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Strong non-equilibrium effects in spin-torque systems

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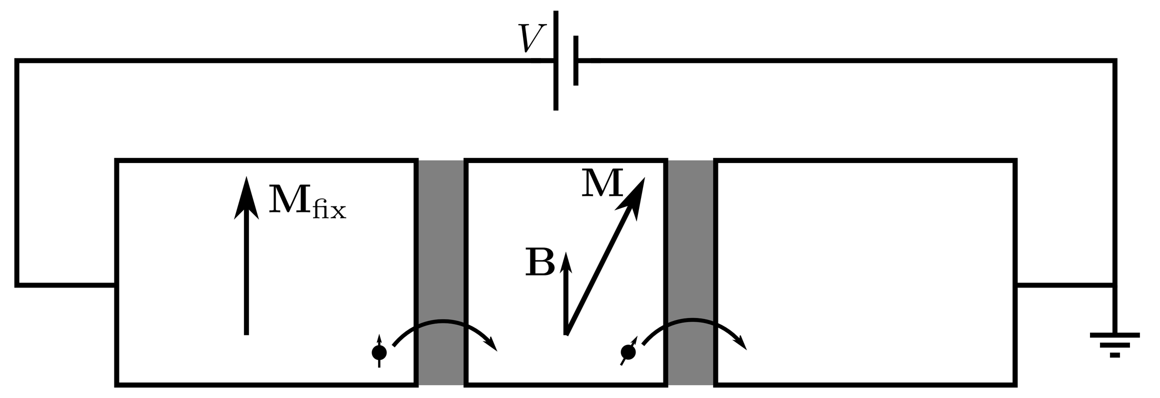
Introduction

We consider a mono-domain ferromagnetic nano-particle in the Stoner regime, which is being driven by spin-transfer torque (STT). We concentrate on the simplest case of a unique stable direction of magnetization at equilibrium. Such systems being driven by STT are known to exhibit persistent precession of magnetization as originally predicted in Refs. [1, 2] and experimentally observed in Ref. [3]. This situation is analyzed in Ref. [4] under the assumption of strong internal spin- and energy-relaxation, which induces an equilibrium distribution on the nano-particle. In contrast, we investigate the limit of vanishing internal energy and spin relaxation, in which the electron distribution function of the nano-particle is driven far from equilibrium. We consider the effect of this non-equilibrium state of the electrons on the semiclassical dynamics of magnetization and voltage. To do so, we derive an SU(2) [magnetization] $\otimes$ U(1) [voltage] Ambegaokar-Eckern-Schön (AES) effective action. In particular, we observe the absence of the regime of persistent precession.

Magnetic quantum dot coupled to two leads



universal Hamiltonian for the dot [5]:

$$H_{dot} = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - \mathbf{BS} - J\mathbf{S}^2 + E_c(N - N_0)^2$$

$$N = \sum_{\alpha,\sigma} a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma}$$
$$\mathbf{S} = \tfrac{1}{2} \sum_{\alpha,\sigma_1,\sigma_2} a^\dagger_{\alpha,\sigma_1} \boldsymbol{\sigma}_{\sigma_1,\sigma_2} a_{\alpha,\sigma_2}$$

lead- and tunneling-Hamiltonian

$$H_{left} = \sum_{\gamma\sigma} (\epsilon_\gamma - M_{\text{fix}}\tfrac{\sigma}{2} + V) c^\dagger_{\gamma,\sigma} c_{\gamma,\sigma} + \sum_{\gamma\alpha\sigma} \left(t_{l,\alpha,\gamma}\, a^\dagger_{\alpha,\sigma} c_{\gamma,\sigma} + h.c. \right)$$

$$H_{right} = \sum_{\tilde\gamma} \epsilon_{\tilde\gamma}\, c^\dagger_{\tilde\gamma,\sigma} c_{\tilde\gamma,\sigma} + \sum_{\tilde\gamma\alpha\sigma} \left(t_{r,\alpha,\tilde\gamma}\, a^\dagger_{\alpha,\sigma} c_{\tilde\gamma,\sigma} + h.c. \right)$$

lead distribution functions

$$n_l(\epsilon) = \frac{1}{e^{\beta(\epsilon - (\mu + V))} + 1}$$
$$n_r(\epsilon) = \frac{1}{e^{\beta(\epsilon - \mu)} + 1}$$

