $\langle O_i O_j \rangle \to M^2$

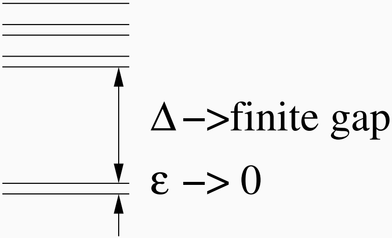
QPT and finite size



- QPT defined only in the thermodynamic limit ($L \to \infty$)
- Opposite regime considered in adatom chains

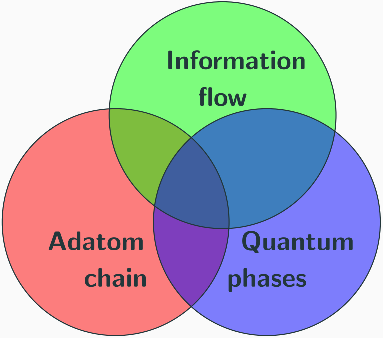


$M_j^z = \langle \varepsilon | S_j^z | \mathrm{G.S.} \rangle$

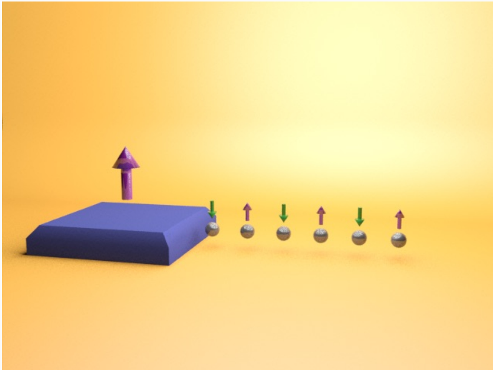


Results

Quantum chains to mediate information flow



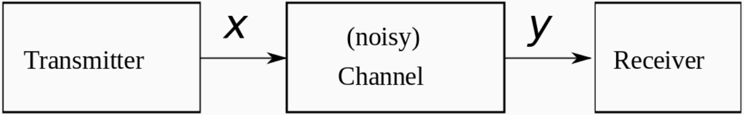
Adatom chain coupled with a magnetic island



$$\mathcal{H} = \left[J \sum_{n=1}^{L-1} \vec{S}_n \cdot \vec{S}_{n+1} + \sum_{n=1}^{L} \mathcal{H}_n \right] + \vec{B}_0 \cdot \vec{S}_1 \ ,$$

$$\mathcal{H}_n = D(S_n^z)^2 + E[(S_n^x)^2 - (S_n^y)^2] + \vec{B} \cdot \vec{S}_n \ , \qquad \vec{B}_0 = J' \left\langle \vec{S}_{\mathrm{island}} \right\rangle$$

Information capacity of adatom chains



- Bit X:

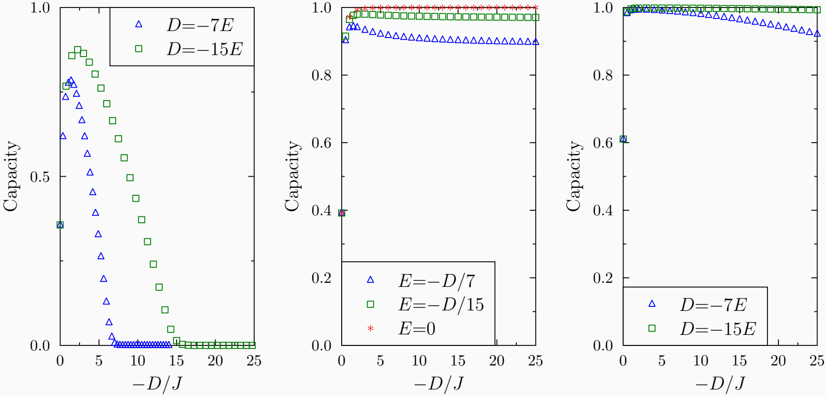
$$0 \;=\; \text{Island}(\uparrow) \qquad\qquad 1 \;=\; \text{Island}(\downarrow)$$

- Bit Y:

$$0 \;=\; \text{positive}\,\langle S_L^z\rangle \qquad\qquad 1 \;=\; \text{negative}\,\langle S_L^z\rangle$$

$$\text{error} \;=\; \langle S_L^z\rangle \simeq 0$$

Information capacity of adatom chains: DMRG, $L = 25$

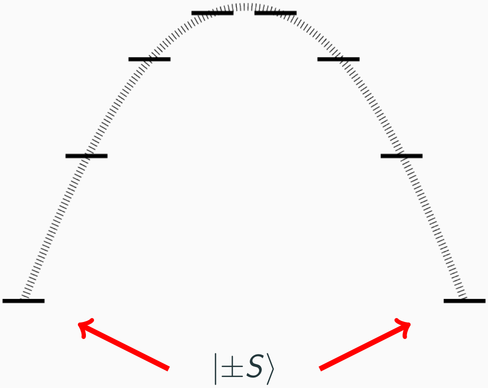


(a) Spin 1 (b) Spin 3/2 (c) Spin 2

Not depend much on $|B_0|$ (only on its sign!)

