Multi-terminal Josephson junctions

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with

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SPICE-Workshop

Exotic New States in Superconducting Devices: The Age of the Interface

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Topological insulators / superconductors:
• gap in the bulk + topologically-protected surface states

Kitaev chain
(1D spinless p-wave SC)
→ Majorana bound states
Kitaev (2001)

quantum spin Hall insulator
→ helical edge states
Fu & Kane (2005), Bernevig et al. (2006), König et al. (2007)

Oreg et al. (2010)
Lutchyn et al.
Mourik et al. (2012)

Weyl points
Wan et al. (2011)
Xu et al. (2015)
Main idea

Materials

Complicated bandstructure in $d$ dimensions

Artificial material in $d = n - 1$ dimensions?

Plissard et al. (2013)
2-terminal junctions:
simplest case: Josephson energy
\[ E = -E_J \cos \phi \quad \rightarrow \quad I_J = \frac{2e \partial E}{\hbar \partial \phi} = I_c \sin \phi \]
in general:
Andreev bound states (ABS)
\[ E = -\sum_i E_A^{(i)}(\phi) \quad (+ \text{continuum}) \]

Delta \( e^{i\phi} \)

\( \Delta \)

discrete Andreev spectrum
in a junction with few channels

Pillet
et al.
(2013)
The analogy

2-terminal junctions:
simplest case: Josephson energy
\[ E = -E_J \cos \phi \quad \rightarrow \quad I_J = \frac{2e}{\hbar} \frac{\partial E}{\partial \phi} = I_c \sin \phi \]
in general:
Andreev bound states (ABS)
\[ E = -\sum_{i} E_A^{(i)}(\phi) \]
\[ \rightarrow \quad n\text{-terminal junctions:} \quad E_A^{(i)}(\phi_1, \phi_2, ..., \phi_{n-1}) \]
ABS energy = periodic fct of \( n - 1 \) phase differences

analogy:

\begin{align*}
\text{n-terminal junction} & \rightarrow \quad d = n - 1 \text{ dimensional material} \\
\text{Andreev spectrum} & \rightarrow \quad \text{band structure} \\
\text{phase differences} & \rightarrow \quad \text{quasi-momenta} \quad k_x, k_y, k_z, ... \\
\end{align*}

Topology: more information in the wavefunctions than in the spectrum!
Main result

topologically-protected Weyl singularities in the ABS spectrum of junctions with $n \geq 4$ terminals
Main result

topologically-protected Weyl singularities in the ABS spectrum of junctions with \( n \geq 4 \) terminals

manifestations:
quantized transconductance between 2 voltage-biased terminals
Outline

• Weyl singularities
• Andreev bound state (ABS) spectrum of multi-terminal junctions
• Quantized transconductance
• Beyond the adiabatic regime
• Conclusion
Weyl singularities

- topologically protected zero-energy states

3D Weyl Hamiltonian: \[ H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j \]

where \( \sigma_i \) 2 x 2 Pauli matrices:

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]
Weyl singularities

- topologically protected zero-energy states

3D Weyl Hamiltonian: \( H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j \)

Weyl points carry a topological charge:

- Weyl points are monopoles of Berry curvature

\[
\chi = \frac{1}{2\pi} \oint dS(k) \cdot B(k) = -\nabla \times \mathcal{S} \langle \psi(k) | \nabla_k | \psi(k) \rangle
\]

\[
= \text{sign } \det \{ v_{ij} \} = \pm 1
\]

Weyl semimetals have been discovered recently (TaAs …)
Weyl singularities

- topologically protected zero-energy states

$3D$ Weyl Hamiltonian:  \( H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j \)

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\[
\chi = \frac{1}{2\pi} \oint dS(k) \cdot B(k) = \text{sign} \det[\{v_{ij}\}] = \pm 1
\]

vs Chern number:  \( C = \frac{1}{2\pi} \int_{2D \text{ BZ}} (dk) B_z(k) \in \mathbb{Z} \)
Weyl singularities

- topologically protected zero-energy states

3D Weyl Hamiltonian: \[ H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j \]

Weyl points carry a topological charge:

