Momentum space signatures of Anderson localization of spin waves

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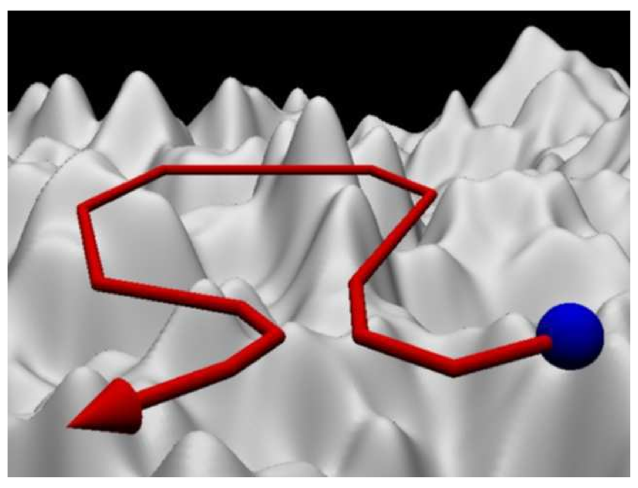
Introduction

- Wave propagation in random media: interference leads to deviations from classical diffusion (e. g. localization phenomena)
- Classical transport: phase information lost at scattering events
- Coherent transport: multiscattering interference effects, e. g. Anderson Localization (AL) [1]
- AL is ubiquitous for wave transport (experimentally proven for electron gases, ultrasound, microwaves, light waves, matter waves) and should also exist for spin waves.



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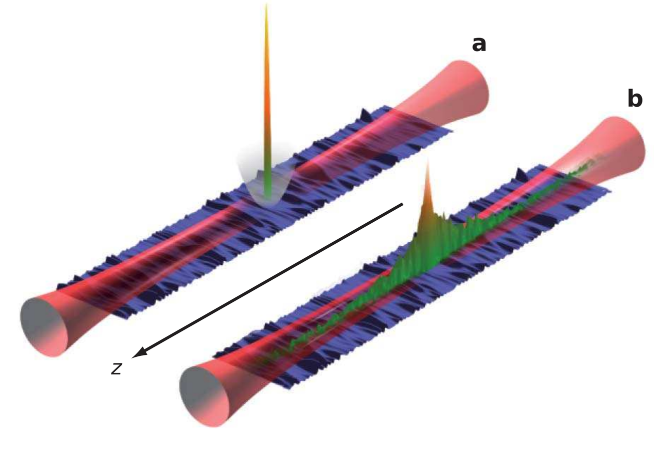
Localization phenomena

Strong localization (also called AL):

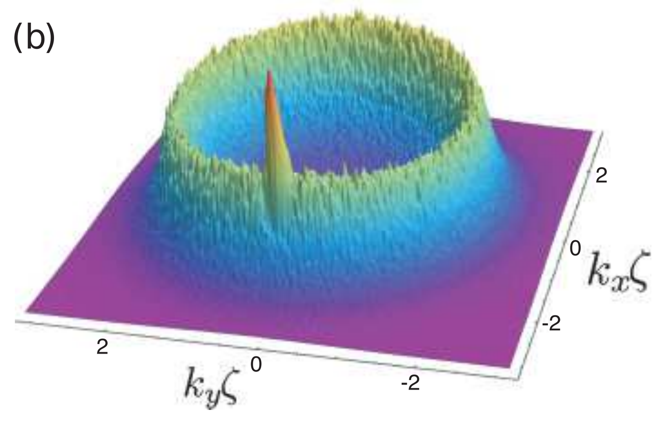
- Complete suppression of wave transport, diffusion constant $D = 0$
- Exponential decay of wave intensity with distance on length scale of localization length ξ_{loc}: $I \propto \exp\left(-|\mathbf{x}-\mathbf{x}_0|/4\xi_{\text{loc}}\right)$
- Further possible features, e. g. coherent forward scattering (CFS) [3, 4]



J. Billy et al. *Nature* **453**, 891–894 (2008)

Weak localization:

