Probing non-integral spin magnons via spin current noise

Akashdeep Kamra and Wolfgang Belzig

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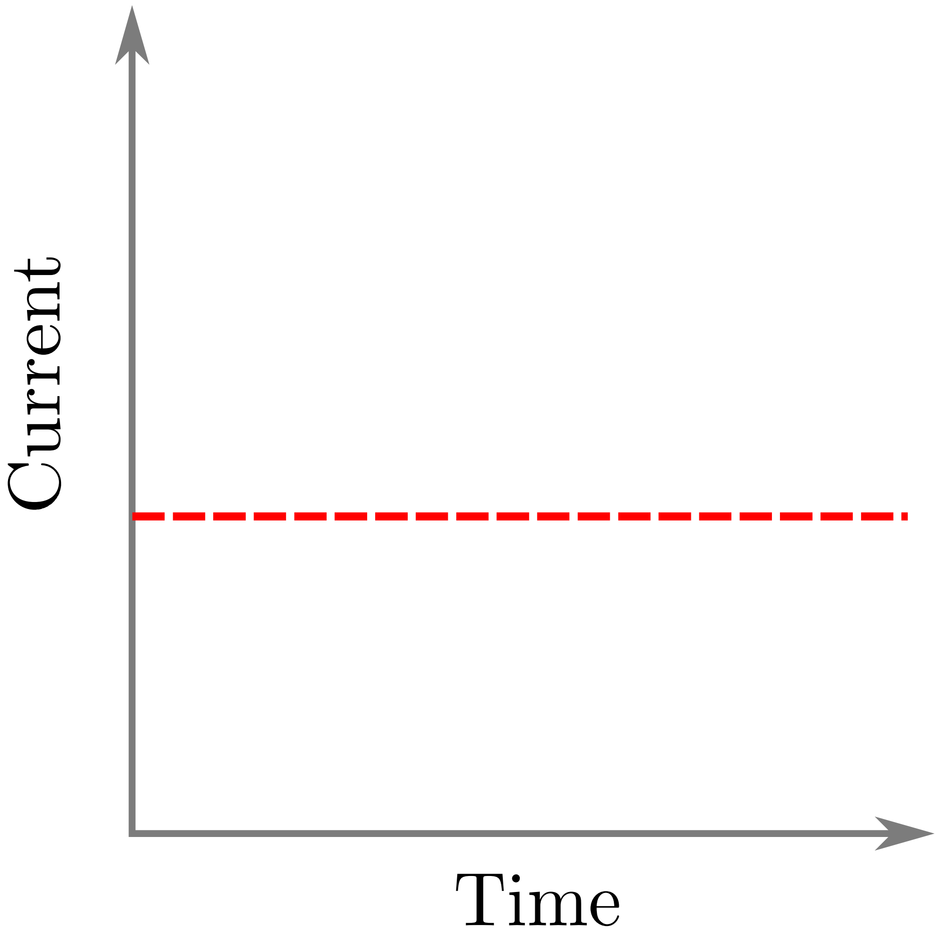
78457 Konstanz, Germany

Reference: Phys. Rev. Lett. 116, 146601 (2016).

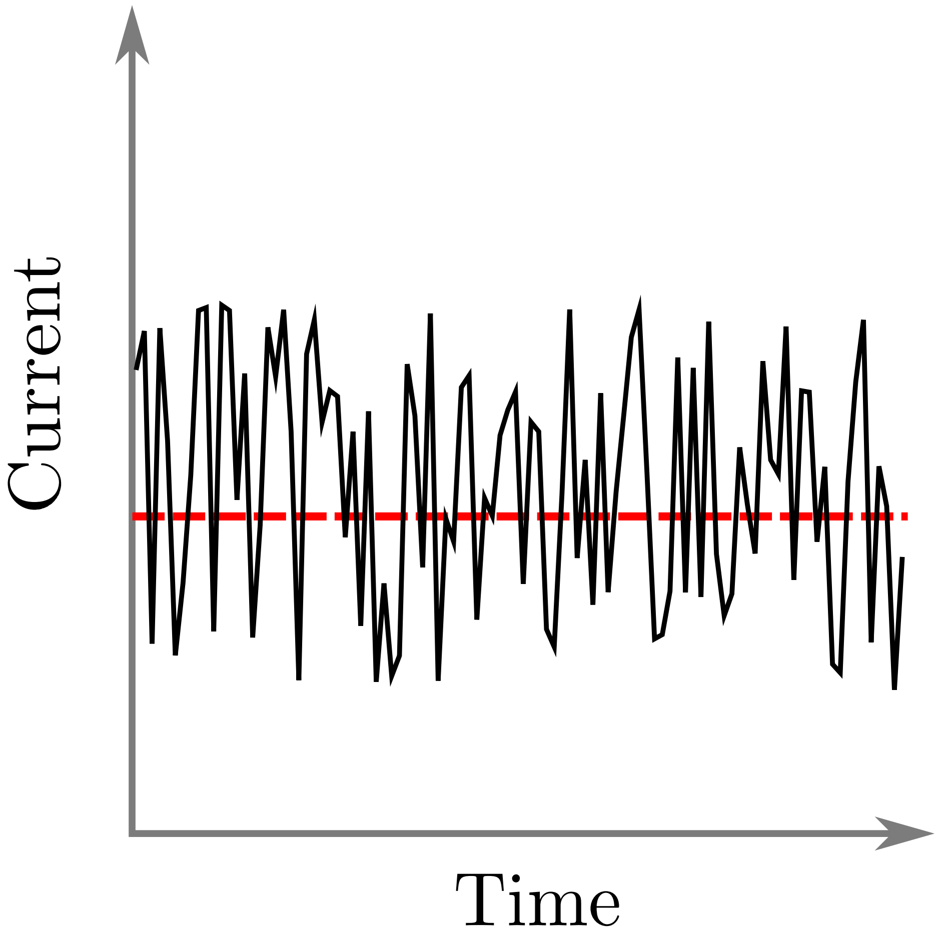
What's that noise!

DC current through a resistor *vs.* time



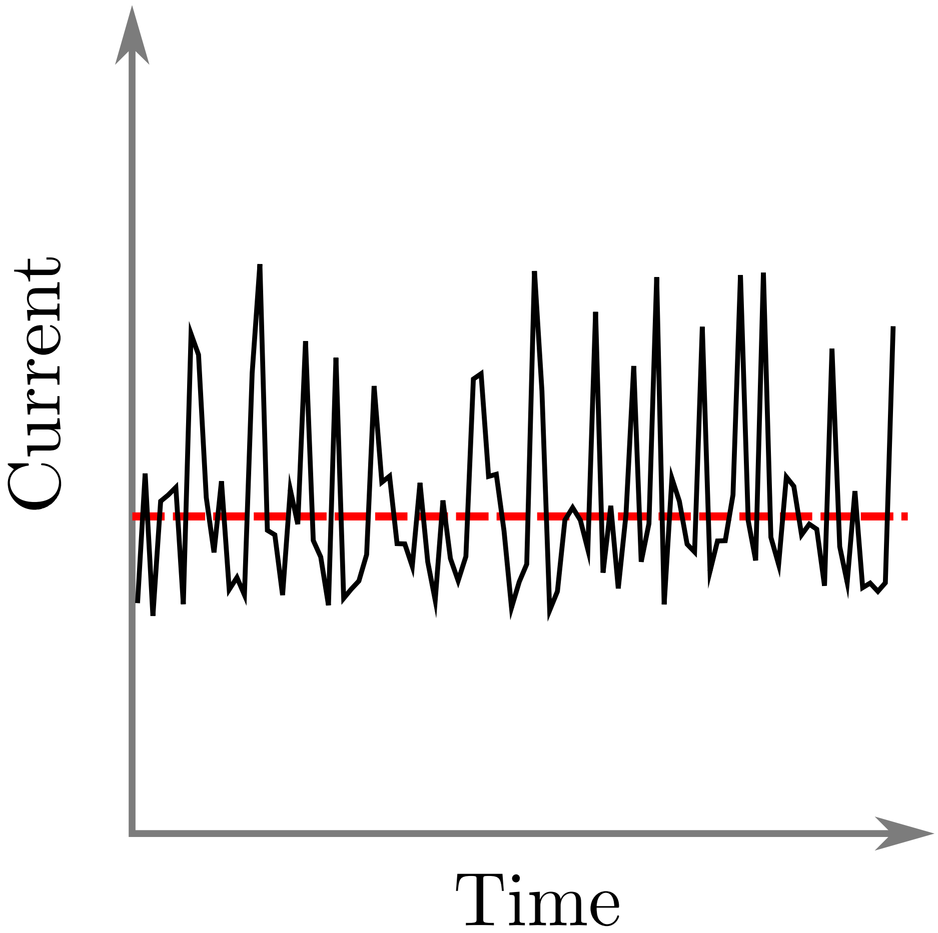
What's that noise!

DC current through a resistor *vs.* time



What's that noise!

DC current through a resistor *vs.* time



What's that noise!