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\[ C_1 = C_0 + \chi_1 \]
\[ C_2 = C_1 + \chi_2 = C_0 + \chi_1 + \chi_2 \]

→ Chern number changes when crossing a Weyl Point:
Weyl singularities

- topologically protected zero-energy states

3D Weyl Hamiltonian: \( H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j \)

Weyl points carry a topological charge:

- Weyl points are monopoles of Berry curvature

\[ C = \frac{1}{2\pi} \oint_{2D BZ} (dk) B_z(k) \]

non-zero Chern number
→ edge states
→ quantum Hall effect

Thouless et al. (1982)
charge transfer between a “non-superconductor” and a superconductor at $\varepsilon < \Delta$:

- no quasi-particle states
- only transfer of Cooper pairs

Andreev reflection

Andreev 1964
ABS spectrum of multi-terminal junctions

generalization of the scattering problem:
ABS spectrum of multi-terminal junctions

• ABS spectrum determined by $|\psi\rangle = S_N S_A |\psi\rangle$
  
  Beenakker (1991)

• normal scattering in the contact:
  
  scattering matrix $S_N$

• Andreev reflection:
  
  scattering matrix $S_A (\phi_1, \ldots, \phi_{n-1}; E)$

• particle-hole symmetry: states come in pairs at energies $\pm E$

• Weyl singularities: doubly degenerate zero-energy states at $\Phi^{(0)}$
Weyl-Hamiltonian

- Weyl singularities: doubly degenerate zero-energy states at $\Phi^{(0)}$

$$E_{\text{ABS}}$$

- in the vicinity of the zero-energy solution at $\Phi^{(0)}$:
  effective low-energy Weyl Hamiltonian
  in the subspace of the 2 orthogonal eigenstates:

$$H_W = \sum_{\alpha, i} M_{\alpha i} \delta \phi_\alpha \tau_i$$

where $\tau_i$ 2x2 Pauli matrices
Weyl-Hamiltonian

- Weyl singularities: doubly degenerate zero-energy states at $\Phi^{(0)}$

$$H_W = \sum_{\alpha, i} M_{\alpha i} \delta \phi_\alpha \tau_i$$

- topological charge of the Weyl point in a 3D subspace:
  $$\chi = \text{sign det } [\{M_{\alpha i}\}]$$

- total topological charge = 0

- time-reversal symmetry: Weyl point at $\Phi^{(0)}$
  $\rightarrow$ Weyl point with the same topological charge at $-\Phi^{(0)}$

- Weyl points come in multiples of 4
example: 4-terminal junction
Chern number:

\[ C^{12} = \frac{\varphi_1}{2\pi} \int_{-\pi}^{\pi} d\varphi_1 \, d\varphi_2 \, B^{12} \]
4-terminal junctions: Occurrence of Weyl points

- 4 single-channel terminals:
  - ~ 5% of random scattering matrices possess Weyl points
  - simple toy models $X$

- 4 multi-channel terminals:
  example: $N_\alpha = 12, 11, 10, 9$
Consequences of Weyl singularities: The current

- current operator: \( \hat{I}_\alpha = 2e \frac{\partial \hat{H}}{\partial \phi_\alpha} \)

- use instantaneous eigenbasis \( E_{A\nu}(t) |\psi_\nu(t)\rangle = \hat{H}(t) |\psi_\nu(t)\rangle \) to compute expectation value for time-dependent phases:

  contribution of ABS
  \[
  I_{\alpha\nu}(t) = \frac{2e}{\hbar} \frac{\partial E_{A\nu}(t)}{\partial \phi_\alpha} - 4e \sum_\beta \dot{\phi}_\beta \Im \langle \frac{\partial \psi_\nu}{\partial \phi_\alpha} | \frac{\partial \psi_\nu}{\partial \phi_\beta} \rangle
  \]

adiabatic supercurrent \( I_{\alpha\nu}^0(t) \)

first correction: \( \delta I_{\alpha\nu}(t) = -2e \sum_\beta \dot{\phi}_\beta B^\alpha_\nu \)

with \( B^\alpha_\nu = 2 \Im \langle \frac{\partial \psi_\nu}{\partial \phi_\alpha} | \frac{\partial \psi_\nu}{\partial \phi_\beta} \rangle \) Berry curvature
Quantized transconductance