Low energy theory

Effective two level system



Effective Pauli operators τ^x, τ^y, τ^z

Effective Hamiltonian when $\vec{B} \parallel \hat{x}$ and $\vec{B}_0 \parallel \hat{z}$

- $S = 1$:

$$H_1^{\mathrm{eff}} = B_0 \tau_1^z + \sum_n \left[\left(E + \frac{B^2}{2D} \right) \tau_n^x + J \tau_n^z \tau_{n+1}^z \right] + \frac{J^2}{D} V_1 \ ,$$

where $V_1 = \sum_n (\tau_n^x \tau_{n+1}^x + \tau_n^x \tau_{n+1}^x - \tau_n^z \tau_{n+1}^z)/4$.

- $S = 3/2$:

$$H_{\frac{3}{2}}^{\mathrm{eff}} = \frac{3}{2} B_0 \tau_1^z + \sum_n \left[\frac{3BE}{2D} \tau_n^x + \frac{9J}{4} \tau_n^z \tau_{n+1}^z \right] + \frac{J^2}{D} V_{\frac{3}{2}} \ ,$$

where $V_{\frac{3}{2}} = -9/32 \sum_n \tau_n^z \tau_{n+1}^z$.

- $S = 2$:

$$H_2^{\mathrm{eff}} = 2 B_0 \tau_1^z + \sum_n \left(\frac{3E^2}{2D} \tau_n^x + 4J \tau_n^z \tau_{n+1}^z \right) + \frac{J^2}{D} V_2 \ ,$$

where $V_2 = -\sum_n \tau_n^z \tau_{n+1}^z / 3$.

Jia, Banchi, Bayat, Dong, Bose, Sci. Rep. (2015)

Delgado, Loth, Zielinski,and J. Fernández-Rossier, EPL (2015)

Effective Hamiltonian when $\vec{B} \parallel \hat{x}$ and $\vec{B}_0 \parallel \hat{z}$

$$\mathcal{H}' = \mu\tau_1^z + \sum_n \left(\lambda\tau_n^x + \tau_n^z\tau_{n+1}^z\right) \ ,$$

$$\begin{aligned}
&\text{Spin } 1: && \mu \simeq \frac{B_0}{J} && \lambda \simeq \frac{E}{J} + \frac{B^2}{2DJ} \ , \\
&\text{Spin } \frac{3}{2}: && \mu \simeq \frac{2B_0}{3J} && \lambda \simeq \frac{2BE}{3DJ} \ , \\
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\end{aligned}$$

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\end{aligned}$$

Remote magnetization

$$m_L^z \simeq \begin{cases} \mathrm{sign}(\mu)\sqrt{1-\lambda^2} & \text{if } |\lambda| < 1 \\ 0 & \text{if } |\lambda| > 1 \end{cases}$$

$m_L^z = \langle S_L^z \rangle$ NOT $\langle \varepsilon | \, S_L^z \, | \mathrm{G.S.} \rangle$!

Effective Hamiltonian when $\vec{B} \parallel \hat{x}$ and $\vec{B}_0 \parallel \hat{z}$

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\end{aligned}$$

Capacity

$$C = \begin{cases} 1 + \sum_\pm \left(\frac{1 \pm \sqrt{1-\lambda^2}}{2} \right) \log_2 \left(\frac{1 \pm \sqrt{1-\lambda^2}}{2} \right) & \text{if } |\lambda| < 1 \\ 0 & \text{if } |\lambda| > 1 \end{cases}$$

Effective Hamiltonian when $\vec{B} \parallel \hat{x}$ and $\vec{B}_0 \parallel \hat{z}$