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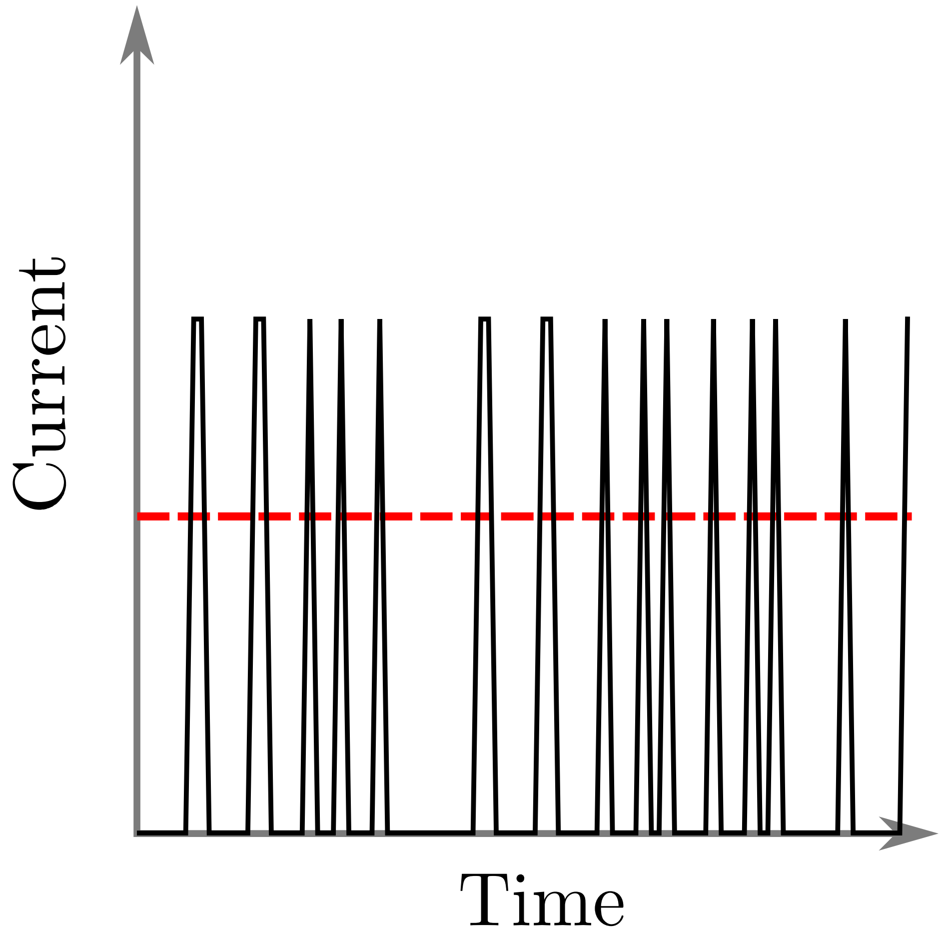
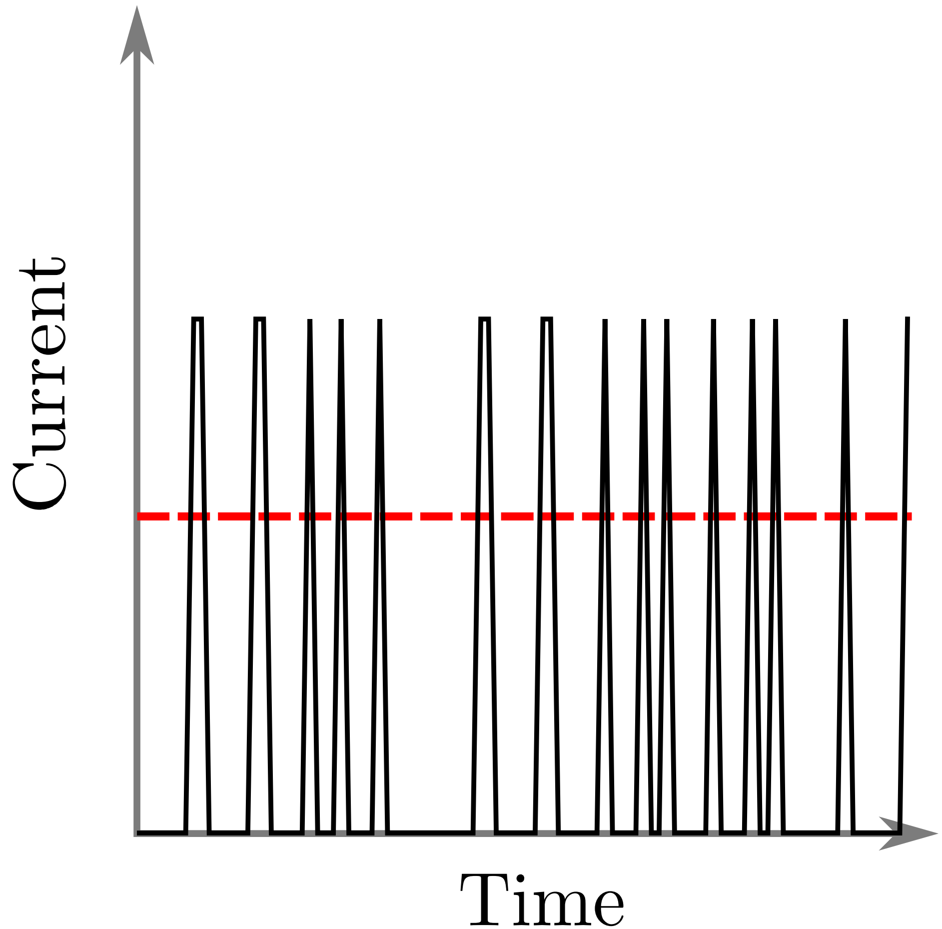


Shot Noise

DC current through a resistor *vs.* time



Discrete/Quantized nature of transport!

Quantifying Noise

Current fluctuation δI

$$\delta I = I - \langle I \rangle$$

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One-sided Noise Power Spectral Density $S_{\delta I}(\Omega)$

$$\langle (\delta I)^2 \rangle = \int_0^{\infty} S_{\delta I}(\Omega)\, d\Omega$$

Quantifying Noise

Current fluctuation δI

$$\delta I = I - \langle I \rangle$$

One-sided Noise Power Spectral Density $S_{\delta I}(\Omega)$

$$\langle (\delta I)^2 \rangle = \int_0^\infty S_{\delta I}(\Omega)\, d\Omega$$

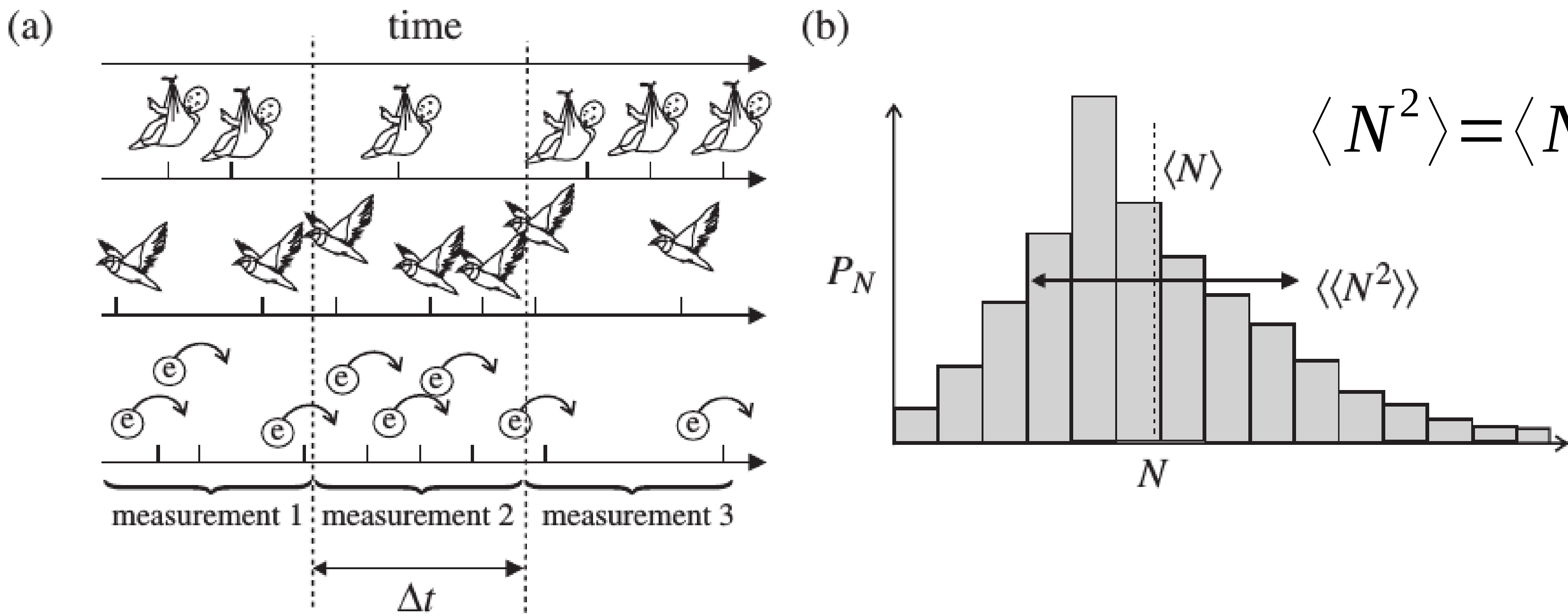
$$S_{\delta I}(\Omega) = \int_{-\infty}^{\infty} \{ \delta I(t)\, \delta I(0) \} e^{i\Omega t}\, dt$$

Poissonian Transport

Probability rate of a charge transfer event is constant.