- **total current:**
  \[
  I_\alpha(t) = \sum_{k,\sigma} I_{\alpha k}(t) \left( n_{k\sigma} - \frac{1}{2} \right) = I_\alpha^0(t) - 2e \sum_{k,\sigma,\beta} \phi_\beta B_{\alpha \beta}^k \left( n_{k\sigma} - \frac{1}{2} \right)
  \]

- **consider 2 voltage-biased leads:** \( \phi_\alpha = 2e V_\alpha t \)
Quantized transconductance

- total current:

\[ I_\alpha(t) = \sum_{k,\sigma} I_{\alpha k}(t) \left( n_{k\sigma} - \frac{1}{2} \right) = I_{\alpha}^0(t) - 2e \sum_{k,\sigma,\beta} \dot{\phi}_\beta B_{k}^{\alpha\beta} \left( n_{k\sigma} - \frac{1}{2} \right) \]

- consider 2 voltage-biased leads: \( \phi_\alpha = 2eV_\alpha t \)

→ phase sweeps 2D “Brillouin zone”

\( (V_{\alpha,\beta} \land \Delta \text{ incommensurate}) \)
Quantized transconductance

- total current:

\[ I_\alpha(t) = \sum_{k,\sigma} I_{\alpha k}(t) \left( n_{k\sigma} - \frac{1}{2} \right) = I_\alpha^0(t) - 2e \sum_{k,\sigma,\beta} \dot{\phi}_\beta B_{k\beta}^{\alpha} \left( n_{k\sigma} - \frac{1}{2} \right) \]

- consider 2 voltage-biased leads: \( \phi_\alpha = 2eV_\alpha t \)

→ phase sweeps 2D “Brillouin zone”

→ time-averaged current in the ground state \((n_{k\sigma} = 0)\):

\[ \overline{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad G^{\alpha\beta} = -\frac{2e^2}{\pi \hbar} C^{\alpha\beta} \]

where \( C^{\alpha\beta} = -\frac{1}{2\pi} \sum_k \int_{-\pi}^{\pi} d\phi_\alpha \, d\phi_\beta \, B_{k\beta}^{\alpha} \quad \text{integer} \)

= Chern number
Multiterminal junctions as topological matter

experimental manifestation: quantized transconductance

\[ \overline{I}_\alpha = G^{\alpha \beta} V_\beta \quad \text{with} \quad G^{\alpha \beta} = \frac{4e^2}{h} C^{\alpha \beta} \]

Chern number

MAIN RESULT
Multiterminal junctions as topological matter

experimental manifestation: quantized transconductance

\[ \bar{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad G^{\alpha\beta} = \frac{4e^2}{h} C^{\alpha\beta} \]

Chern number

\[ \bar{I}_\alpha = -\frac{4e^2}{h} V_\beta \sum_k C^{\alpha\beta}_k (n_{k\uparrow} + n_{k\downarrow} - 1) \]

ground state: \( n_{k\sigma} = 0 \)

→ poisoning? (Landau-Zener …)
Beyond the adiabatic regime

- Landau-Zener processes:
Beyond the adiabatic regime

- Landau-Zener processes:

\[ \phi_0 = 2.21, \quad \phi = 0.45 \]

- Inelastic relaxation necessary to quickly recover equilibrium occupations

- Empty states

- Occupied states
Beyond the adiabatic regime

- multiple Andreev reflections

→ compute the currents using (Floquet) scattering theory

- account for inelastic relaxation with a Dynes parameter $\Gamma$ in the leads
Beyond the adiabatic regime

- multiple Andreev reflections

\[ \rightarrow \text{compute the currents using (Floquet) scattering theory} \]

specific scattering matrix
with Weyl points at \( \pm(1.7, -1.9, -2.8, 0) \) and \( \pm(2.7, -1.8, 1.0, 0) \)