- Regime where transport is not yet completely suppressed, but there are deviations from classical behavior (e. g. coherent backscattering (CBS)).
- Precursor for AL



N. Cherroret et al. *Phys. Rev. A* **85**, 011604 (2012)

Modeling spin waves

- Atomistic spin model with Heisenberg Hamiltonian

$$H = -\frac{1}{2}\sum_{n,m}\left[J\mathbf{S}^n\cdot\mathbf{S}^{nm} + \mathbf{D}^m\cdot\left(\mathbf{S}^n\times\mathbf{S}^{nm}\right)\right] - d_z\sum_n\left(S_z^n\right)^2 - \mu_S\sum_n\mathbf{B}^n\mathbf{S}^n$$

- Time evolution given by Landau-Lifshitz equation (LL)

$$\frac{\partial\mathbf{S}^l}{\partial t} = -\frac{\gamma}{\mu_S}\mathbf{S}^l\times\mathbf{H}^l$$

with effective field: $\mathbf{H}^l = -\frac{\partial H}{\partial\mathbf{S}^l}$.

Defects:

- For AL a disordered potential is needed. Possible defect models are:
  - magnetic field: apply constant $\mathbf{B}$ at random lattice sites
  - zero spins: set spin $\mathbf{S} = 0$ at random lattice sites.

Numerical treatment of model:

- LL solved numerically using common ODE methods (e. g. Heun or implicit Adams) [2].

Dispersion relation of spin waves

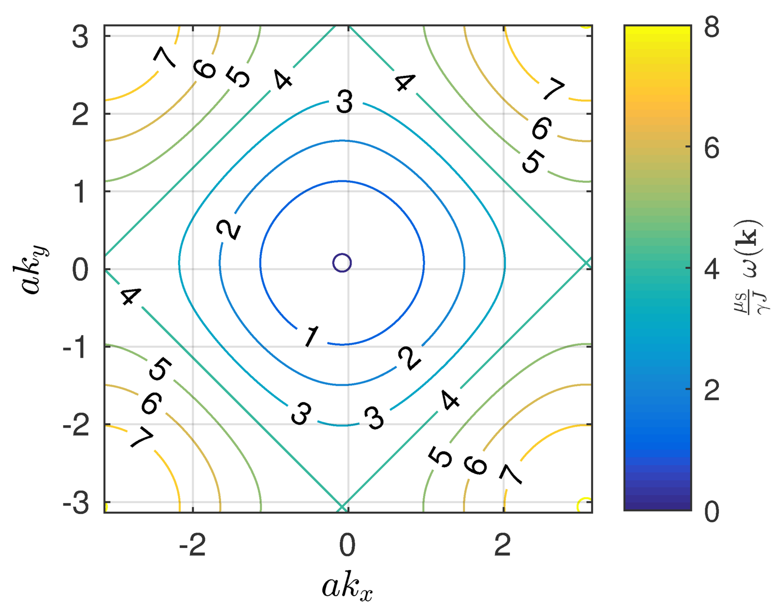
- Localized spin wave at position $\mathbf{r}_0$ with wave vector $\mathbf{k}_0$ and width σ is given by

$$\mathcal{S}^l := S_x^l - i\cdot S_y^l = A\cdot\exp\left(-i\mathbf{k}_0\cdot\mathbf{r}\right)\cdot\exp\left(-(\mathbf{r}-\mathbf{r}_0)^2/2\sigma^2\right).$$

- Dispersion of spin waves:

$$\omega(\mathbf{k}) = \frac{\gamma}{\mu_S}\left[J\sum_m\left(1-\cos\left(\mathbf{k}\cdot\mathbf{a}^m\right) + D_z^m\sin\left(\mathbf{k}\cdot\mathbf{a}^m\right)\right) + 2d_z\right],$$