Y. V. Nazarov and Y. M. Blanter, *Quantum Transport Introduction to Nanoscience* (Cambridge University Press, 2009).

Poissonian Transport



$$\langle N^2 \rangle = \langle N \rangle$$

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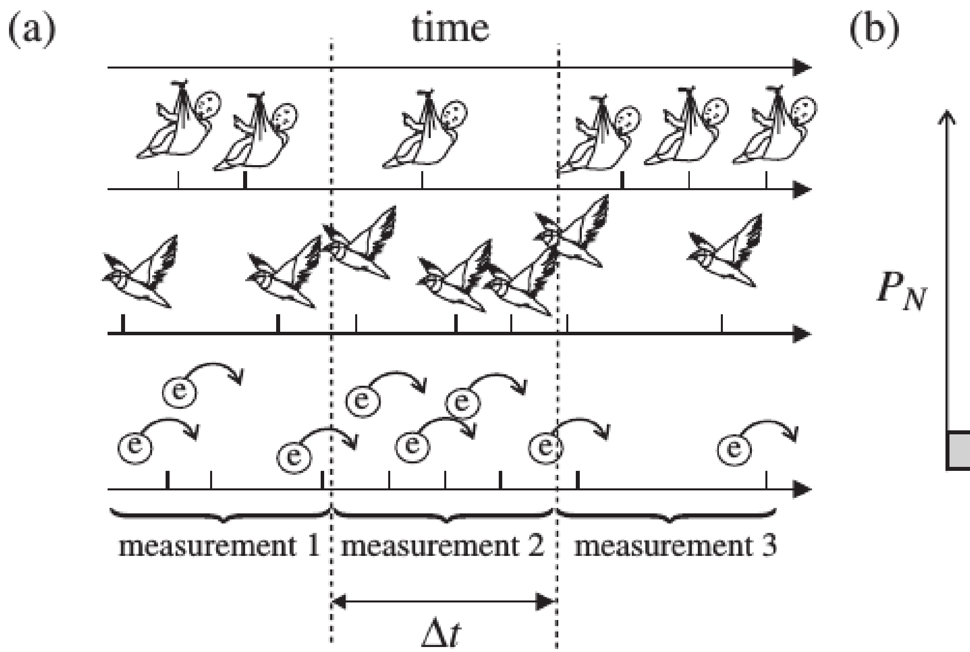


$$\langle N^2 \rangle = \langle N \rangle$$

Probability rate of a charge transfer event is constant.

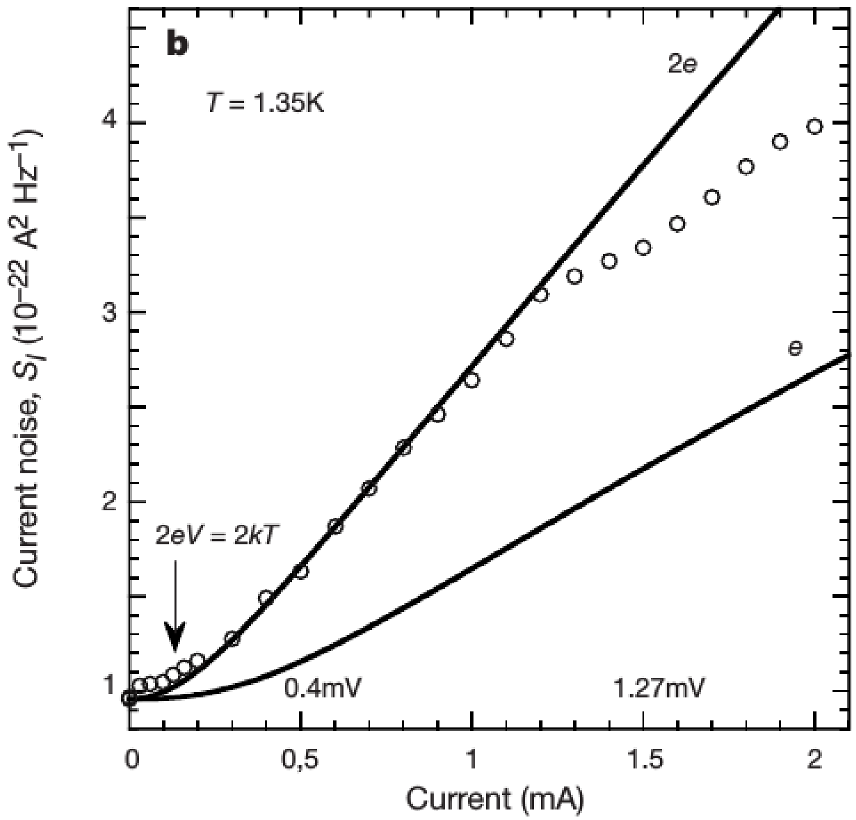
$$S(0) = 2q\langle I \rangle$$

Y. V. Nazarov and Y. M. Blanter, *Quantum Transport Introduction to Nanoscience* (Cambridge University Press, 2009).

Interacting Systems

Interacting Systems

Normal metal/superconductor junction



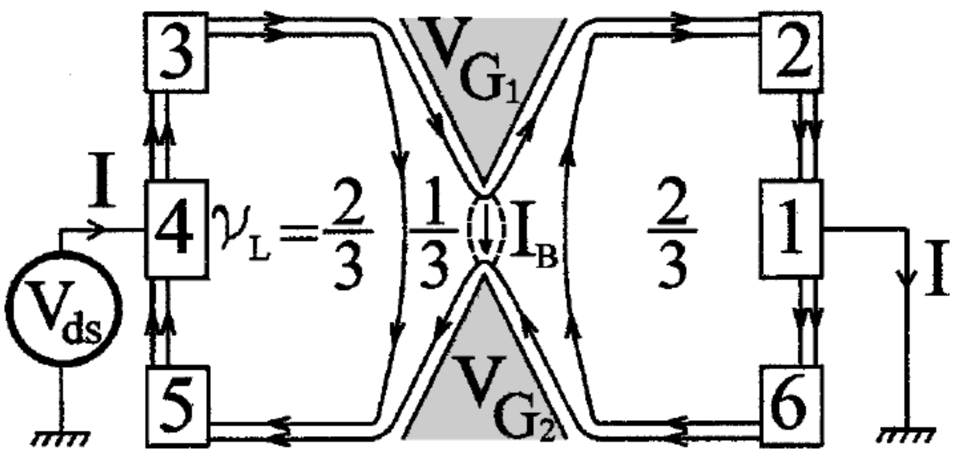
$2e$

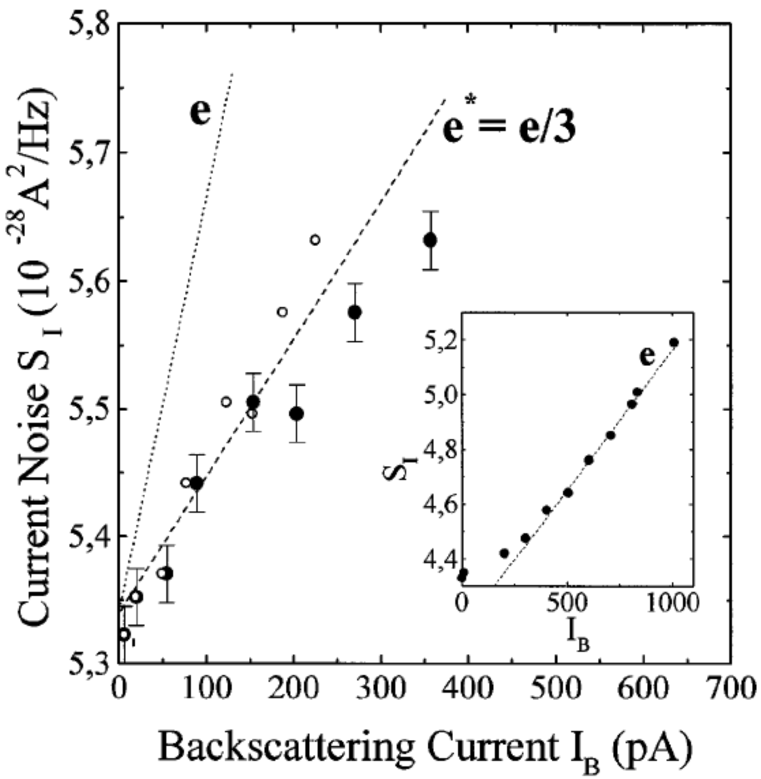
X. Jehl, M. Sanquer, R. Calemczuk, and D. Mailly, *Detection of doubled shot noise in short normal-metal/ superconductor junctions*, Nature 405, 50 (2000).

Interacting Systems

Fractional Quantum Hall Effect

$e/3$





L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, *Observation of the e/3 Fractionally Charged Laughlin Quasiparticle*, Phys. Rev. Lett. 79, 2526 (1997).