- choose \( \begin{align*}
\phi_1 &= 2en_1Vt + \chi \\
\phi_2 &= 2en_2Vt
\end{align*} \)

- commensurate voltages \( \rightarrow \) average over \( \chi \)

- obtain conductances from 2 sets of voltages: \( (n_1, n_2) = (1, 3) \) and \( (2, 3) \)

\[
\begin{pmatrix}
I_1 \\
I_2
\end{pmatrix}
= \begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix}
\]
Beyond the adiabatic regime

currents as a fct of \( V \) at fixed \( \phi_0 \) (\( \Gamma = 0.002\Delta \)):

\[(n_1, n_2) = (1, 3)\]

\( \phi_0 = 2.21 \) (topological)

\[(n_1, n_2) = (2, 3)\]

\( \phi_0 = 0 \) (trivial)
Beyond the adiabatic regime

conductances as a function of $V$ at fixed $\phi_0 = 2.21$ ($\Gamma = 0.002\Delta$):

- Multiple Andreev reflections
- Normal state conductances ($G_{12} = G_{21}$)
- Quantized transconductances

\[ \log_{10} \left( eV/\Delta \right) \]
Beyond the adiabatic regime

conductances as a fct of $V$ at fixed $\phi_0 = 2.21$ ($\Gamma = 0.002\Delta$):

quantization requires fixed parity:

$$\frac{1}{\tau_{LZ}} = eV e^{-E_A/eV} < \Gamma$$

$$\rightarrow eV < eV_* \sim \frac{E_A}{\log(E_A/\Gamma)}$$

where

$$E_A = \min_{\phi_1, \phi_2} E_A(\phi_0, \phi_1, \phi_2)$$
conductances as a fct of $V$ at fixed $\phi_0 = 2.21$ ($\Gamma = 0.002\Delta$):

for comparison: $\phi_0 = 0$

quantized transconductances
conductances as a function of $\phi_0$ at fixed $V = 0.0003\Delta/e$:

Beyond the adiabatic regime

large dissipation close to Weyl points

topological regime

topological regime
3-terminal junctions

- only 2 independent phases
- add magnetic flux through the junctions area → break time-reversal symmetry

\[ t e^{i 2\pi \Phi / (3\Phi_0)} \equiv e^{i s / 3} \]

example: \( U = 0.1, t = 1 \)
minimal gap as a function of \( s \) → 4 Weyl points
3-terminal junctions

- only 2 independent phases
- add magnetic flux through the junctions area
  → break time-reversal symmetry

Preliminary results:

\[ V = 0.01\Delta, \Gamma = 0.01\Delta \]

(using only 1 voltage \( V_1 = \frac{\hbar}{2} \) & averaging over \( \phi_2 \))
3-terminal junctions

- only 2 independent phases
- add magnetic flux through the junctions area
  \( \rightarrow \) break time-reversal symmetry

preliminary results:

\[ V = 0.01\Delta, \Gamma = 0.01\Delta \]

(using only 1 voltage \( V_1 = \$V \)

& averaging over \( \phi_2 \))

\[ C_{12}^{\text{ABS}} \]

= Chern number of the Andreev bound state

\[ -1 \rightarrow \text{continuum contributes to the topological properties of the junction!} \]
Conclusion

- Weyl singularities in ABS spectrum of multi-terminal Josephson junctions without any fine-tuning
- Superconducting phases = quasi-momenta
- Transconductance between 2 voltage-biased terminals probes Chern number

\[ I_\alpha = G^{\alpha \beta} V_\beta \quad \text{with} \quad G^{\alpha \beta} = - \frac{2e^2}{\pi \hbar} C^{\alpha \beta} \]

Multi-terminal Josephson junction = topological material

R.-P. Riwar et al., Nat. Commun. 7, 11167 (2016); E. Eriksson et al., PRB 95, 075417 (2017); JSM & M. Houzet, PRL ... (2017)
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InSb nanocrosses?
Plissard et al. (2013)
Outlook

• specific realizations ?

• higher-dimensional “materials” ?

• more complex topologies ?

• edges ?
Thank you!
Conclusion

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• superconducting phases = quasi-momenta
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