Action and transition to rotating frame

Decoupling of the interaction and integration over fermionic fields
$\longrightarrow$ **action for Hubbard-Stratonovich fields:**

$$i\mathcal{S}_{\mathbf{M},V_d} = \operatorname{tr}\ln\left[-i\left(G^{-1}_{d,0} + \mathbf{M}\frac{\boldsymbol{\sigma}}{2} - V_d - \Sigma\right)\right] - i\oint_K dt\, \frac{(\mathbf{M} - \mathbf{B})^2}{4J} + i\oint_K dt \left(\frac{V_d^2}{4E_c} + V_d N_0\right)$$

Rotation [SU(2)]: $\mathbf{n}\boldsymbol{\sigma} = R\sigma_z R^\dagger$

U(1) gauge-trafo: $e^{-i\psi}$

Euler angle rep.: $R = e^{-i\frac{\phi}{2}\sigma_z} e^{-i\frac{\theta}{2}\sigma_y} e^{i\frac{\phi-\chi}{2}\sigma_z}$

Berry Phase: $Q = -iR^\dagger \dot R = [\dot\phi(1 - \cos\theta) - \dot\chi]\frac{\sigma_z}{2} + Q_{LZ}$

$$i\mathcal{S}_{\mathbf{M},V_d} = \operatorname{tr}\ln\Big[-i\Big(\underbrace{G^{-1}_{d,0} + \frac{M_0}{2}\sigma_z}_{G^{-1}_{d,z}} - Q + (\dot\psi - V_d) - U^\dagger\Sigma U\Big)\Big] - i\oint_K dt\, \frac{(\mathbf{M} - \mathbf{B})^2}{4J} + i\oint_K dt \left(\frac{V_d^2}{4E_c} + V_d N_0\right)$$

Gauge fixing [6]: choose ψ and χ such that the effect of $(\dot\psi - V_d)$ and Q is minimized, the boundary conditions give rise to non-gaugeable zero-modes [7]

Expansion around stationary trajectories

Split the U-rotated self-energy term $U^\dagger\Sigma U$ into a saddle-point part and the deviation

$$i\mathcal{S}_{\mathbf{M},V_d} = \operatorname{tr}\ln\left[-i\left(\tilde G^{-1}_{d,z} - \tilde U^\dagger_{sp}\tilde\Sigma\tilde U_{sp} - \frac{Q_q}{2}\tau_0 + \frac{V^q_{d0}}{2}\tau_0 - \left(\tilde U^\dagger\tilde\Sigma\tilde U - \tilde U^\dagger_{sp}\tilde\Sigma\tilde U_{sp}\right)\right)\right] + \\ + i\int dt\, \frac{\mathbf{BM}_q}{2J} + i\int dt \left(\frac{V^c_d V^q_d}{2E_c} + V^q_d N_0\right)$$

Trial saddle point $\tilde U_{sp} \to U_0$:
persistent precession
$\theta \to \theta_0,\ \dot\phi \to -B_0,\ V_d \to V^0_d$

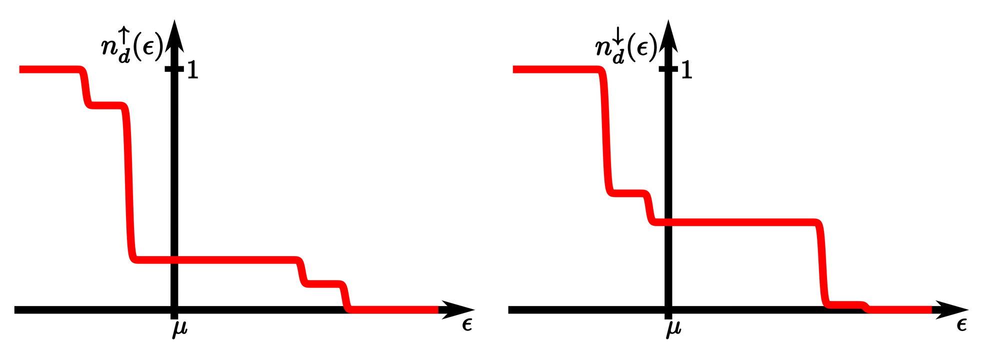
$\Longrightarrow$ **Kinetic equation:**

$$\left(\tilde G^{-1}_{d,z} - U^\dagger_0\tilde\Sigma U_0\right)\begin{pmatrix} G^K_d & G^R_d \\ G^A_d & 0 \end{pmatrix} = \mathbf{1}$$