Shot Noise

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Physics Reports 336 (2000) 1–166

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Shot noise in mesoscopic conductors

Ya.M. Blanter*, M. Büttiker

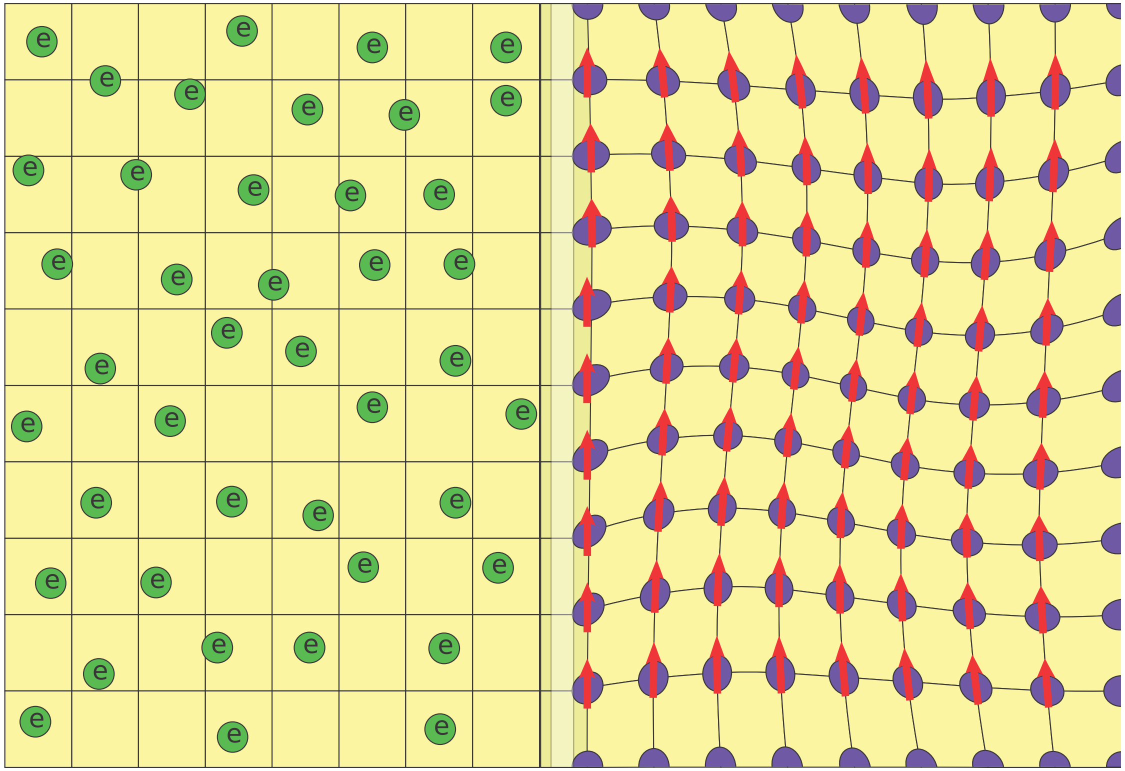
Département de Physique Théorique, Université de Genève, CH-1211, Genève 4, Switzerland

Received October 1999; editor: C.W.J. Beenakker

Spin Currents

Spin Currents

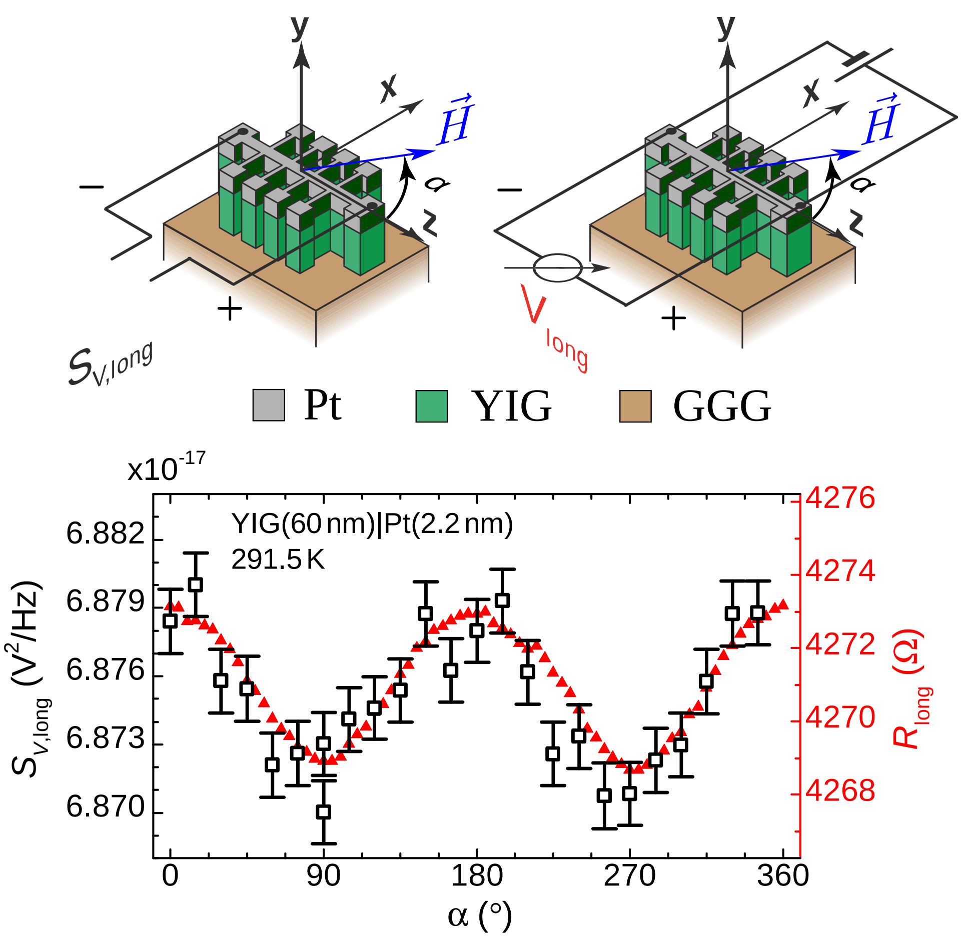
Non-magnetic metal (N)/Ferromagnetic insulator (FI)



V. V. Kruglyak, S. O. Demokritov, and D. Grundler, *Magnonics*, Journal of Physics D: Applied Physics 43, 264001 (2010).

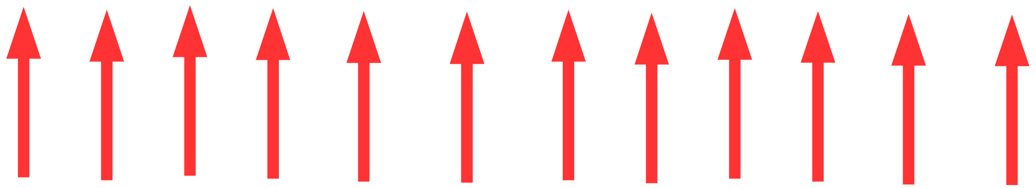
Thermal Spin Current Noise

Non-magnetic metal (N)/Ferromagnetic insulator (FI)



A. Kamra, F. P. Witek, S. Meyer, H. Huebl, S. Geprägs, R. Gross, G. E. W. Bauer, and S. T. B. Goennenwein, *Spin Hall Noise*, Phys. Rev. B 90, 214419 (2014).

Magnon



Considering only exchange interaction and Zeeman energy!

C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1953)

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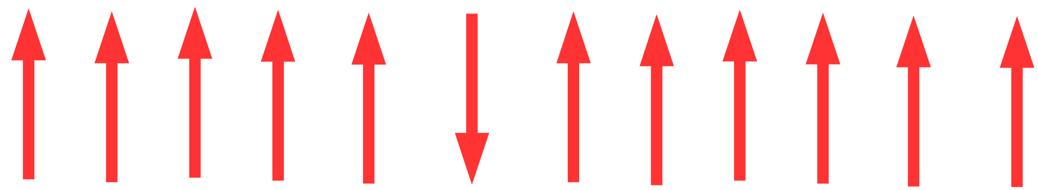

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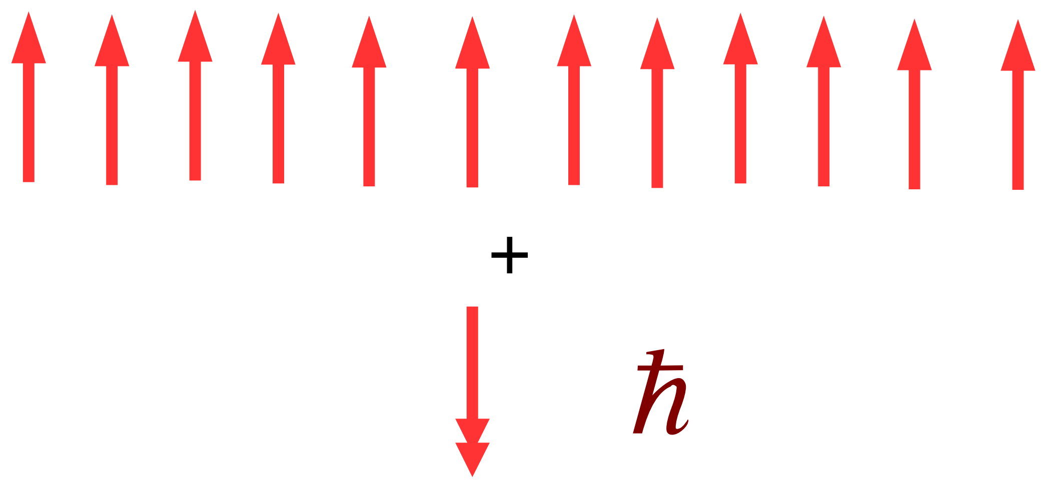

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Classical Hamiltonian

$$\mathcal{H}_{\mathrm{F}} = \int_{V_{\mathrm{F}}} d^3r \left(H_{\mathrm{Z}} + H_{\mathrm{aniso}} + H_{\mathrm{ex}} + H_{\mathrm{dip}} \right)$$

C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, London 1963).

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Linearization about equilibrium orientation:
Magnetization saturated along z direction

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$$H_{\mathrm{Z}} + H_{\mathrm{aniso}} = \frac{\omega_{\mathrm{za}}}{2|\gamma| M_s} \left(M_x^2 + M_y^2 \right)$$

$$H_{\mathrm{ex}} = \frac{A}{M_s^2} \left[\left(\boldsymbol{\nabla} M_x \right)^2 + \left(\boldsymbol{\nabla} M_y \right)^2 \right]$$

C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, London 1963).