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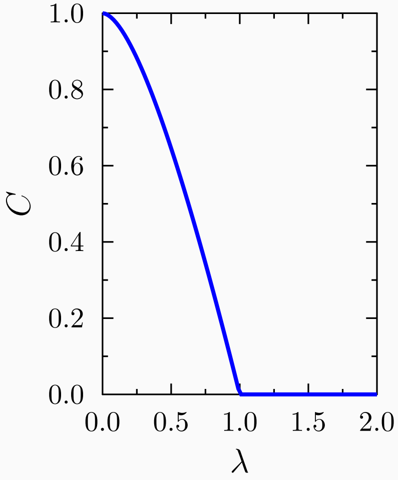
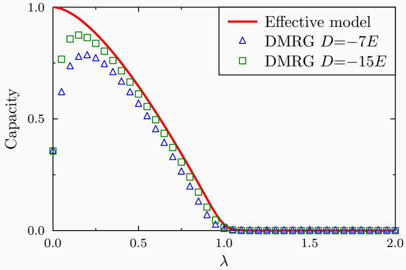
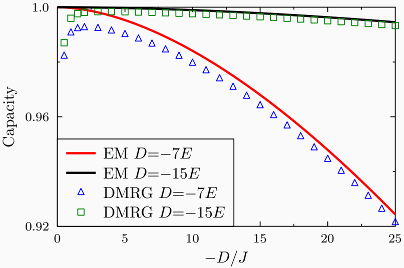


Effective description

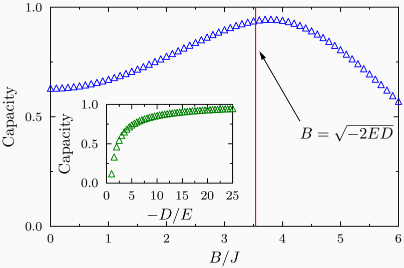


(a) Spin 1

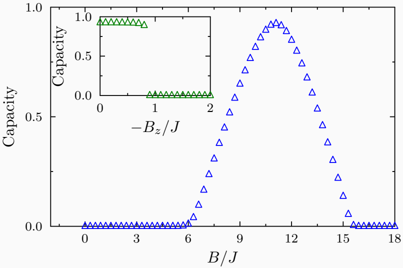


(b) Spin 2

Gating information: DMRG, $L = 25$

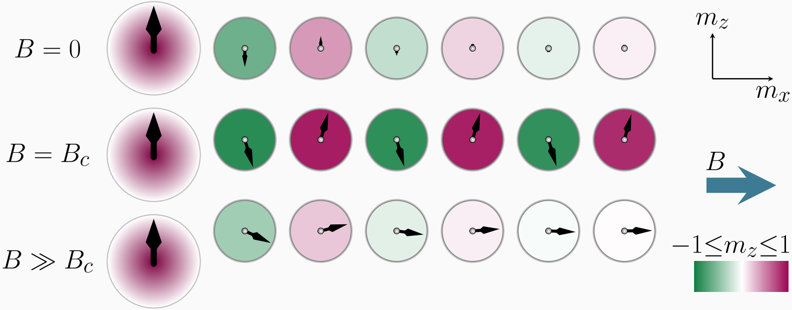


(a) Spin 1: E=0.5J



(b) Spin 1: E=1.5J

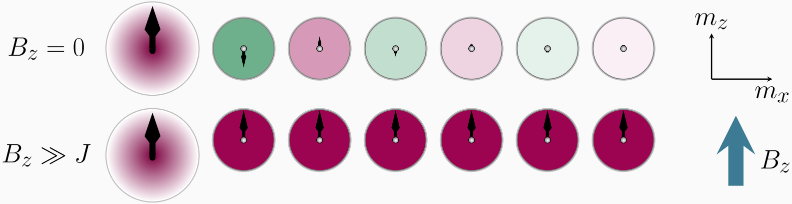
Gating information



$\lambda = 1.5$

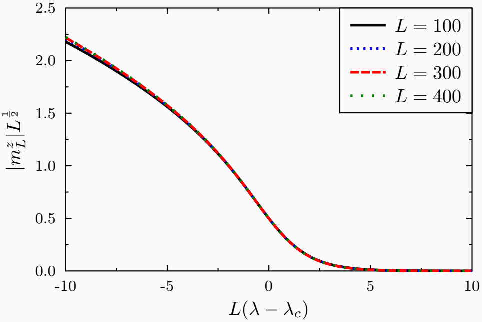
Gating information

Straightforward alternative.



. . . but makes the system almost non-interacting!

Finite size scaling for order parameters



$$m \approx L^{-\beta/\nu} f(L^{1/\nu} |\lambda - \lambda_c|)$$

Critical exponents:

Surface magnetization	$\rightarrow$	$\beta_s = 1/2$
Bulk magnetization	$\rightarrow$	$\beta = 1/8$
Capacity	$\rightarrow$	$\beta_C = 1$

Conclusions

Summary and perspectives

- **Spin 1** is interesting
- **Gating** of classical information dependent on **quantum phases**
  - Strongly interacting regime
  - Flow on: ordered phase
  - Flow off: disordered phase
- Controllable phases
  - Effective field due to anisotropy E
  - **Tunable with an external field** B_x

Acknowledgements

UCL

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Collaborators:

- C. F. Hirjibehedin
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