$\Longrightarrow$ **Green's functions:**

$$G^{R/A}_d = \frac{1}{\epsilon - \epsilon_\alpha - (M_0/2)\sigma \pm i\Gamma_\sigma(\theta_0)}$$
$$G^K_d = G^R_d F_d - F_d G^A_d$$

non-equilibrium distribution functions



$$F^\sigma_d(\epsilon) = \frac{1}{\Gamma_\sigma(\theta_0)}\Big[\cos^2\frac{\theta_0}{2}\Gamma^\sigma_l F(\epsilon - \sigma B_- + V^0_d - V) \\ + \sin^2\frac{\theta_0}{2}\Gamma^{\bar\sigma}_l F(\epsilon - \bar\sigma B_+ + V^0_d - V) \\ + \cos^2\frac{\theta_0}{2}\Gamma_r F(\epsilon - \sigma B_- + V^0_d) \\ + \sin^2\frac{\theta_0}{2}\Gamma_r F(\epsilon - \bar\sigma B_+ + V^0_d)\Big]$$

expansion in $\tilde U^\dagger\tilde\Sigma\tilde U - \tilde U^\dagger_{sp}\tilde\Sigma\tilde U_{sp}$:

$$iS_{AES} = -\operatorname{tr}[\tilde G_d \tilde U^\dagger\tilde\Sigma\tilde U]$$

$\longrightarrow$ dissipation and currents

expansion in Q_q:

$$iS_{WZNW} = -\tfrac{1}{2}\operatorname{tr}[G^K_d Q_q]$$

$\longrightarrow$ Berry-phase

Equations of motion and zero-mode equation

Landau-Lifshitz-Gilbert equation:

$$\frac{d\mathbf{M}}{dt} = -\mathbf{B} \times \mathbf{M} - \alpha(\theta)\frac{\mathbf{M}}{M_0} \times \frac{d\mathbf{M}}{dt} + \frac{1}{S}\frac{\mathbf{M}}{M_0} \times (\mathbf{I}_s \times \mathbf{M})$$

Kirchhoff's law:

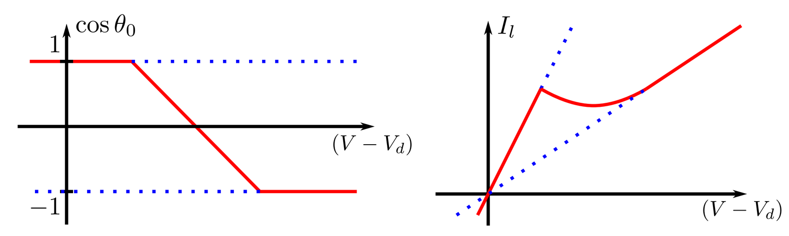
$$C\dot V_d = I_l - I_r$$

Gilbert dissipation coefficient:
$\alpha(\theta) = \frac{1}{S}(\tilde g_l(\theta) + \tilde g_r)$

Spin-torque current:
$I_s = g_s(V - V_d) + \Delta I_s(\theta_0)$

Charge currents:
$I_l = 4g_l(V - V_d) - g_s \sin^2\theta\,\dot\phi + \Delta I_l(\theta, \theta_0)$
$I_r = 4g_r V_d + \Delta I_r(\theta, \theta_0)$

disregarding the right lead and non-eq. effects we obtain known results [4, 8]



$\Delta I_s(\theta_0)$, $\Delta I_l(\theta, \theta_0)$, $\Delta I_r(\theta, \theta_0)$ are significant current-modifications due to the strong non-equilibrium character of the distribution function

zero-mode equation:
$CV^0_d = -\frac{i}{2}\operatorname{tr}\left[G^K_d\right] - N_0$

V^0_d is determined by capacity and excessive charges

$\longleftarrow$

$\Delta I_l(\theta, \theta_0), \Delta I_r(\theta, \theta_0)$ eliminate V^0_d from the equations of motion
$\Downarrow$
calls for a separate equation that determines V^0_d