Classical Hamiltonian

Mean-field description of dipolar interaction:
Demagnetization field

$$H_{\text{dip}} = -\frac{1}{2}\mu_0 \boldsymbol{H}_m \cdot \boldsymbol{M}$$

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Classical Hamiltonian

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$$\boldsymbol{H}_m = \boldsymbol{H}_u + \boldsymbol{H}_{nu} \qquad \boldsymbol{M} = \boldsymbol{M}_u + \boldsymbol{M}_{nu}$$

$$\boldsymbol{H}_u = -N_x M_{ux}\,\hat{\boldsymbol{x}} - N_y M_{uy}\,\hat{\boldsymbol{y}} - N_z M_{uz}\,\hat{\boldsymbol{z}}$$

C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, London 1963).

Quantization: HP transformations

With $\tilde{a}^\dagger(\boldsymbol{r}), \tilde{a}(\boldsymbol{r})$ the magnon ladder operators in real space

$$\tilde{M}_\pm = \tilde{M}_x \pm i(\gamma/|\gamma|)\tilde{M}_y$$

$$\tilde{M}_+ = \sqrt{2|\gamma|\hbar M_s}\left(1 - \frac{|\gamma|\hbar}{2M_s}\tilde{a}^\dagger\tilde{a}\right)^{\frac{1}{2}}\tilde{a}$$

$$\tilde{M}_- = \sqrt{2|\gamma|\hbar M_s}\tilde{a}^\dagger\left(1 - \frac{|\gamma|\hbar}{2M_s}\tilde{a}^\dagger\tilde{a}\right)^{\frac{1}{2}}$$

$$\tilde{M}_z = M_s - |\gamma|\hbar\tilde{a}^\dagger\tilde{a}.$$

T. Holstein and H. Primakoff, *Field Dependence of the Intrinsic Domain Magnetization of a Ferromagnet*, Phys. Rev. 58, 1098 (1940).

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Quantum Hamiltonian

With $\tilde{b}_{\boldsymbol{q}}$ the magnon annihilation operators in Fourier space

$$\tilde{\mathcal{H}}_{\mathrm{F}} = \sum_{\boldsymbol{q}} \left[A_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}}^{\dagger} \tilde{b}_{\boldsymbol{q}} + B_{\boldsymbol{q}}^{*} \tilde{b}_{\boldsymbol{q}}^{\dagger} \tilde{b}_{-\boldsymbol{q}}^{\dagger} + B_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}} \tilde{b}_{-\boldsymbol{q}} \right]$$

$$A_{\boldsymbol{q}} \;=\; A_{-\boldsymbol{q}} = \hbar \left(\omega_{\mathrm{za}} - \omega_s N_z + D q^2 + \frac{\omega_s}{2} (N_x + N_y) \delta_{\boldsymbol{q},\boldsymbol{0}} + \frac{\omega_s}{2} \sin^2 \theta_{\boldsymbol{q}} \right)$$

$$B_{\boldsymbol{q}} \;=\; B_{-\boldsymbol{q}} = \hbar \left(\frac{\omega_s}{4} N_{xy} \delta_{\boldsymbol{q},\boldsymbol{0}} + \frac{\omega_s}{4} \sin^2 \theta_{\boldsymbol{q}} \; e^{i2\phi_{\boldsymbol{q}}} \right)$$

$$D = 2A|\gamma|/M_s \qquad \omega_s = |\gamma| \mu_0 M_s \qquad N_{xy} = N_x - N_y$$

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$$B_{\boldsymbol{q}} = B_{-\boldsymbol{q}} = \hbar \left(\frac{\omega_s}{4} N_{xy} \delta_{\boldsymbol{q},\boldsymbol{0}} \qquad \right)$$

$$\omega_s = |\gamma| \mu_0 M_s \qquad N_{xy} = N_x - N_y$$

Ferromagnet Eigenmodes

With magnon annihilation operators $\tilde{b}_{\boldsymbol{q}}$

$$\tilde{\mathcal{H}}_{\mathrm{F}} = \sum_{\boldsymbol{q}} \left[A_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}}^{\dagger} \tilde{b}_{\boldsymbol{q}} + B_{\boldsymbol{q}}^{*} \tilde{b}_{\boldsymbol{q}}^{\dagger} \tilde{b}_{-\boldsymbol{q}}^{\dagger} + B_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}} \tilde{b}_{-\boldsymbol{q}} \right]$$

Effect of dipolar interactions!

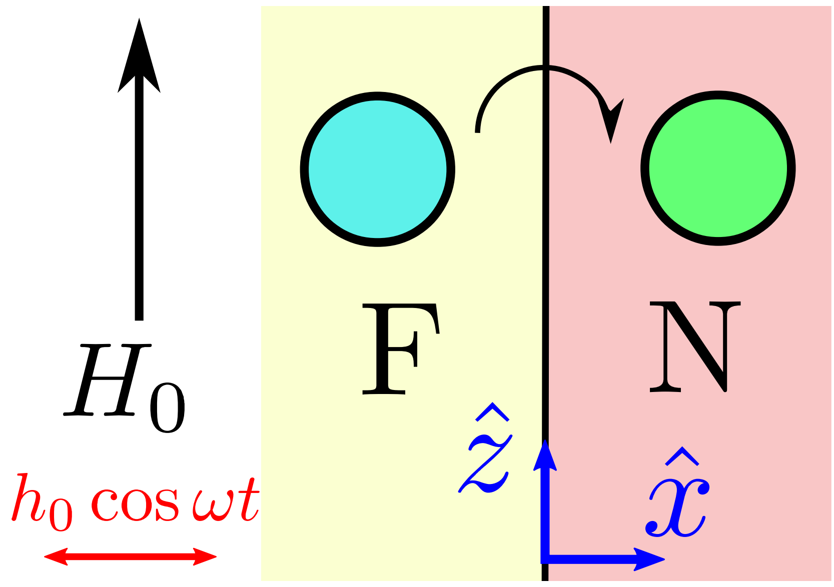
Squeezed-magnons

Bogoliubov transformation to new quasi-particles

$$\tilde{\beta}_{\boldsymbol{q}} = u_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}} - v_{\boldsymbol{q}}^{*} \tilde{b}_{-\boldsymbol{q}}^{\dagger}$$

$$\tilde{\mathcal{H}}_{\mathrm{F}} = \sum_{\boldsymbol{q}} \hbar \omega_{\boldsymbol{q}} \tilde{\beta}_{\boldsymbol{q}}^{\dagger} \tilde{\beta}_{\boldsymbol{q}}$$

Spin Current Injection



$$\tilde{\mathcal{H}} = \tilde{\mathcal{H}}_{\mathrm{F}} + \tilde{\mathcal{H}}_{\mathrm{N}} + \tilde{\mathcal{H}}_{\mathrm{int}} + \tilde{\mathcal{H}}_{\mathrm{drive}}$$

Hamiltonian

$$\tilde{\mathcal{H}}_{\mathrm{N}} = \sum_{\boldsymbol{k},s=\pm} \hbar\omega_{\boldsymbol{k}} \tilde{c}^{\dagger}_{\boldsymbol{k},s} \tilde{c}_{\boldsymbol{k},s}$$

$$\tilde{\mathcal{H}}_{\mathrm{int}} = \sum_{\boldsymbol{k}_1 \boldsymbol{k}_2 \boldsymbol{q}} \hbar W_{\boldsymbol{k}_1 \boldsymbol{k}_2 \boldsymbol{q}} \; \tilde{c}^{\dagger}_{\boldsymbol{k}_1 +} \tilde{c}_{\boldsymbol{k}_2 -} \tilde{b}_{\boldsymbol{q}} \; + \; \mathrm{h.c.}$$

$$\tilde{\mathcal{H}}_{\mathrm{drive}} = -\mu_0 h_0 \cos(\omega t) B \left(\tilde{\beta}_{\mathbf{0}} + \tilde{\beta}^{\dagger}_{\mathbf{0}} \right)$$

$$\tilde{I}_z = \frac{1}{i\hbar} [\tilde{\mathcal{S}}_z, \tilde{\mathcal{H}}_{\mathrm{int}}] = \sum_{\boldsymbol{k}_1 \boldsymbol{k}_2 \boldsymbol{q}} -i\hbar W_{\boldsymbol{k}_1 \boldsymbol{k}_2 \boldsymbol{q}} \; \tilde{c}^{\dagger}_{\boldsymbol{k}_1 +} \tilde{c}_{\boldsymbol{k}_2 -} \tilde{b}_{\boldsymbol{q}} + \mathrm{h.c.}$$

Ferromagnetic Resonance

Under coherent excitation $\beta(t) = \langle \tilde{\beta}_{\mathbf{0}}(t) \rangle$

$$\beta(t) = \frac{\mu_0 h_0 B}{2\hbar} \frac{1}{(\omega_{\mathbf{0}} - \omega) - i\Gamma(u_{\mathbf{0}}^2 + v_{\mathbf{0}}^2)} e^{-i\omega t}$$

$$\langle M_x \rangle \propto \Re(\beta) \qquad \langle M_y \rangle \propto \Im(\beta)$$