$\mathbf{a}^m$ lattice vectors



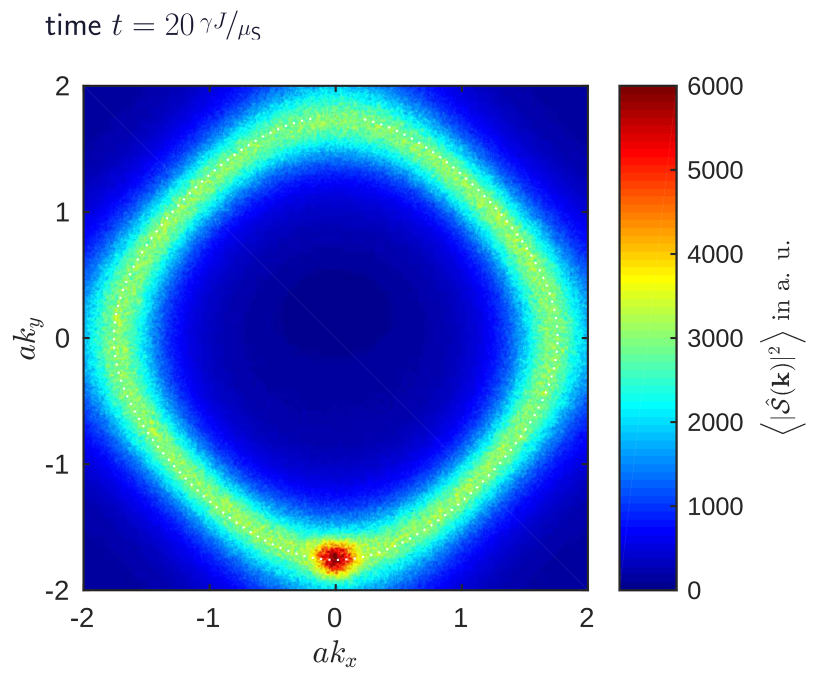
Coherent backscattering in 2D ($\mathbf{D}^m = 0$)

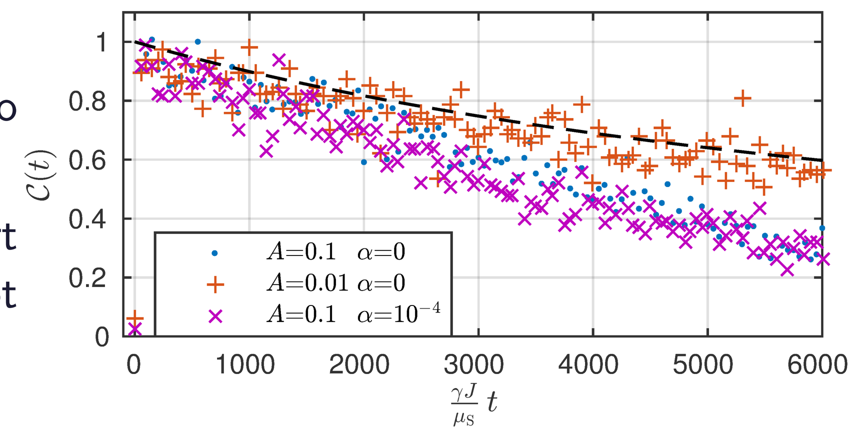
- The elastic scattering leads to incoherent (diffusive) and coherent parts of intensity: $I(\mathbf{k}) = I_{\text{bg}}(\mathbf{k}) + I_{\text{CBS}}(\mathbf{k})$.
- CBS contrast $\mathcal{C} = I_{\text{bg}}(-\mathbf{k}_0)/I_{\text{CBS}}(-\mathbf{k}_0)$ is unity for pure plane-wave.
- Theoretical prediction for time evolution of CBS contrast of finite width (linear waves) [6]:

$$\mathcal{C}(t) = \left(1 + \left|\mathbf{v}(\mathbf{k}_0)\right|^2\tau_{\text{tr}}\Delta k^2\cdot t\right)^{-1}$$

$\mathbf{v}(\mathbf{k})$: group velocity, τ_{tr}: transport time

- Nonlinearities (larger amplitude A) lead to faster decay of CBS contrast.
- Finite Gilbert damping α (using LL-Gilbert equation) reduces overall intensity I, but not contrast. (All modes damped uniformly.)