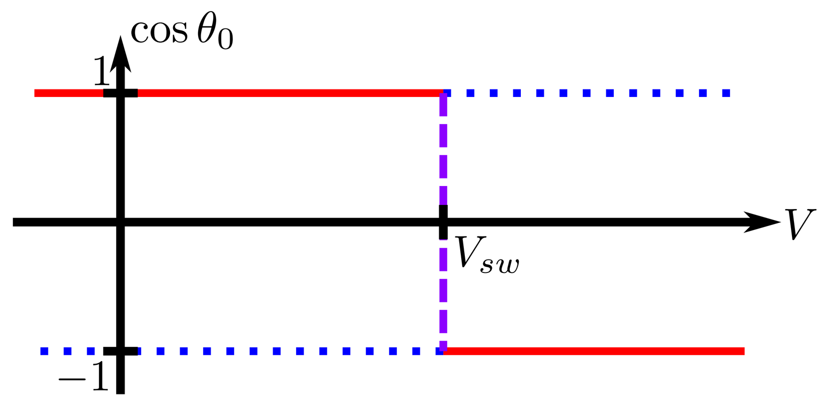
Stationary solutions and stability

LLG-equation in terms of Euler angles:

$$\sin\theta\,\dot\phi = -\sin\theta B - \alpha(\theta)\,\dot\theta$$
$$\dot\theta = \sin\theta\left[\alpha(\theta)\dot\phi - I_s/S\right]$$

"stationary" solutions: $B_0 \simeq B$

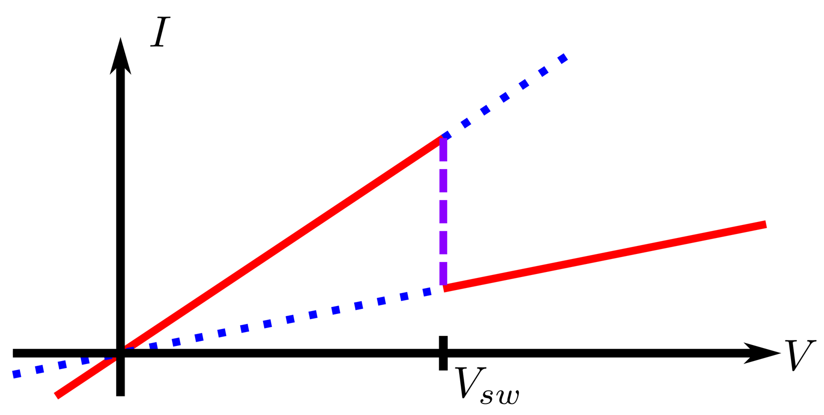
$$0 = \sin\theta_0\left[\alpha(\theta_0)B + \frac{g_s}{S}(V - V^0_d) + \frac{\Delta I_s(\theta_0)}{S}\right]$$



The non-equilibrium current $\Delta I_s(\theta_0)$ shifts all the θ_0-dependence to a prefactor

$$0 = \frac{\sin\theta_0\,\alpha(\theta_0)}{\Gamma^2_\Sigma - \cos^2\theta_0\Gamma^2_\Delta}\left[(\Gamma^2_\Sigma - \Gamma^2_\Delta)B + 2\Gamma_\Delta\Gamma_r V\right]$$

$\rightarrow$ **Except for a critical voltage V_{sw}, only the poles, where $\sin\theta_0 = 0$, are possible stationary solutions.**



relaxation dynamics close to stationary solutions:

$$\delta\dot\theta = -\cos\theta_0\,\alpha(\theta_0)\left[B + \frac{2\Gamma_\Delta\Gamma_r}{\Gamma^2_\Sigma - \Gamma^2_\Delta}V\right]\delta\theta$$
$$C\,\delta\dot V_d = -4\,(g_l + g_r)\,\delta V_d$$

- $\delta\theta$ and δV_d are decoupled
- at V_{sw} the poles switch stability
- δV_d shows usual RC-relaxation

critical slowing down: When the voltage V approaches the switching voltage V_{sw}, we predict critical slowing down for the dynamics of $\delta\theta$. Analysis of the critical region requires special care, since long living deviations $\delta\theta$ could affect the distribution function.

Summary

We want to emphasize three key results:

- the **intimate link between spin-torque current** [1, 2] $g_s(V - V_d)$ **and pumping current** [8] $-g_s\sin^2\theta\,\dot\phi$ is demonstrated, as both effects are derived from a single AES-like action
- **state of persistent precession**, usually arising from a balance between dissipation and spin-torque [4], **disappears due to strong non-equilibrium modification of the electronic distribution function**
- **it is important to take both leads into account, even in the limit of very transparent right contact $\Gamma_r \gg \Gamma^\sigma_l$** — although the height of the left-contact step tends to zero, the integral weight stays important when the spin-torque becomes large enough to compete with dissipation

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