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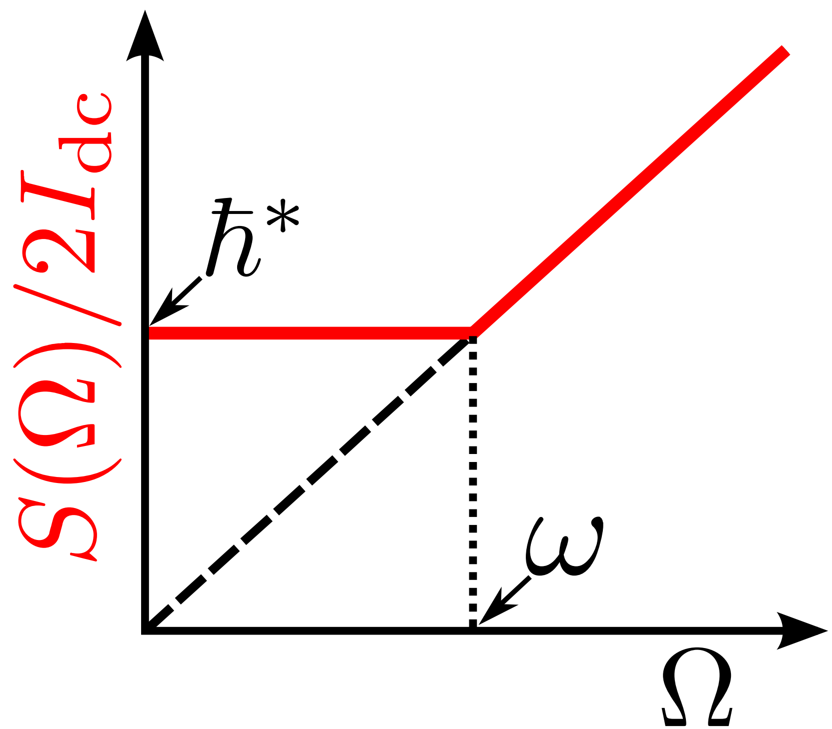
$$\langle M_x \rangle \propto \Re(\beta) \qquad \langle M_y \rangle \propto \Im(\beta)$$

$$I_z(t) = \langle \tilde{I}_z(t) \rangle = I_{\mathrm{dc}} = 2\hbar\alpha'\omega|\beta|^2$$

Resonant excitation of the uniform mode and spin current injection!

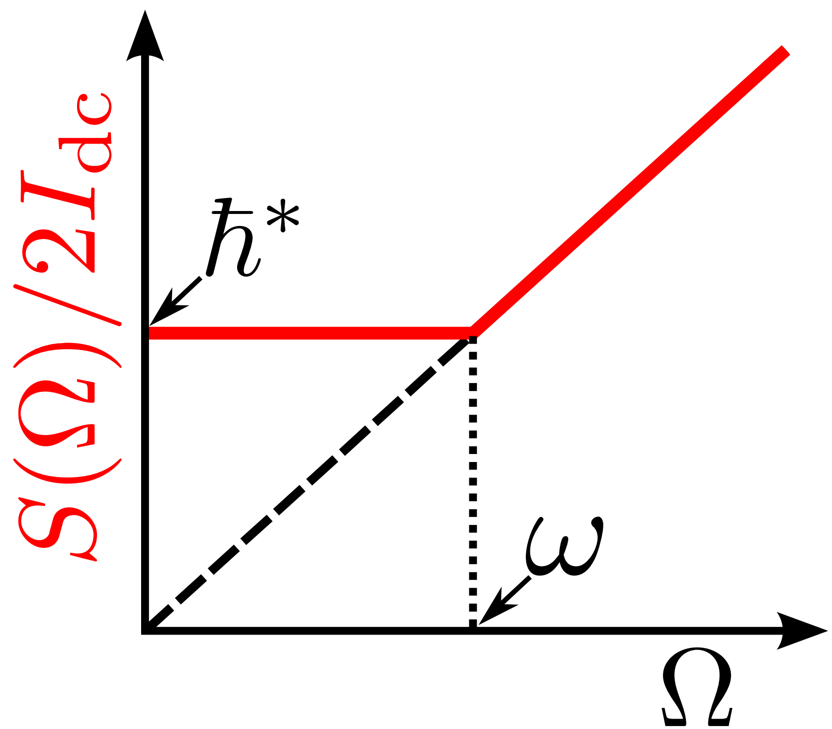
Spin Current Shot Noise

Spin Current Shot Noise



$$S(\Omega) = \hbar^* \frac{I_{\mathrm{dc}}}{\omega} \left(|\omega + \Omega| + |\omega - \Omega| \right)$$

Spin Current Shot Noise



$$S(0) = 2\hbar^* I_{\mathrm{dc}}$$

Squeezed-magnon

$$\hbar^* = \hbar(1+\delta)$$

$$\int_{V_{\mathrm{F}}} \langle \tilde{S}^z_{\mathrm{F}}(\boldsymbol{r}) \rangle d^3 r = -\frac{\mathcal{M}_0}{|\gamma|} + \sum_{\boldsymbol{q}} \hbar (1 + 2|v_{\boldsymbol{q}}|^2) n^{\beta}_{\boldsymbol{q}} + \sum_{\boldsymbol{q}} \hbar |v_{\boldsymbol{q}}|^2$$

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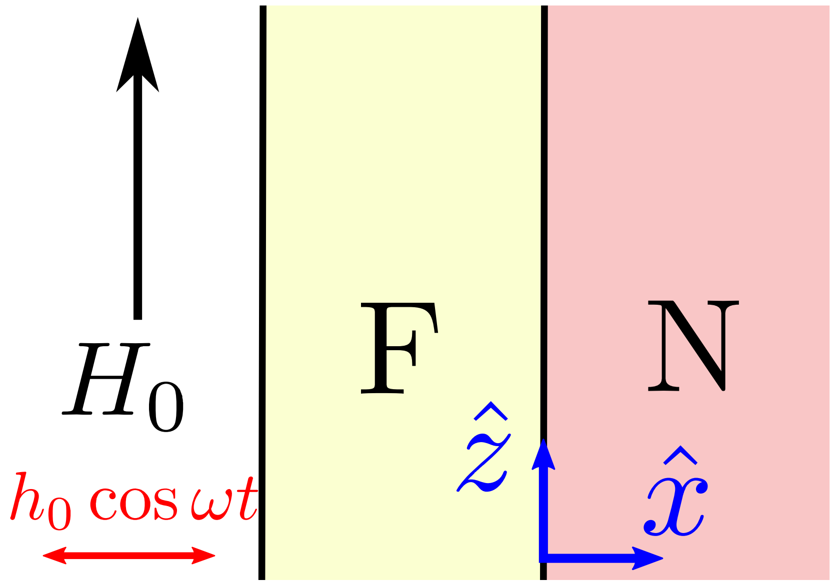

Specimen in the shape of a film

Squeezed-magnon

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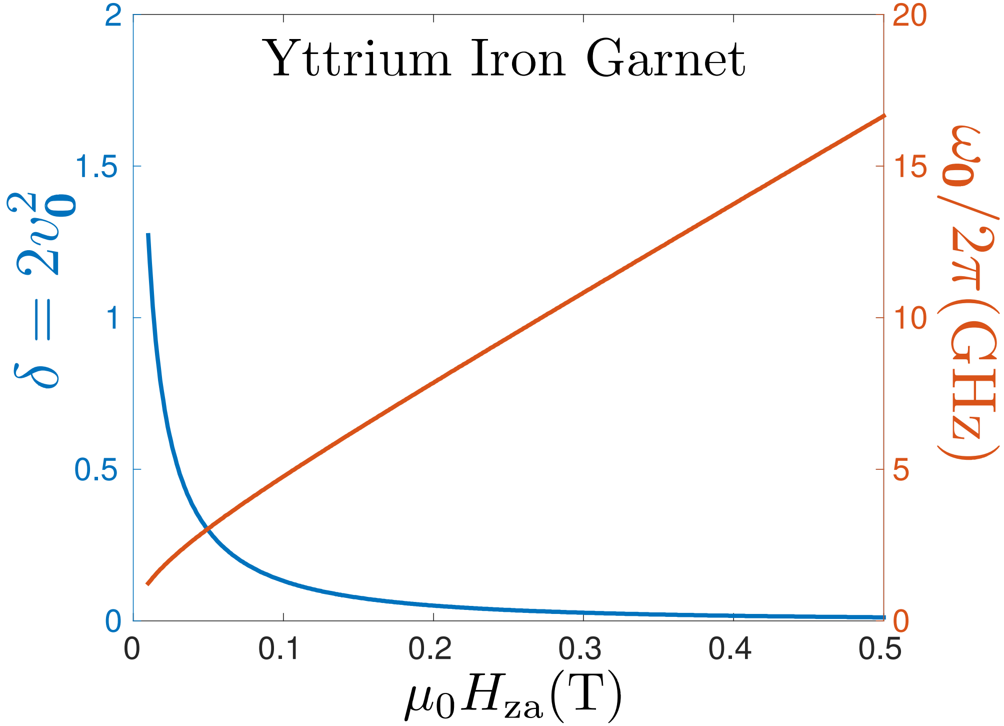