CBS in 2D with DMI

Dzyaloshinskii-Moriya interaction (DMI):

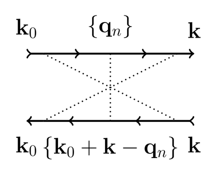
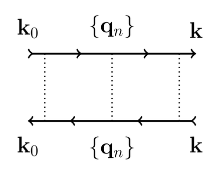
- weak DMI: $|\mathbf{D}^m| < 0.1J$, for ferromagnetic ground state
- breaks inversion symmetry $\omega(\mathbf{k}) = \omega(2\mathbf{K}-\mathbf{k}) \neq \omega(-\mathbf{k})$, $\mathbf{K}$: center of inversion

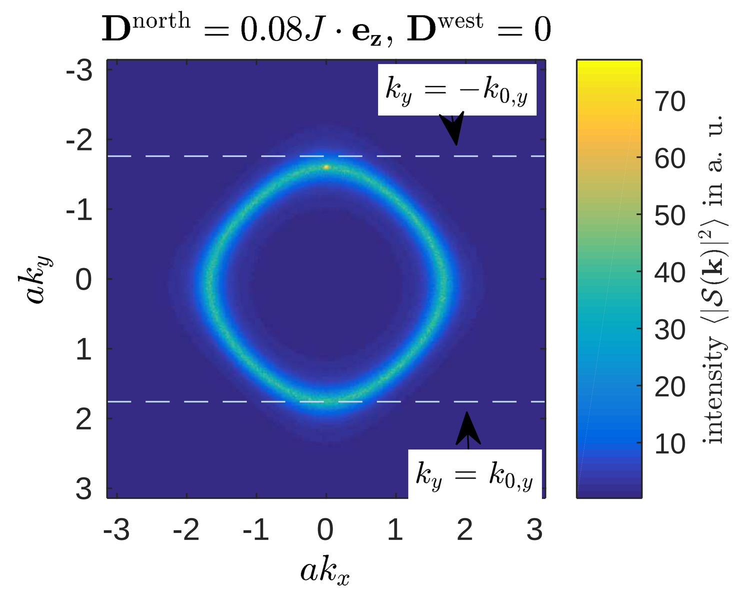
Effect on CBS:

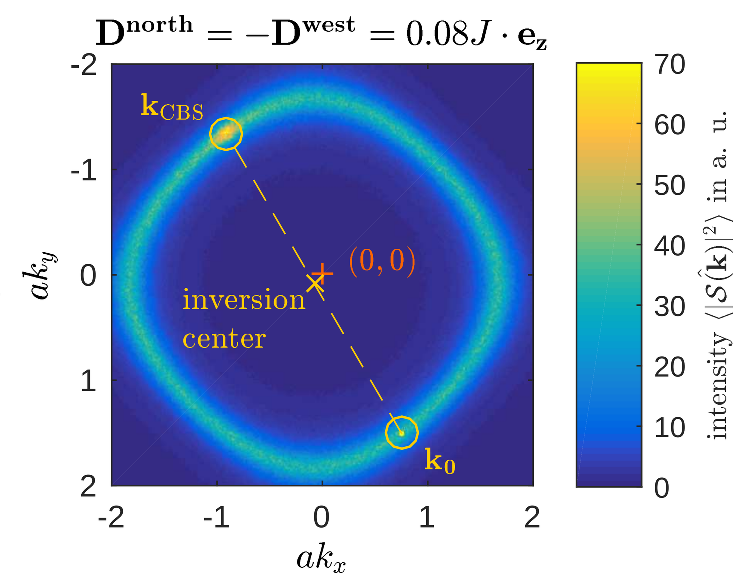
- CBS contrast not effected
- position of CBS peak: $\mathbf{k}_{\text{CBS}} = 2\mathbf{K} - \mathbf{k}_0$, especially $-\mathbf{k}_0 \nparallel \mathbf{k}_{\text{CBS}}$