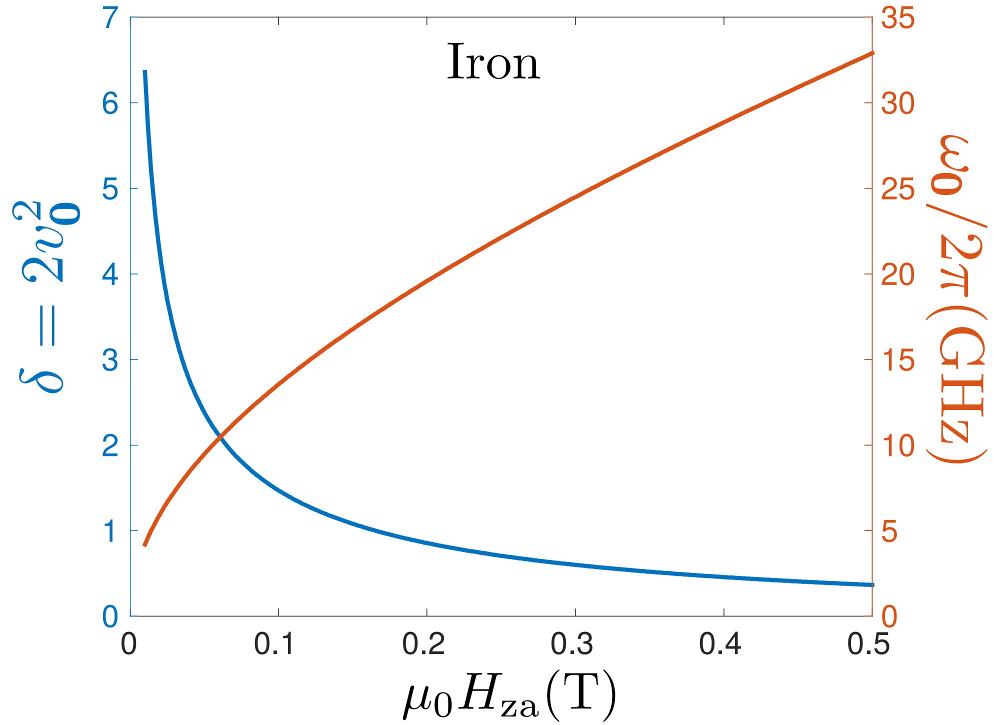

Specimen in the shape of a film

A Primer on Squeezing

C. Gerry and P. Knight, *Introductory Quantum Optics* (Cambridge University Press, 2005).

Uncertainty Relations

$$[\hat{A}, \hat{B}] = i\hat{C}$$

$$\left\langle (\Delta \hat{A})^2 \right\rangle \left\langle (\Delta \hat{B})^2 \right\rangle \geq \frac{1}{4} |\langle \hat{C} \rangle|^2$$

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$$\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger) \qquad \hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$$

$$[\hat{X}_1, \hat{X}_2] = i/2$$

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M_x

$$[\hat{X}_1, \hat{X}_2] = i/2$$

M_y

Squeezed Vacuum

Define (single mode) Squeeze operator:

$$\hat{S}(\xi) = \exp\left[\frac{1}{2}(\xi^* a^2 - \xi a^{\dagger 2})\right]$$

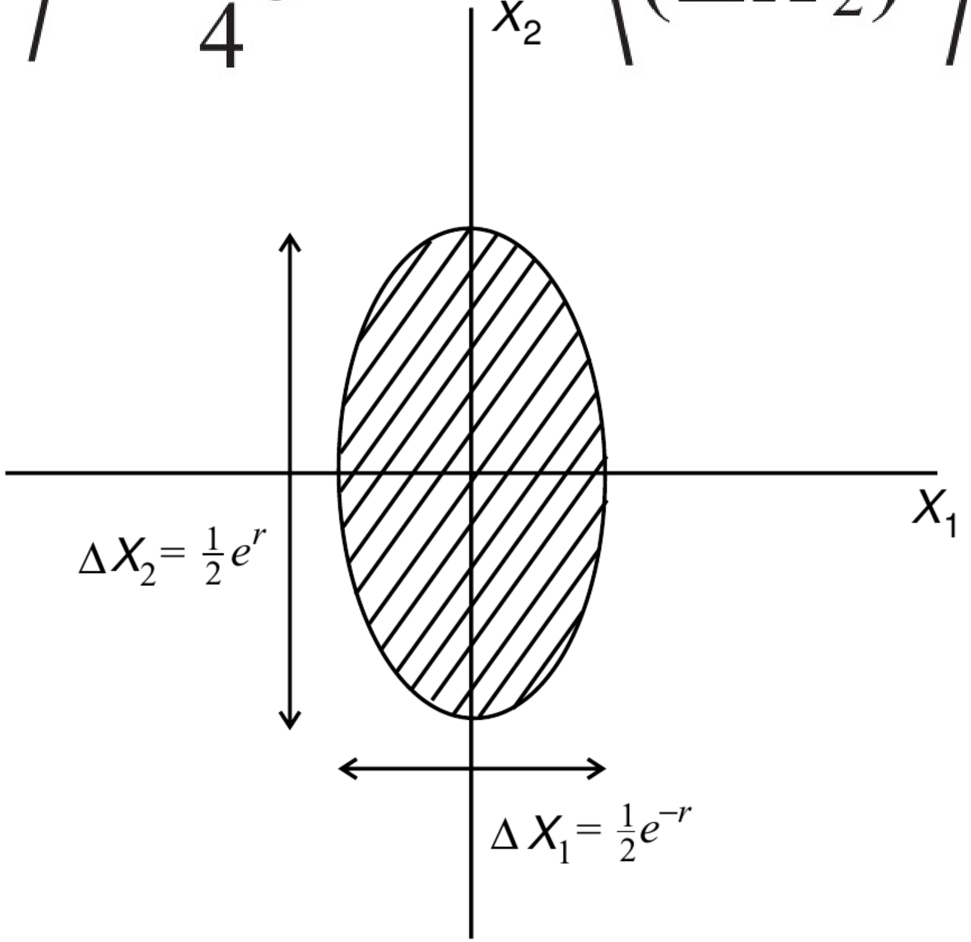
$$\xi = r\ e^{i\ \theta}$$

Squeezed vacuum

$$|\xi\rangle = \hat{S}(\xi)\,|0\rangle$$

Squeezed Vacuum

$$\left\langle (\Delta \hat{X}_1)^2 \right\rangle = \frac{1}{4} e^{-2r} \qquad \left\langle (\Delta \hat{X}_2)^2 \right\rangle = \frac{1}{4} e^{2r}$$



Squeezed Number States

Squeezed zero number state

$$|\xi\rangle = \hat{S}(\xi)\,|0\rangle$$

Alternate (equivalent) approach to squeezing:

$$\hat{a}\,|0\rangle = 0$$

$$\left(\hat{a}\mu + \hat{a}^{\dagger}\nu\right)|\xi\rangle = 0$$

$$\mu = \cosh r \qquad \nu = e^{i\theta}\sinh r$$

Squeezed-magnon

What's in the name – relation to squeezed light

C. Gerry and P. Knight, *Introductory Quantum Optics* (Cambridge University Press, 2005).

Squeezed-magnon

What's in the name – relation to squeezed light

$$\left\langle (\Delta \hat{X}_1)^2 \right\rangle = \frac{1}{4} e^{-2r} \qquad \left\langle (\Delta \hat{X}_2)^2 \right\rangle = \frac{1}{4} e^{2r}$$

$$\left\langle \left(\delta \tilde{\mathcal{M}}_{x,y} \right)^2 \right\rangle_0 = \frac{|\gamma| \hbar \mathcal{M}_0}{2} \exp\left(\mp 2 \xi_{\mathbf{0}} \right)$$

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Two mode squeezing

$$\left(u_{\boldsymbol{q}} \tilde{b}_{\boldsymbol{q}} - v_{\boldsymbol{q}}^{*} \tilde{b}_{-\boldsymbol{q}}^{\dagger} \right) \, |0\rangle_{\beta} = 0$$

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Finite Temperature Noise

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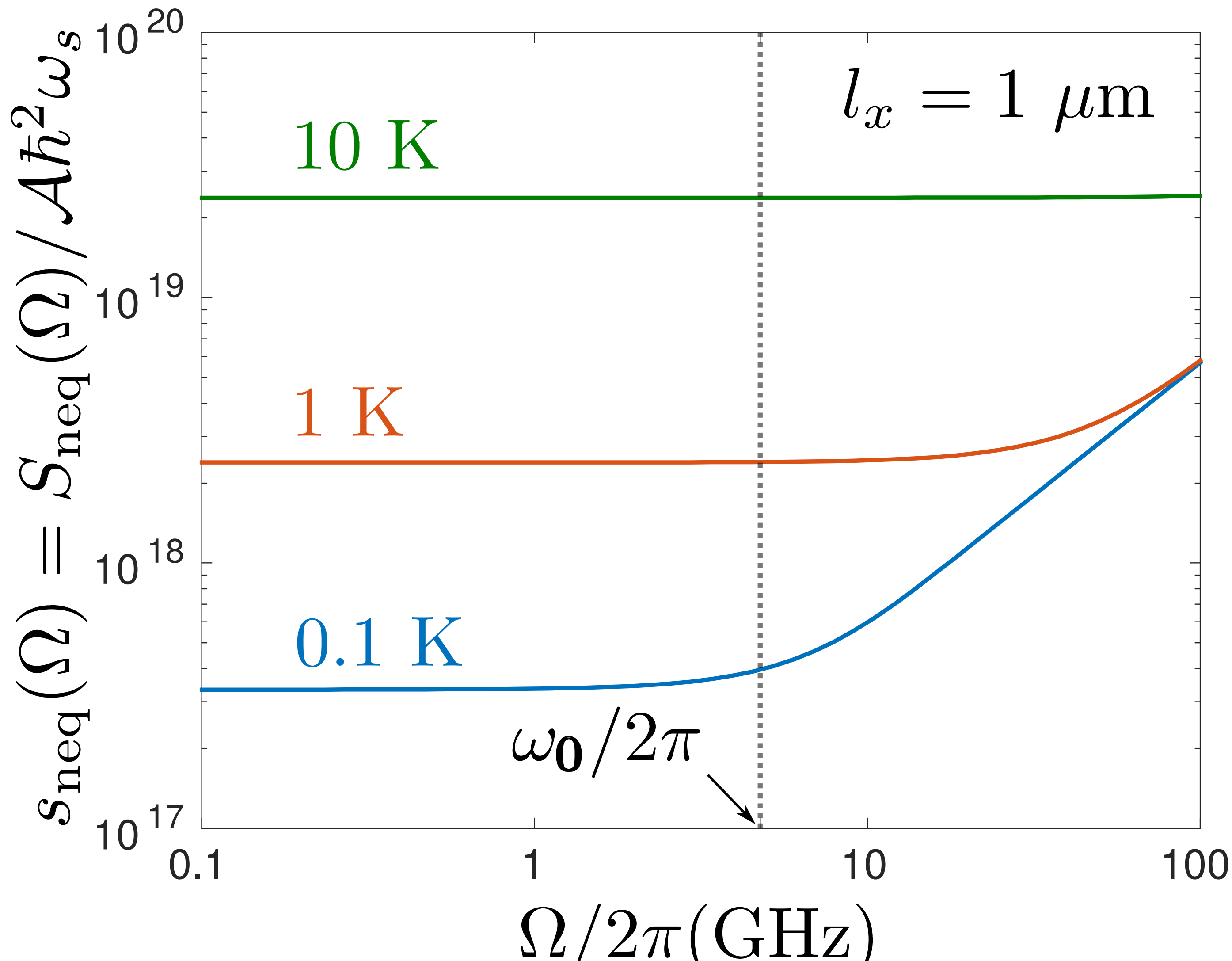
Finite Temperature Noise