Explanation:

- CBS contribution described by Cooperon diagram C, that is only equivalent to a ladder diagram L, if $\omega(\mathbf{k}_0 + \mathbf{k} - \mathbf{q}_n) = \omega(\mathbf{q}_n)$, which is achieved by $\mathbf{k}_0 + \mathbf{k} = 2\mathbf{K}$

$C(\mathbf{k}_0, \mathbf{k}) =$  $, L(\mathbf{k}_0, \mathbf{k}) =$ 



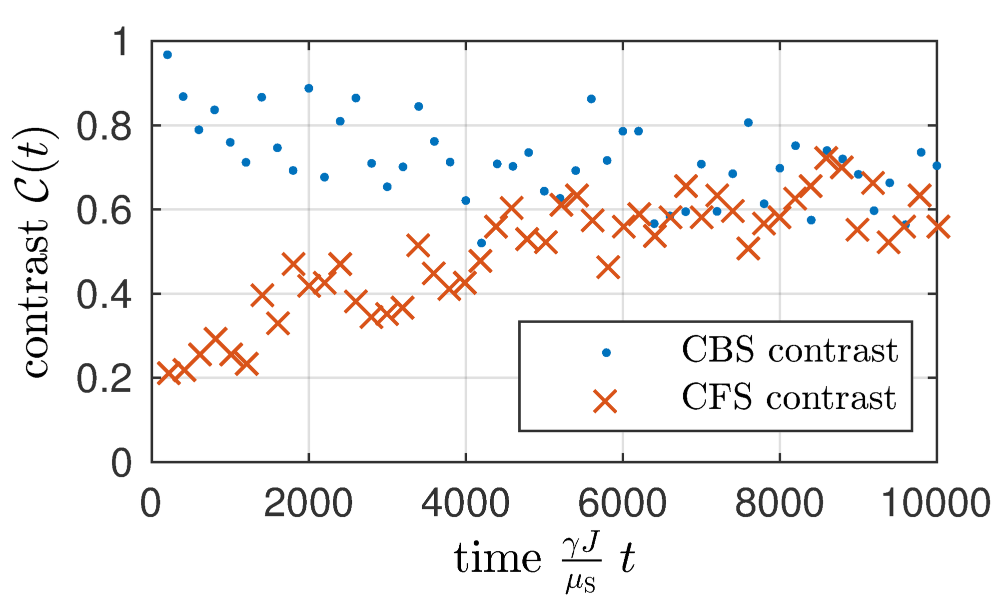


Coherent forward scattering in quasi 1D systems

- CFS: interference peak at $\mathbf{k}_0$ position, in contrast to CBS strong localization feature
- CFS peak arises on Heisenberg time scale [3] $\tau_{\text{H}} = h\,\langle\text{DOS}(E(\mathbf{k}_0))\rangle\,\xi_{\text{loc}}^d$, where $\langle\text{DOS}(E(\mathbf{k}_0))\rangle$ is the ensemble averaged density of states per unit volume
- CFS peak has same height and width as CBS peak in long time limit.

Studied here:

- quasi one-dimensional system ($N_x \gg N_y = 21$), $\mathbf{D}^m = 0$ → local scattering is two-dimensional, but much smaller ξ_{loc}
- localization length in x-direction: $\xi_{\text{loc}} = N_y\cdot\xi_{\text{loc}}^{1\text{D}}$
- estimation of $\tau_{\text{H}} \approx 2\pi\cdot\text{DOS}(\omega(\mathbf{k}_0))\cdot N_y a\xi_{\text{loc}} \approx 1\times 10^3\,\frac{\mu_S}{\gamma J}$
- estimate $\langle\text{DOS}(E(\mathbf{k}_0))\rangle$ with DOS of system without defects (only valid for weak disorder) → estimation for τ_{H} to small, but gives right order of magnitude



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