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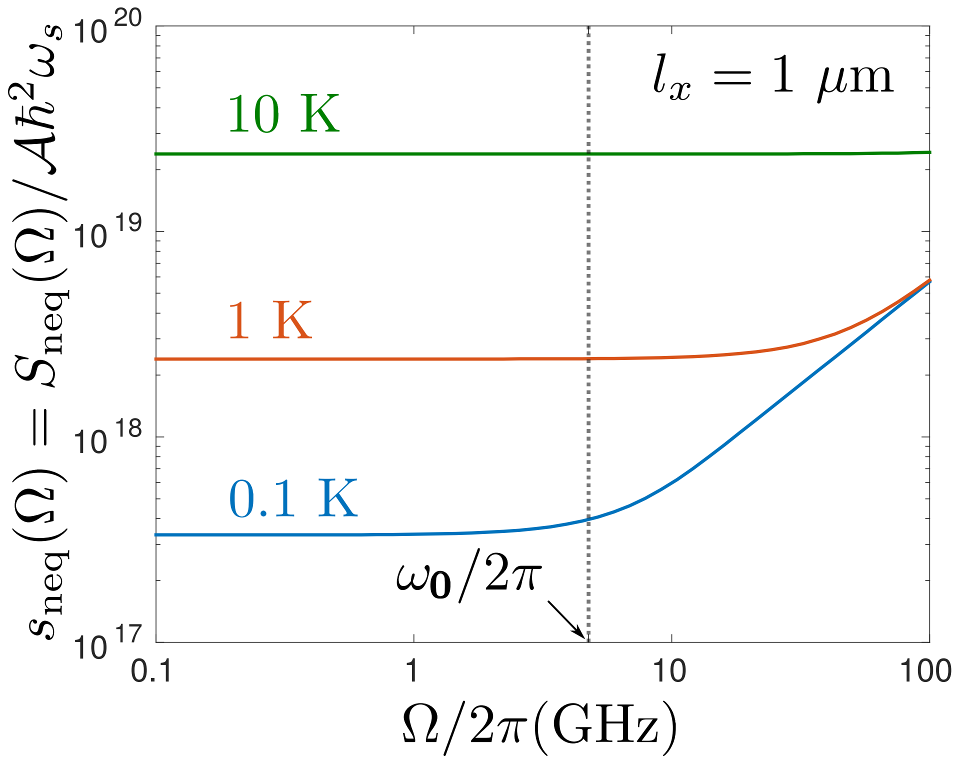
Finite Temperature Noise

$$S_{\text{neq}}(\Omega) = 2\hbar^* I_{\text{dc}} \frac{2k_B T}{\hbar\omega}, \quad k_B T \gg (\hbar\omega, \hbar\Omega)$$



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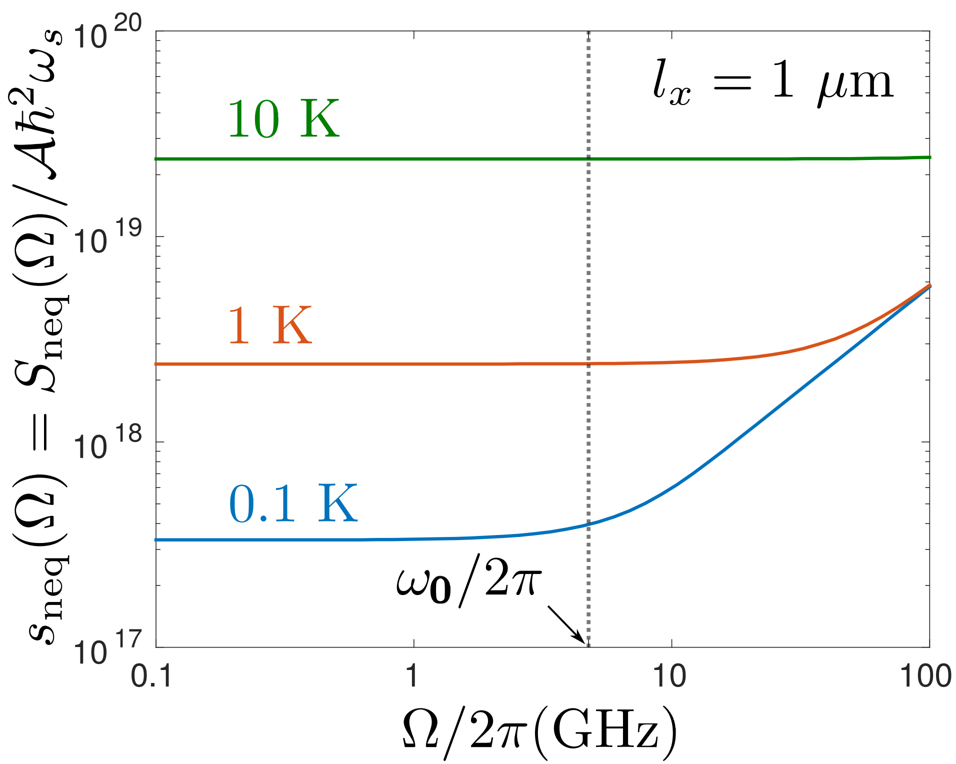
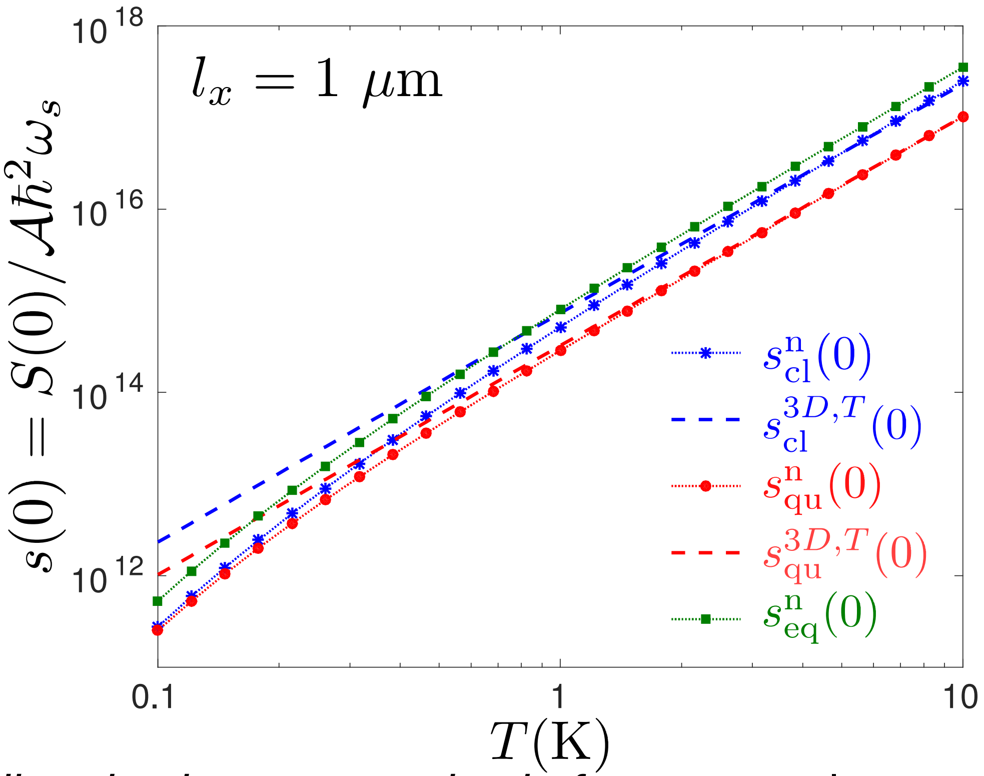
Finite Temperature Noise

$$S_{\mathrm{neq}}(\Omega) = 2\hbar^* I_{\mathrm{dc}} \frac{2k_B T}{\hbar\omega}, \quad k_B T \gg (\hbar\omega, \hbar\Omega)$$





A. Kamra and W. Belzig, *Magnon-mediated spin current noise in ferromagnet|nonmagnetic conductor hybrids*, Phys. Rev. B 94, 014419 (2016).

Summary

- Dipolar interaction mediated squeezing of magnons
- Non-integer spin quasiparticles
- Dominance of shot over thermal noise
- Quantum effects of squeezing